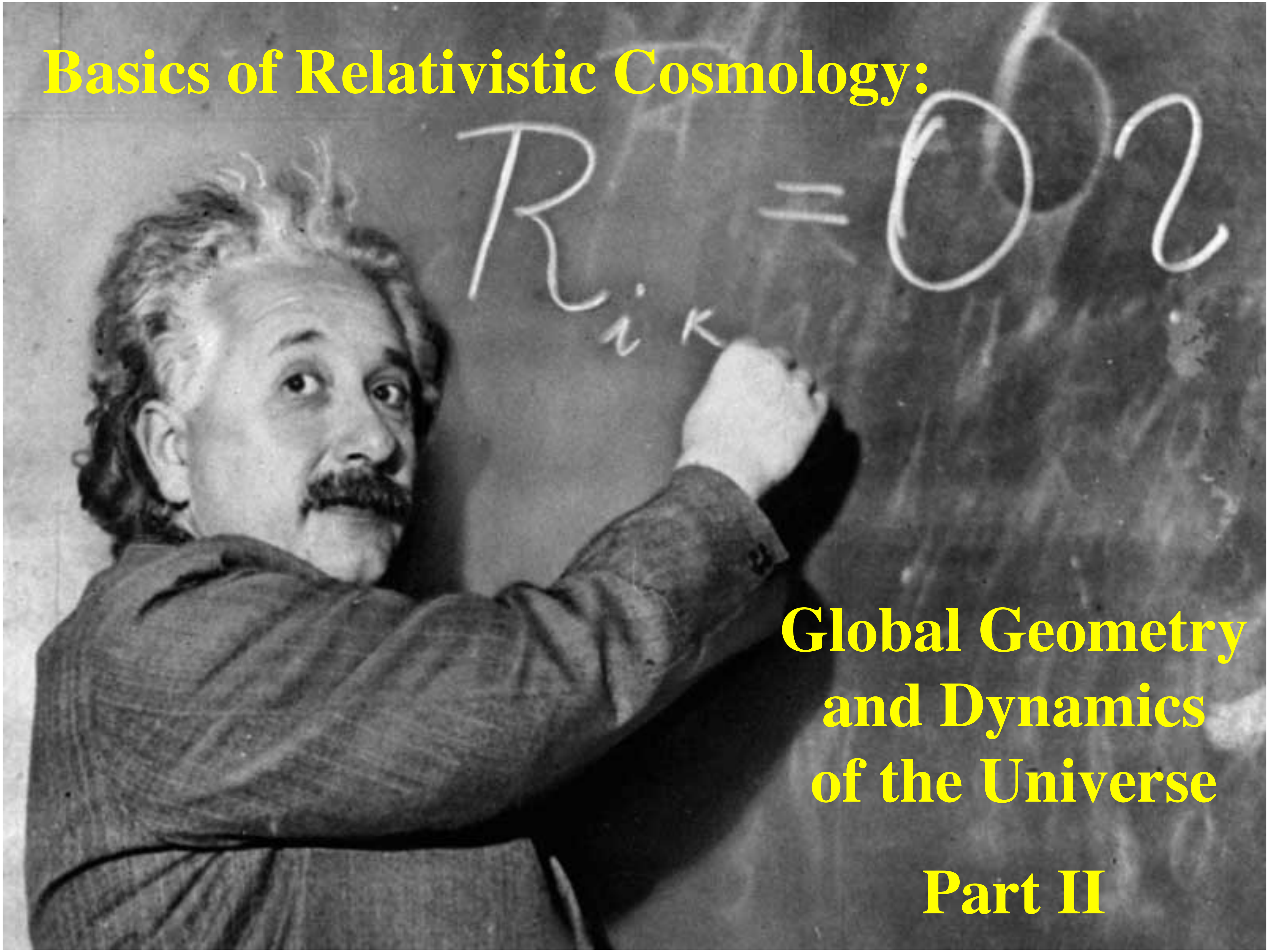
Basics of Relativistic Cosmology:



Global Geometry and Dynamics of the Universe

Part II

The Cosmological Principle

Relativistic cosmology uses some symmetry assumptions or principles in order to make the problem of "solving the universe" viable. The **Cosmological Principle** states that

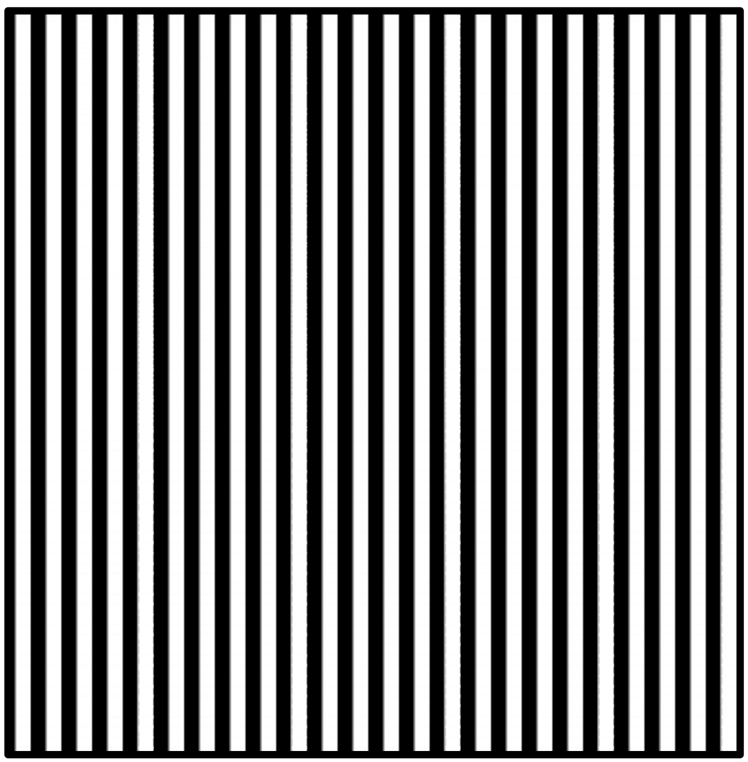
At each epoch, the universe is the same at all locations and in all directions, except for local irregularities

Therefore, globally the Universe is assumed to be **homogeneous** and **isotropic** at any given time; and its dynamics should be the same everywhere

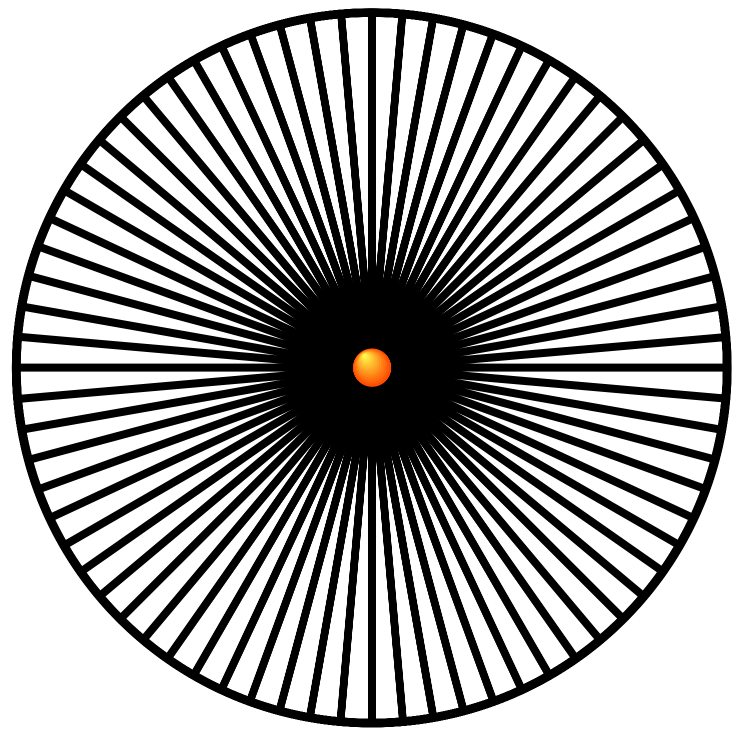
Note: the **Perfect Cosmological Principle** states that the Universe appears the same at all times and it is unchanging - it is also homogeneous in time - this is the basis of the "Steady State" model

Homogeneity and Isotropy

Homogeneous but not isotropic

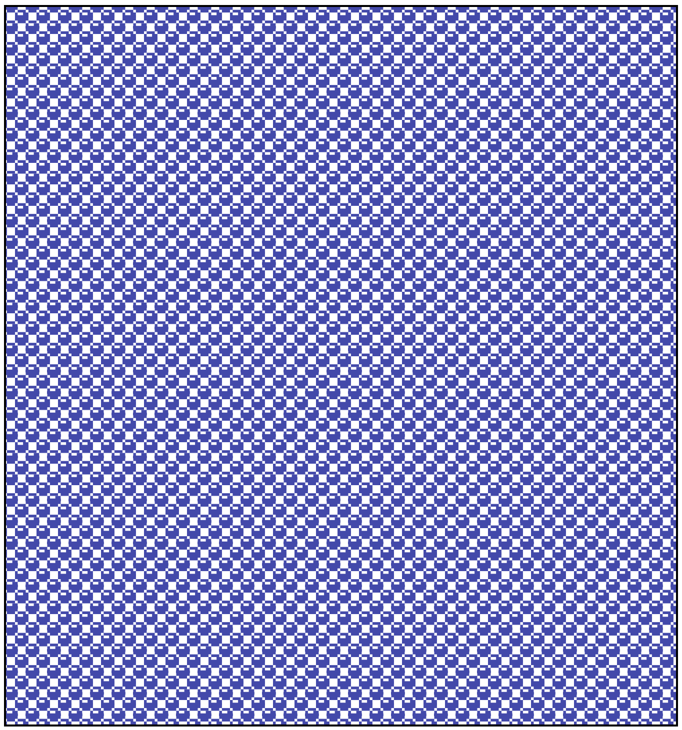


Isotropic at but not homogeneous

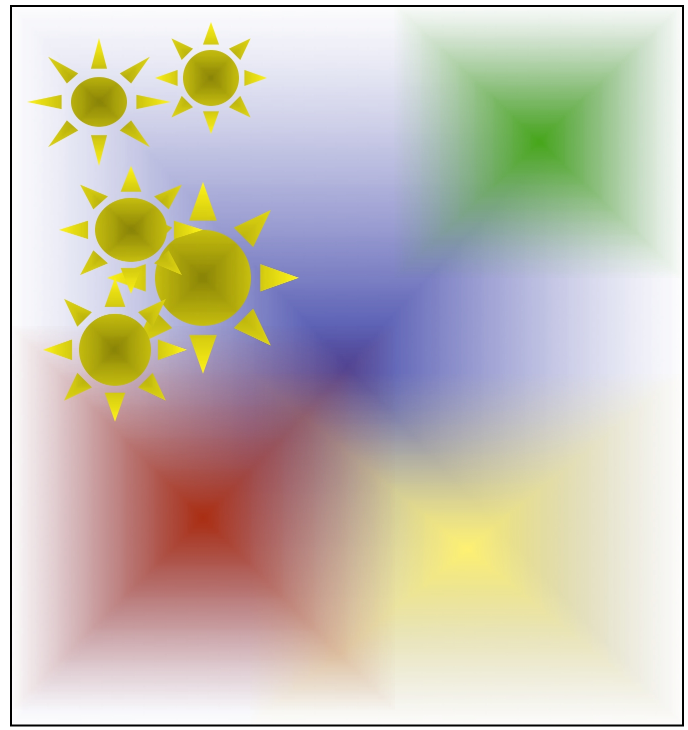


Homogeneity and Isotropy

Homogeneous *and* Isotropic



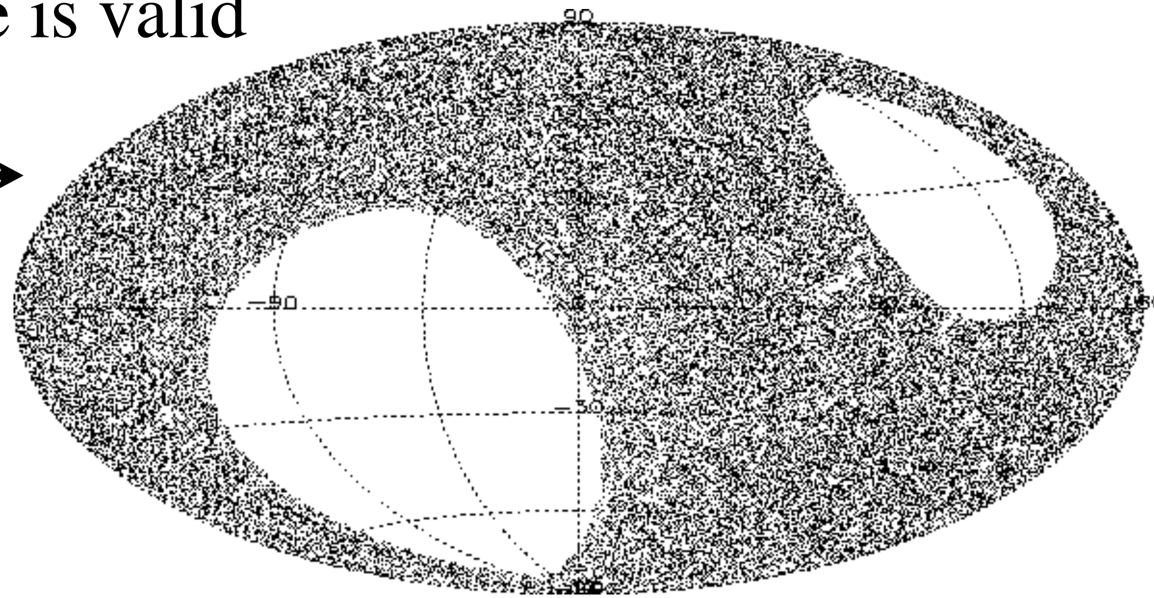
Neither

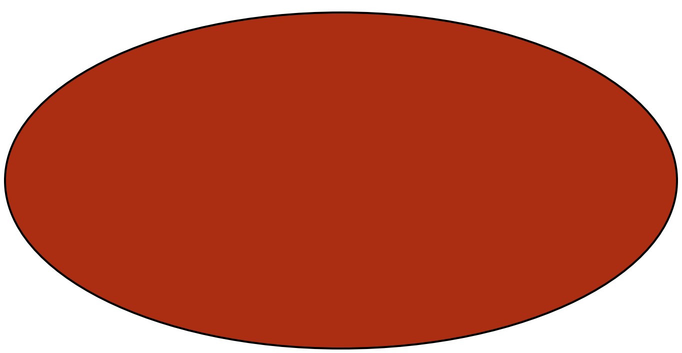


So, is the Universe Really Homogeneous and Isotropic?

Globally, on scales larger than ~ 100 Mpc, say, it is - so the cosmological principle is valid

Distribution on the sky ➞ of 65000 distant radio sources from the Texas survey, a cosmological population

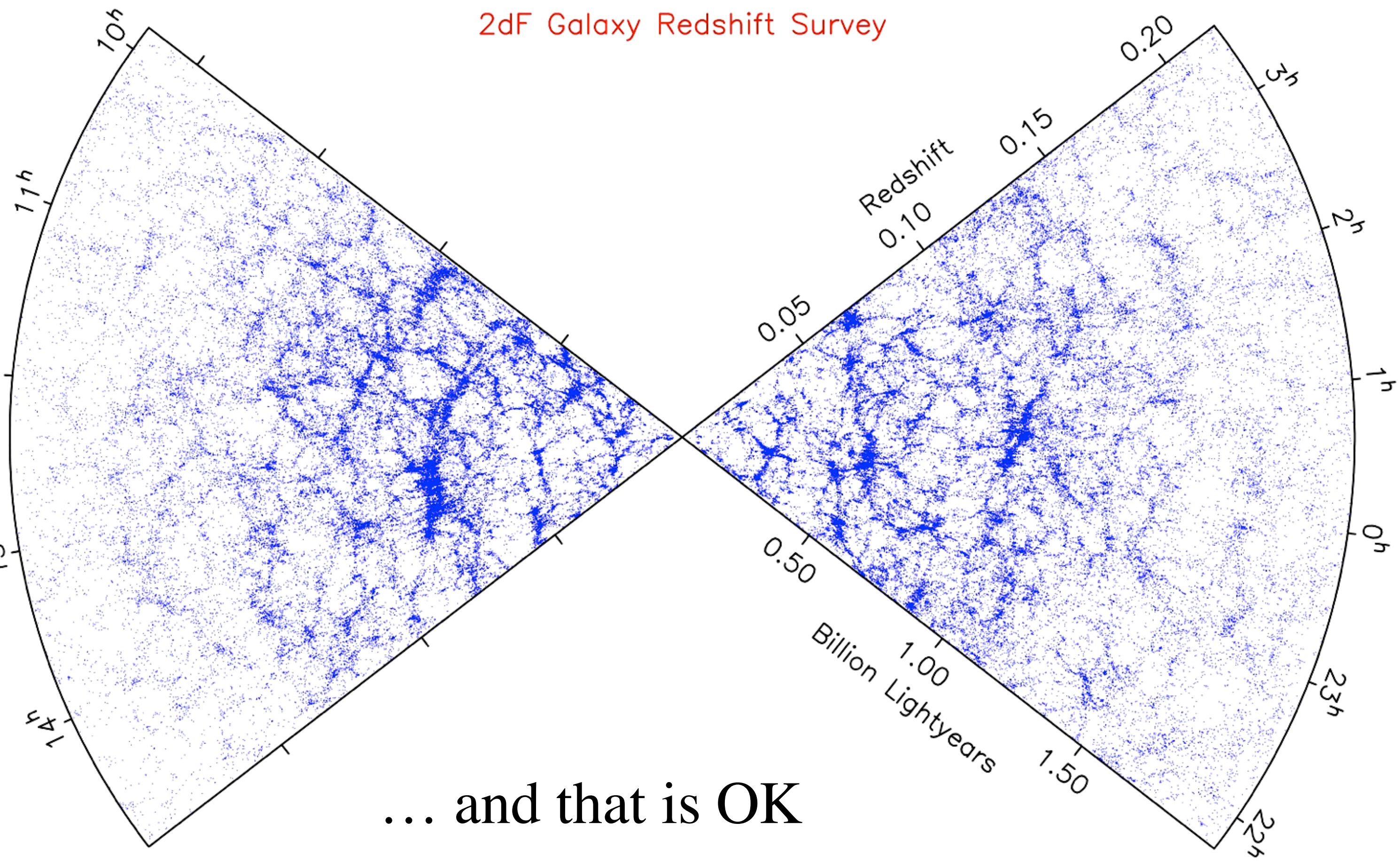




… and of course the CMBR, uniform to better than $\Delta T/T < 10^{-5}$, after taking the dipole out

So, is the Universe Really Homogeneous and Isotropic?

But not so on scales up to ~ 100 Mpc, as shown by the large-scale structure



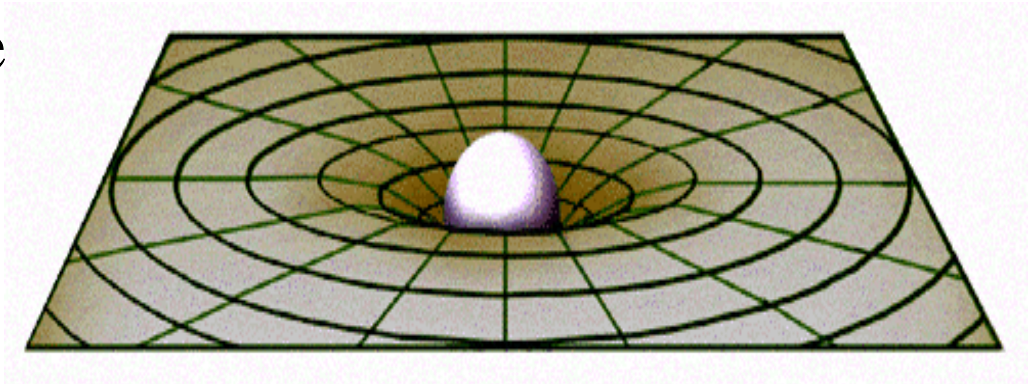
… and that is OK

General Relativity

Remember the key notion:

Presence of mass/energy determines the geometry of space
Geometry of space determines the motion of mass/energy

Thus, the distribution of the matter and energy in space must be consistent with its spatial geometry



Mathematical expression of that statement is given by the **Einstein equation(s)**

Their derivation is well beyond the scope of this class, but here is just a little flavor…

The Einstein Equations

Poisson equation connects gravitational potential ϕ with the matter density ρ:

$$\nabla^2 \phi = 4\pi G \rho$$

Gravitational potential specified by the metric:

$$\nabla^2 g_{tt} = -8\pi G T_{tt}$$

The matter/energy density is specified is by the stress-energy tensor

$$T^{\mu\nu} = \sum_i m u^\mu u^\nu \delta(x - x_i)$$

A tensor equation - a shorthand for 16 partial differential equations, connecting the geometry and mass/energy density is :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

The Einstein Equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

Spacetime geometry

Matter distribution

where:

$G_{\mu\nu}$ = The Einstein tensor

$R_{\mu\nu}$ = The Ricci tensor

$g_{\mu\nu}$ = The metric tensor

R = The Ricci scalar

$T_{\mu\nu}$ = The stress-energy tensor

Homogeneity and isotropy requirements reduce this set of 16 eqs. to only 1, $G_{00} = T_{00}$, which becomes the **Friedmann Equation**

Introducing the Cosmological Constant

Gravitation is an attractive force, so what is to prevent all matter and energy falling to one gigantic lump?

Einstein introduced a negative potential term to balance the attractive gravity:

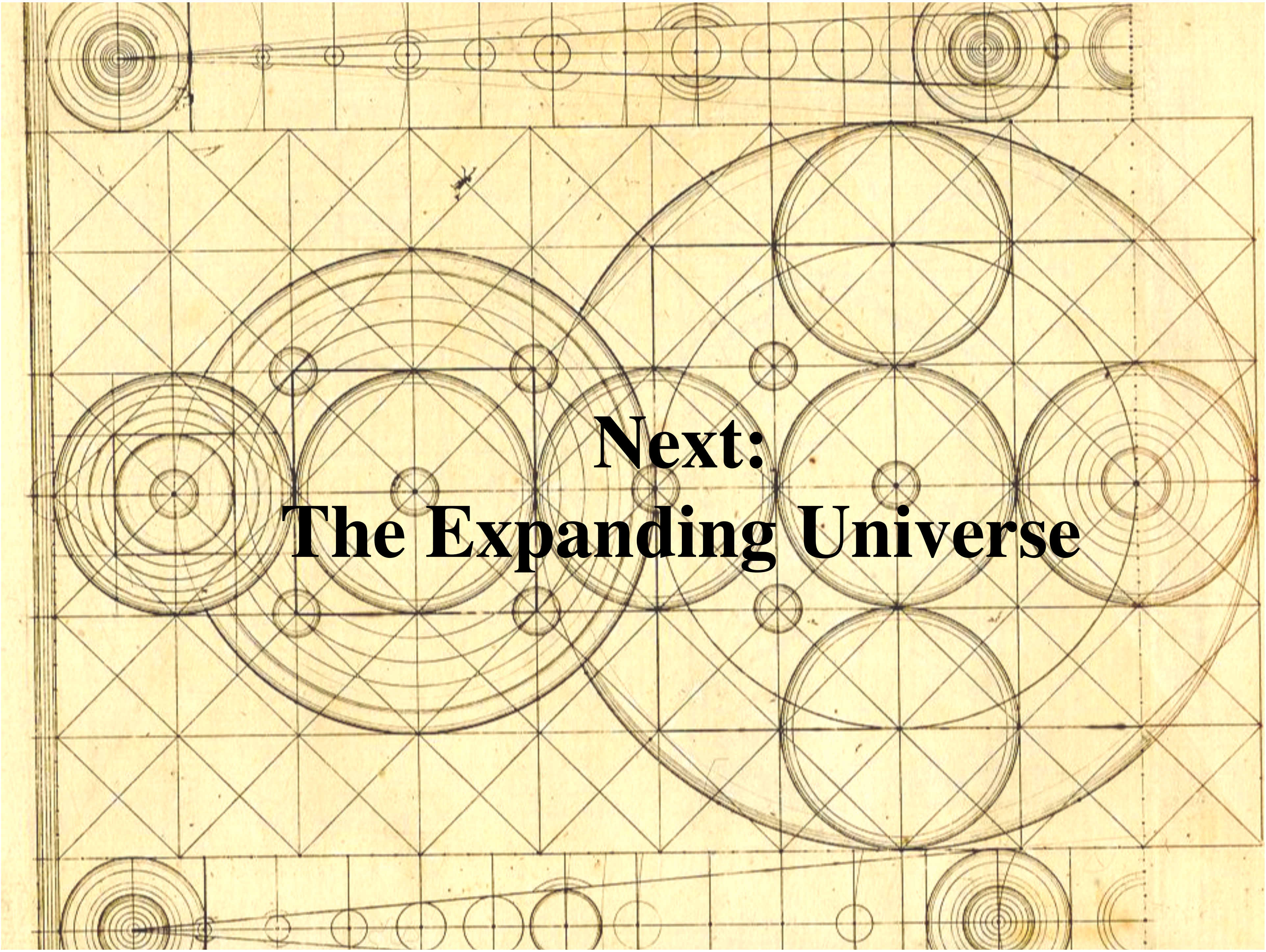
$$\nabla^2\phi - \lambda\phi = 4\pi G\rho$$

λ could be thought of as a screening length, an integration constant, a new constant of nature, a new aspect of gravity

The Einstein Equations now become:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$
$$\Lambda = 1/L^2,\ \rho = \Lambda/4\pi G$$

This is the **cosmological constant**. Note that the theory does not specify its value, or even the sign!



Next:
The Expanding Universe