Linear Circuits

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An introduction to linear electric circuit elements and a study of circuits containing such devices.

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Transfer Functions

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Understand how linear systems react to inputs of different frequencies.

Previous Lesson

- AC analysis
- Sinusoidal response lab

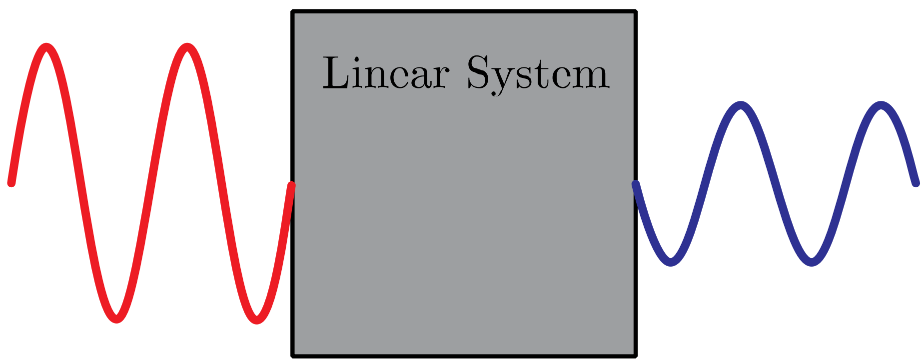
Module 4: Frequency Analysis

- Sinusoids and phasors
- Impedance
- Analysis of sinusoidal systems
- Transfer functions
- Freqency response
- Low-pass and high-pass filters
- Bandpass and notch filters

Lesson Objectives

- Describe the definition of a transfer function and how they are used
- Plot transfer functions in both linear and logarithmic spaces and interpret the results

Behavior of Sinusoids in Linear Systems



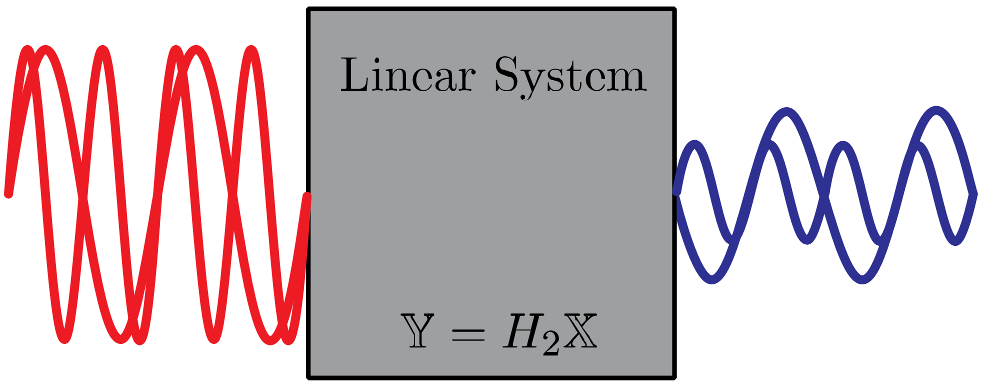
$v_{\text{in}} = A_{\text{in}} \cos(\omega t + \theta_{\text{in}})$

$\mathbb{V}_{\text{in}} = A_{\text{in}} \angle \theta_{\text{in}}$

$v_{\text{out}} = A_{\text{out}} \cos(\omega t + \theta_{\text{out}})$

$\mathbb{V}_{\text{out}} = A_{\text{out}} \angle \theta_{\text{out}}$

Transfer Function



$x(t) = X \cos(\omega_2 t)$

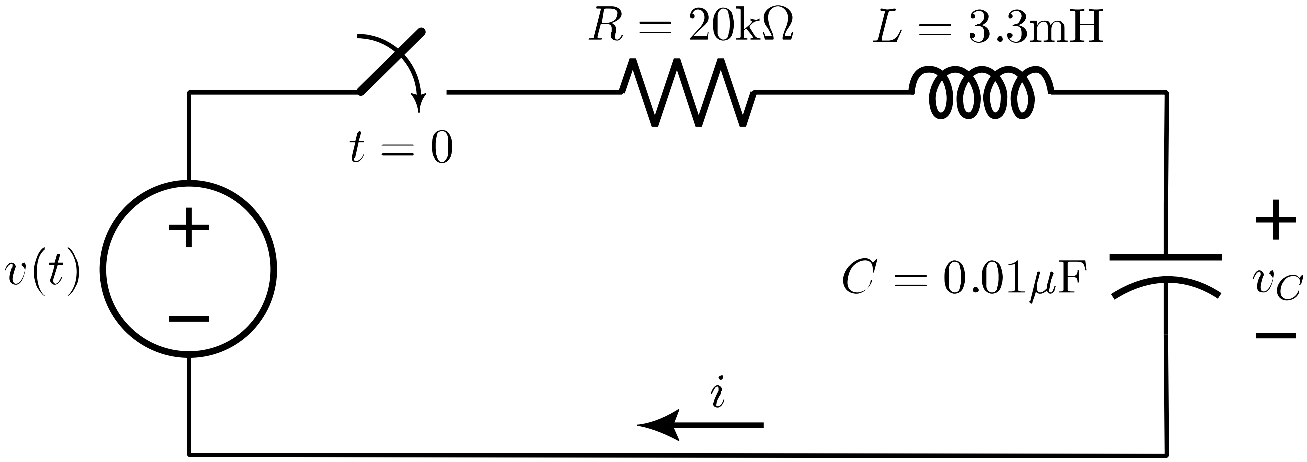
$\mathbb{X} = X \angle 0$

$y(t) = Y_2 \cos(\omega_2 t + \theta_2)$

$\mathbb{Y} = Y_2 \angle \theta_2$

$$H(f) = \frac{\mathbb{Y}_f}{\mathbb{X}_f} = \frac{Y_f}{X_f} \angle \theta_{y,f} - \theta_{x,f}$$

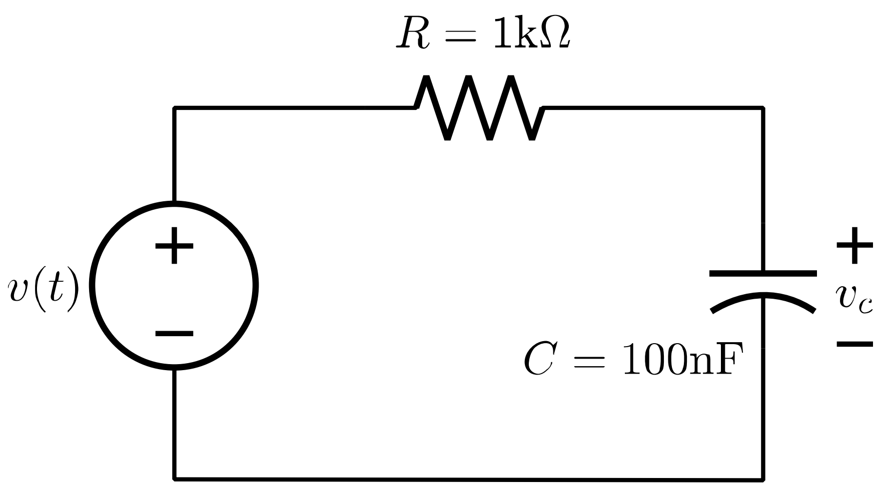
Example 1: Effect of Frequency



$$v(t) = \cos\left(2\pi(780)t\right)$$

$$H(780) = 0.51 - j0.50 = \frac{1}{\sqrt{2}} \angle -\frac{\pi}{4}$$

Example 2: RC



$$v(t) = 5\cos(2\pi f t)$$

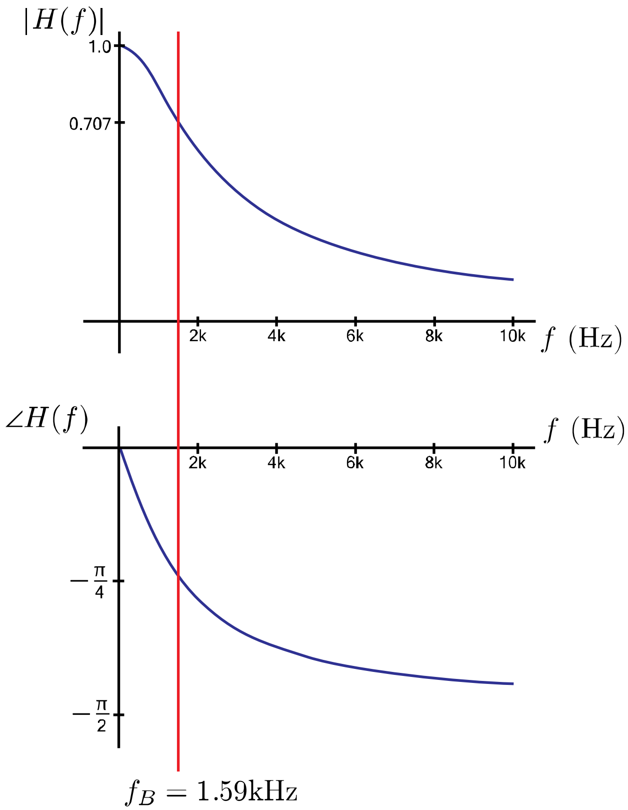
$$H(f) = \frac{1}{1+j2\pi f RC} \qquad \text{Let } f_B = \frac{1}{2\pi RC}$$

$$H(f) = \frac{1}{1+j(f/f_B)}$$

$$|H| = \frac{1}{\sqrt{1+(f/f_B)^2}}$$

$$\angle H = -\text{atan}\left(f/f_B\right)$$

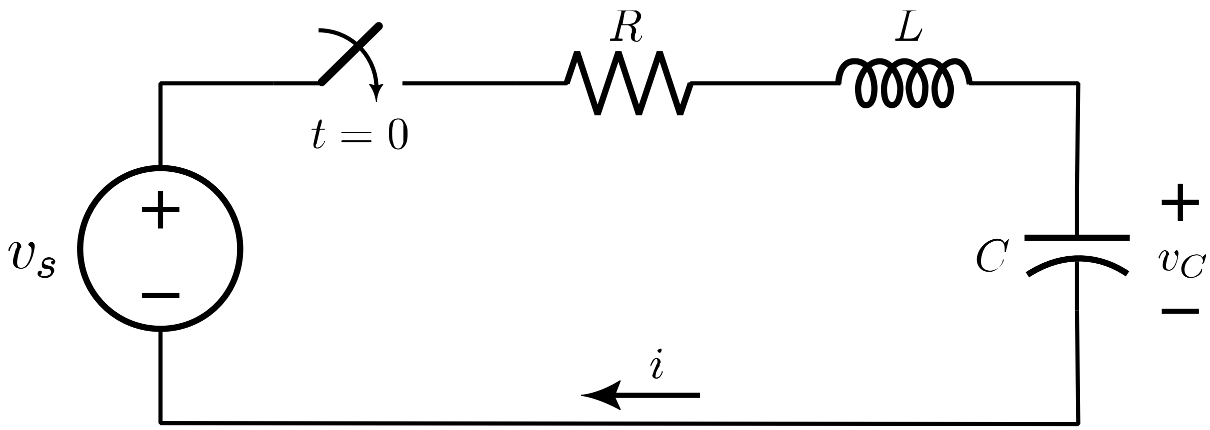
Transfer Function Plots



$$|H| = \frac{1}{\sqrt{1+(f/f_B)^2}}$$

$$\angle H = -\text{atan}\left(f/f_B\right)$$

Example 3: RLC



$$H(\omega) = \frac{1}{(1-\omega^2 LC) + j\omega RC}$$

$$|H(\omega)| = \frac{1}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

$$\angle H(\omega) = -\text{atan}\left(\frac{\omega RC}{1-\omega^2 LC}\right)$$

Summary

- Introduced the concept of a transfer function
- Showed how to calculate a transfer function for a particular system
- Demonstrated how to graph the magnitude and phase response

Next Lesson

- Time and frequency domain
- Demo with a guitar string to show frequency spectrum