Georgia Institute of Technology

Linear Circuits

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An introduction to linear electric circuit elements and a study of circuits containing such devices.

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Impedance

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Identify impedances – a mathematical tool to analyze reactive circuits with sinusoidal inputs.

Previous Lesson

- Sinusoids and phasors

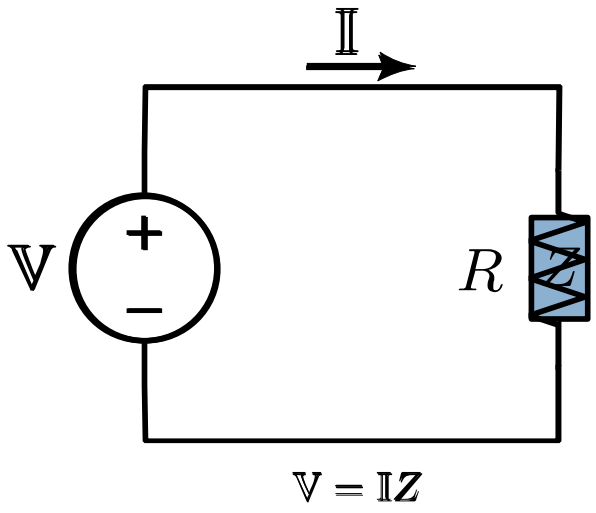
Module 4: Frequency Analysis

- Sinusoids and phasors
- Impedance
- Circuit analysis in AC
- Transfer functions
- Frequency response
- High/low-pass filters
- Bandpass/notch filters

Lesson Objectives

- Be able to describe impedance
- Calculate impedances of resistors, capacitors, and inductors
- Identify the relationship between voltage and current based on and impedance value

Definition of Impedance

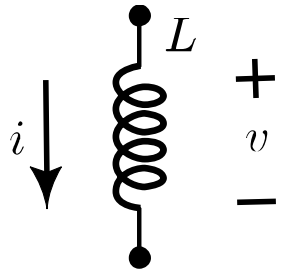


$$v(t) = V_m \cos(\omega t + \theta) \quad \Leftrightarrow \quad V_m \angle\theta$$

$$i(t) = \frac{V_m}{R} \cos(\omega t + \theta) \quad \Leftrightarrow \quad \frac{V_m}{R} \angle\theta$$

$$\boxed{Z_R = R}$$

Impedance of an Inductor



$$v_L = L\frac{di_L}{dt}$$

$$i = I_m \cos(\omega t + \theta) \quad \Leftrightarrow \quad \mathbb{I} = I_m \angle\theta$$

$$v = LI_m\omega \cos\left(\omega t + \theta + \tfrac{\pi}{2}\right) \quad \Leftrightarrow \quad \mathbb{V} = LI_m\omega \angle\theta + \tfrac{\pi}{2}$$

$$\mathbb{V} = \mathbb{I}Z$$

$$Z = \frac{\mathbb{V}}{\mathbb{I}} = \frac{LI_m\omega\angle\theta + \frac{\pi}{2}}{I_m\angle\theta} = L\omega\angle\tfrac{\pi}{2} = j\omega L$$

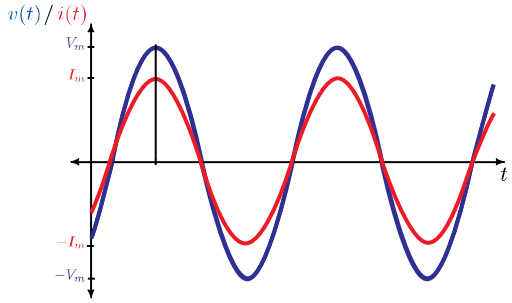
$$\boxed{Z_L = j\omega L}$$

- Inductor impedance purely imaginary
- Scales based on frequency
- Positive imaginary, so current **lags** voltage

Impedances

R

$$Z_R = R$$

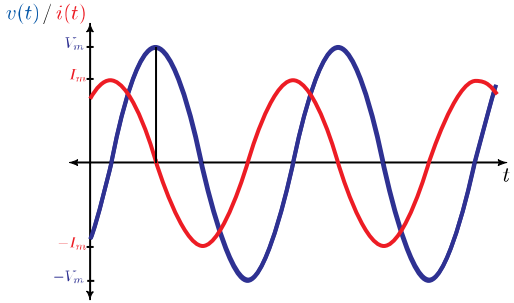


In-phase

Frequency invariant

C

$$Z_C = \frac{1}{j\omega C}$$

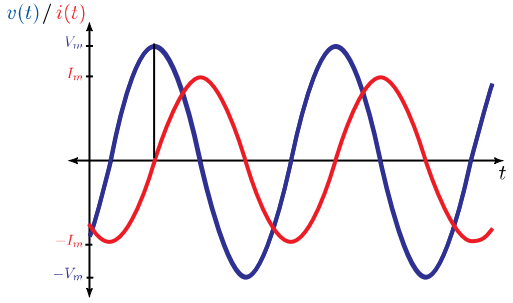


Current leads voltage

Voltage attenuates for high frequency

L

$$Z_L = j\omega L$$



Current lags voltage

Current attenuates for high frequency

Summary

- Defined impedance and calculated impedance of linear devices
- Described the relationship between the current and the voltage given impedance

Next Lesson

- Using impedances, apply analysis techniques to AC systems