Georgia Institute of Technology

Linear Circuits

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An introduction to linear electric circuit elements and a study of circuits containing such devices.

RLC Circuits Part 2

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- *Use differential equations to show the behavior of an RLC circuit as the system changes.*

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Previous Lesson

- Analyzed an overdamped second-order system

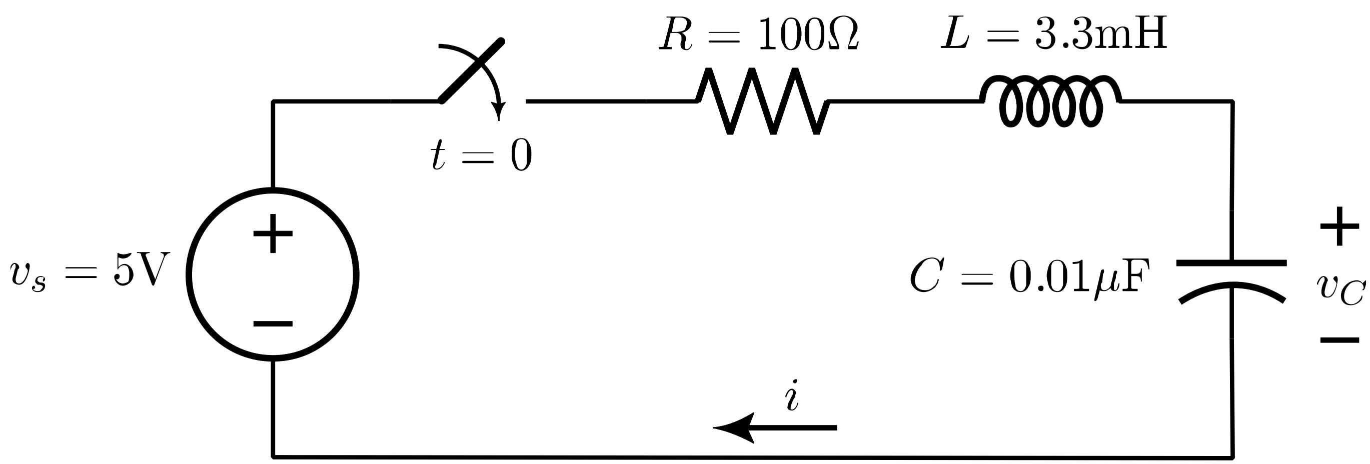
Module 3: Reactive Circuits

- Capacitors
- Inductors
- First-order differential equations
- RC Circuits
- RL Circuits
- Second-order differential equations
- RLC Circuits

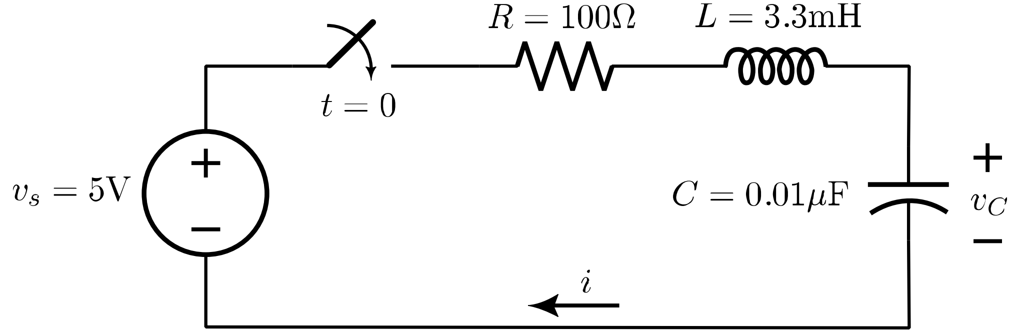
Lesson Objectives

- Generate a second-order differential equation from an underdamped RLC circuit
- Identify initial and final conditions
- Solve the differential equation
- Recognize if a system is underdamped/overdamped
- Identify the effect of damping on a second-order system

Example 2: Initial and Final Conditions



Example 2: Differential Equation



$$\frac{d^2 v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = \frac{v_s}{LC}$$

$$\frac{d^2 v_c}{dt^2} + 30.3\text{e}3\frac{dv_c}{dt} + 30.3\text{e}9 v_c = 151.5\text{e}9$$

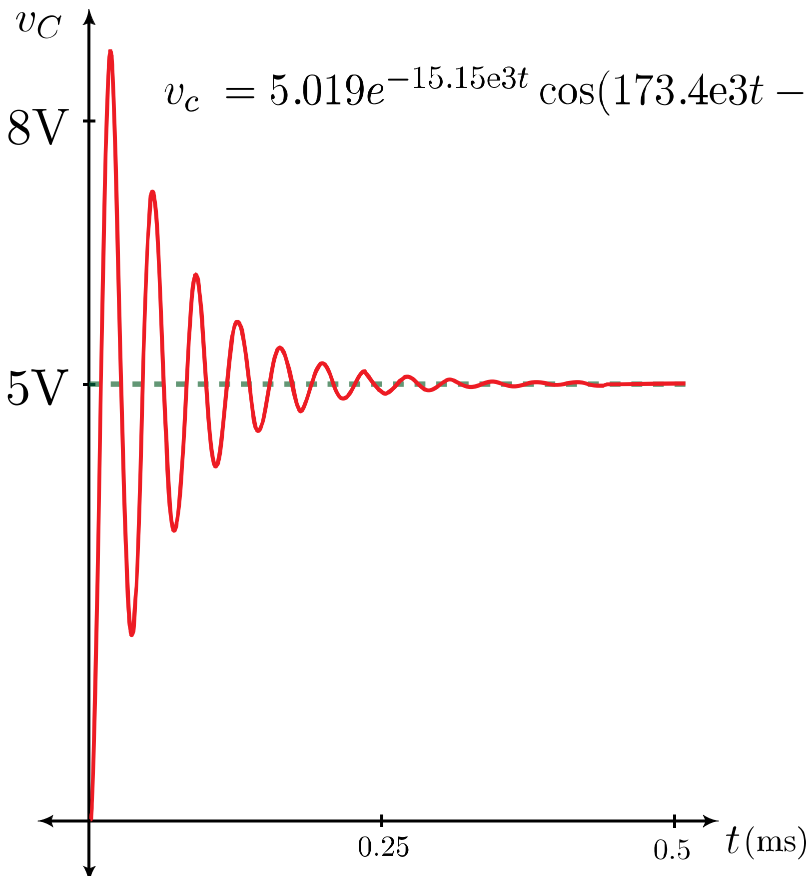
$$s^2 + 30.3\text{e}3 s + 30.3\text{e}9 = 0$$

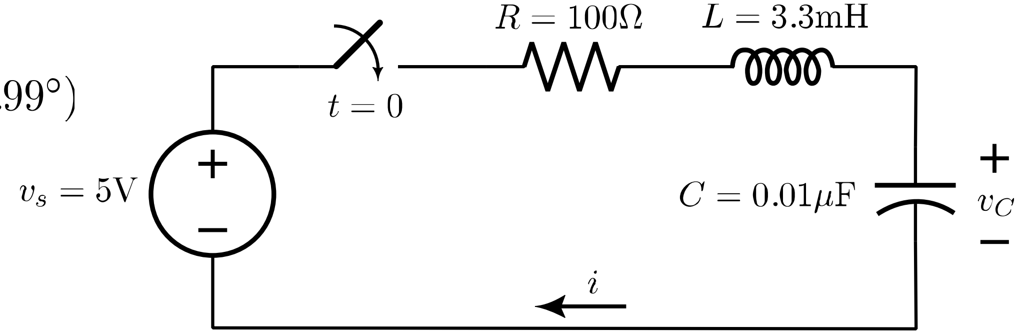
Roots: $-15.15\text{e}3 \pm j173.4\text{e}3$

$$\boxed{v_{c,\,\text{t}} = K e^{-15.15\text{e}3t}\cos(173.4\text{e}3t + \theta)}$$

$$\boxed{v_{c,\,\text{s}} = 5}$$

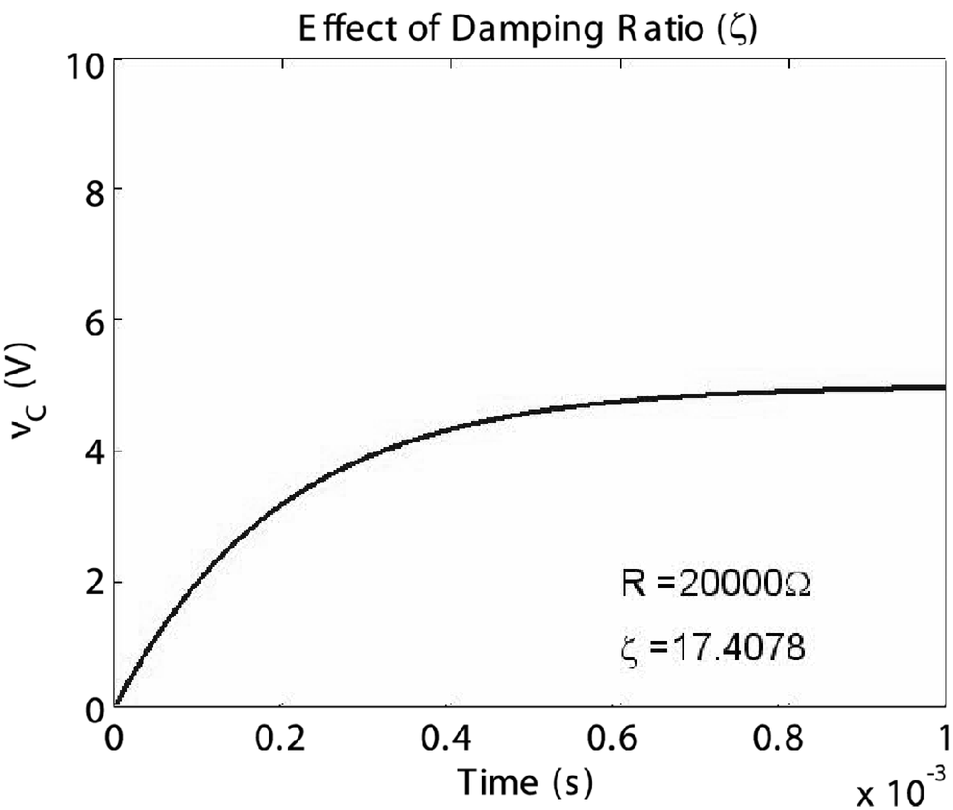
Example 2: Final Solution





Damping Ratio

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n\frac{dy}{dt} + \omega_n^2 y = K$$



$\zeta > 1$ Overdamped

$\zeta = 1$ Critically damped

$\zeta < 1$ Underdamped

$$\omega_n = \sqrt{\frac{1}{LC}} \qquad \zeta = \frac{R}{2\omega_n L}$$

Summary

- Got some intuition about how RLC circuits behave and contrasted overdamped and underdamped cases
- Identified initial and final conditions
- Found and solved representative differential equations
- Plotted the results
- Animated response as the resistance changes to show the effect of damping on the system

Next Lesson

- Lab to demonstrate RLC systems.