Linear Circuits

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An introduction to linear electric circuit elements and a study of circuits containing such devices.

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RLC Circuits Part 1

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- *Use differential equations to show the behavior of an RLC circuit as the system changes.*

Previous Lesson

- Learned about solving second-order differential equations

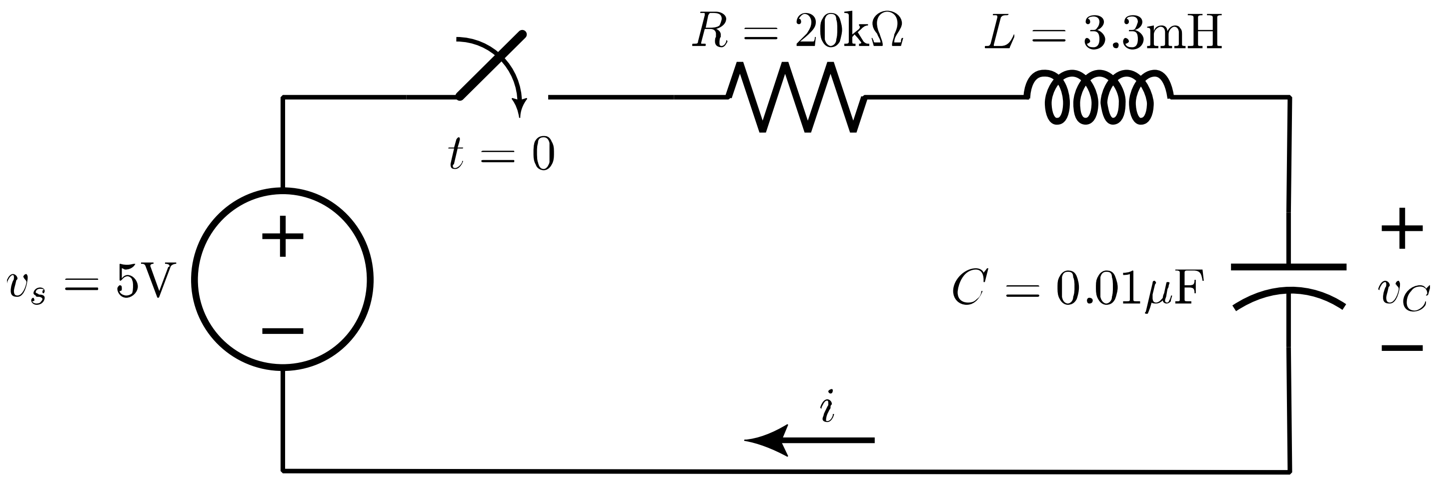
Module 3: Reactive Circuits

- Capacitors
- Inductors
- First-order differential equations
- RC Circuits
- RL Circuits
- Second-order differential equations
- RLC Circuits

Lesson Objectives

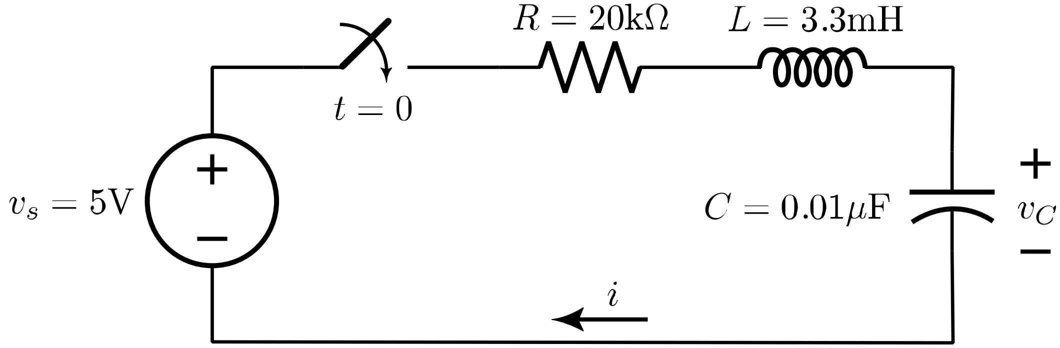
- Generate a second-order differential equation from a RLC circuit
- Identify initial and final conditions
- Solve the differential equation
- Recognize if a system is underdamped/overdamped

Example 1: Initial and Final Conditions



$$v_C(0^-) = 0$$

Example 1: Differential Equation



$$LC\frac{d^2v_C}{dt^2} + RC\frac{dv_C}{dt} + v_C = v_s$$

$$\frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = \frac{v_s}{LC}$$

$$\frac{d^2y}{dt^2} + a_1\frac{dy}{dt} + a_2y = K$$

$$v_s = v_R + v_L + v_C$$

$$v_R = iR \qquad v_L = L\frac{di}{dt} \qquad i = C\frac{dv_c}{dt}$$

Example 1: Transient

$$\frac{d^2v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = \frac{v_s}{LC}$$

$$\frac{d^2v_c}{dt^2} + 6.06\text{e}6\frac{dv_c}{dt} + 30.3\text{e}9 v_c = 151.5\text{e}9$$

Characteristic Equation:

$$s^2 + 6.06\text{e}6\ s + 30.3\text{e}9 = 0$$

Roots: $-5.00\text{e}3$ $-6.06\text{e}6$

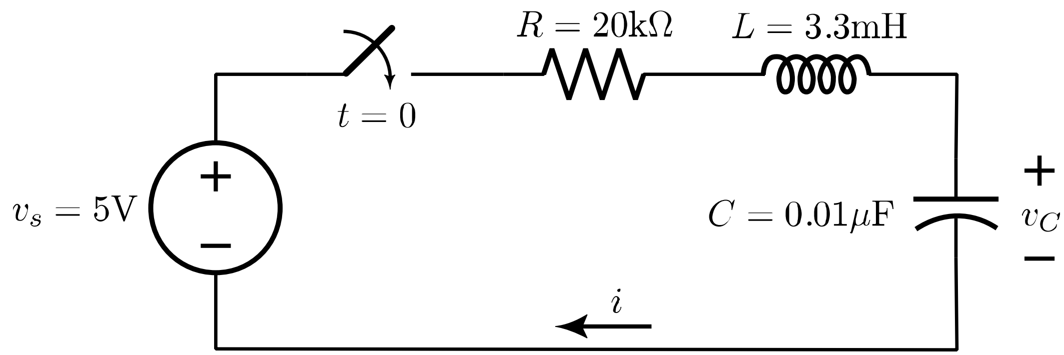
$$v_{c,\,\text{t}} = K_1 e^{-5.00\text{e}3t} + K_2 e^{-6.06\text{e}6t}$$

Example 1: Steady State

$$\frac{d^2 v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = \frac{v_s}{LC}$$

$$v_c \rightarrow \frac{K}{a_2} = LC\frac{v_s}{LC} = v_s$$

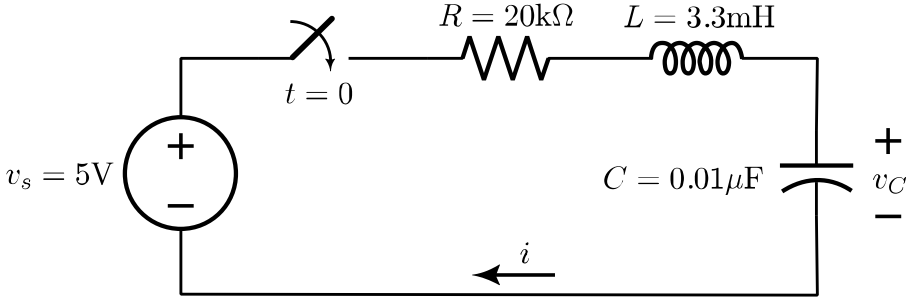
$$v_{c,\,\mathrm{s}} = 5$$



Example 1: Solving for Constants

$$v_c = v_{c,t} + v_{c,s}$$

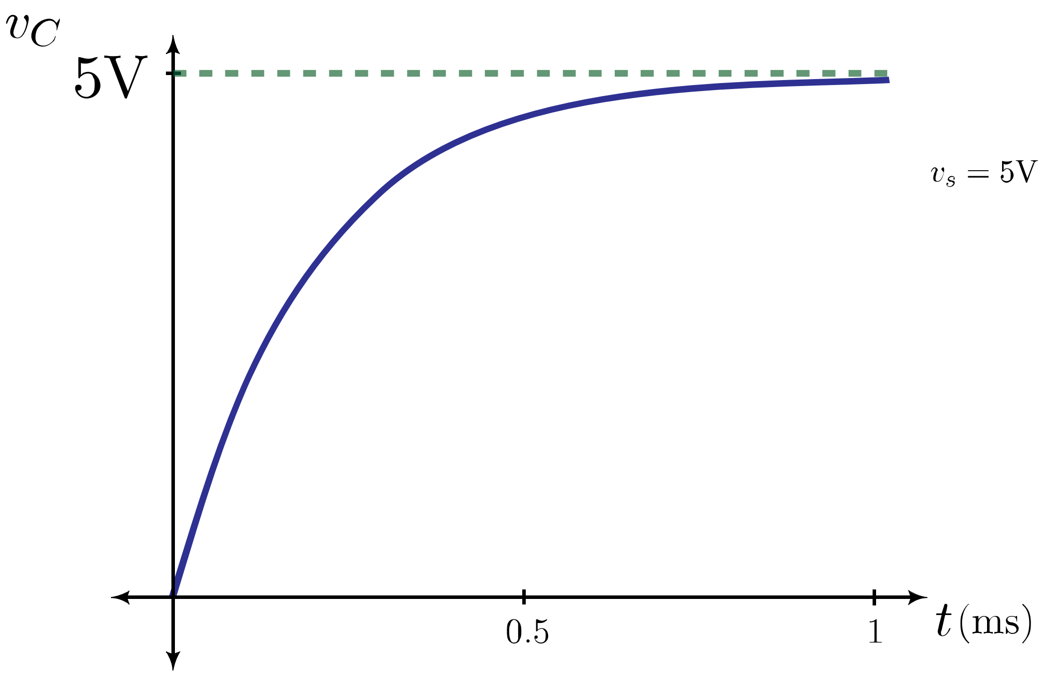
$$v_c = K_1 e^{-5.00e3t} + K_2 e^{-6.06e6t} + 5$$

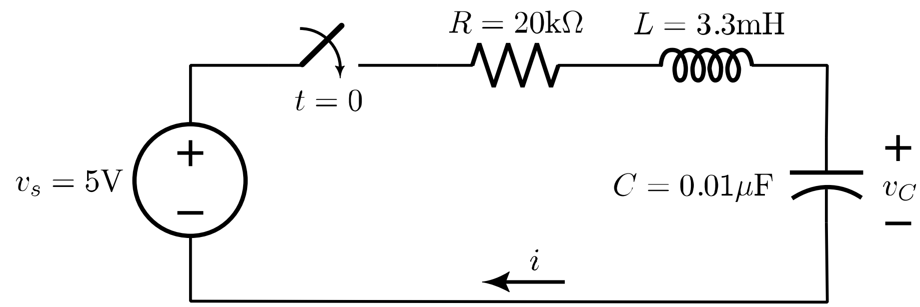


$v_C(0^-) = v_C(0) = 0$	$i(0^-) = i(0) = 0$
$K_1 + K_2 + 5 = 0$	$i = C\frac{dv_C}{dt}$
	$-50\text{e}(-6)K_1 - 60.6\text{e}(-3)K_2 = 0$

$$K_1 = -5.00413 \qquad K_2 = 0.00413$$

Example 1: Final Solution





$$v_C = -5.00413e^{-5.00\mathrm{e}3t} + 0.00413e^{-6.06\mathrm{e}6t} + 5$$

Summary

- Looked at an overdamped case
- Identified initial and final conditions
- Found and solved representative differential equation
- Plotted the results

Next Lesson

- Consider an underdamped example