Linear Circuits

Dr. Bonnie H. Ferri
Professor and Associate Chair
School of Electrical and
Computer Engineering

An introduction to linear electric components and a study of circuits containing such devices.

Georgia Institute of Technology

Second-Order Differential Equations

Dr. Bonnie H. Ferri
Professor and Associate Chair
School of Electrical and
Computer Engineering

Recognize types of second-order responses

Previous Lessons

- Solutions to first-order differential equations and application to RC and RL circuits

Module 3: Reactive Circuits

- Capacitance
- Inductance
- First-Order Differential Equations
- RC and RL Circuits
- Second-Order Differential Equations
- RLC Circuits
- Applications

Lesson Objectives

Examine second-order differential equations with a constant input:

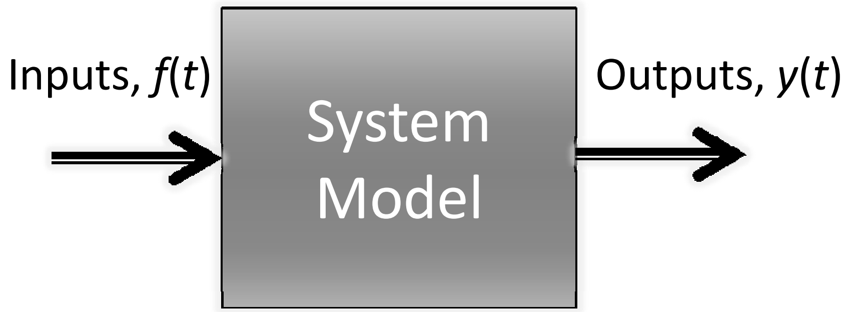
- Determine the steady-state solution
- Determine the type of transient response
- Recognize the characteristics of the plot of the solution

Ordinary Differential Equations

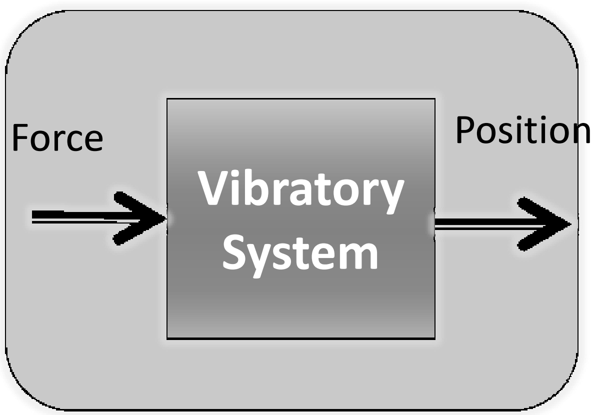
- ODE: Include functions of variables and their derivatives

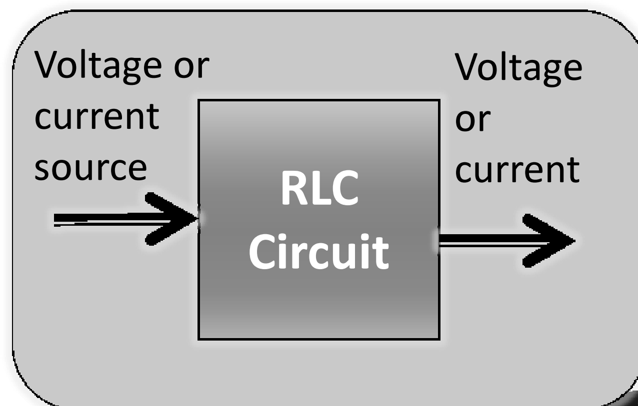
$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 4y = f(t)$$

Models of Physical Systems



$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 4y = f(t)$$





Solutions to Second-Order Differential Equation

$$\frac{d^2y}{dt^2}+a_1\frac{dy}{dt}+a_2y=K, \quad y(0)\,and\,\left.\frac{dy}{dt}\right|_{t=0}$$

Solution: y(t) = steady-state + transient

STEADY-STATE: $y(t)\rightarrow\frac{K}{a_2}$

TRANSIENT: Three possible forms depending on roots of $(s^2+a_1s+a_2)=0$.

Transient Response

$$(s^2+a_1 s+a_2)=0.$$

OVERDAMPED: ((real and distinct roots, r_1 and r_2)

$$K_1 e^{r_1 t} + K_2 e^{r_2 t}$$

CRITICALLY DAMPED: ((real and equal roots, r and r)

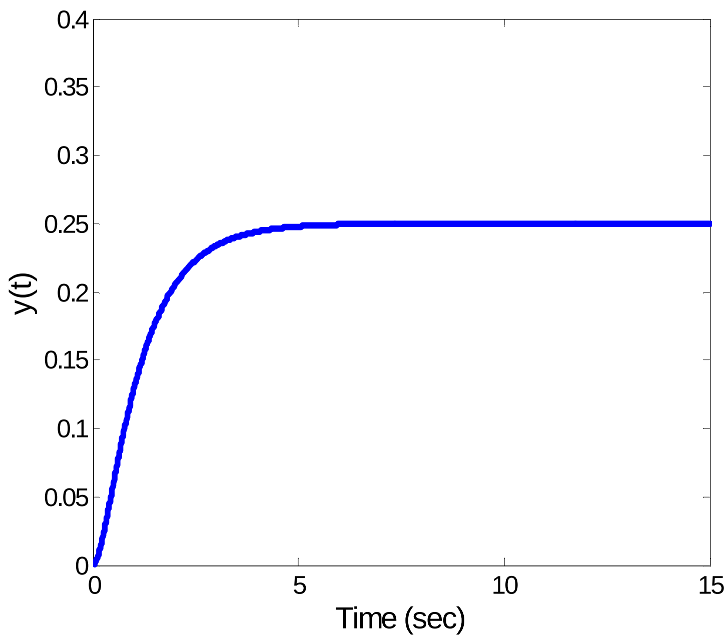
$$K_1 e^{rt} + K_2 t e^{rt}$$

UNDERDAMPED: ((complex roots, $a \pm jb$)

$$K e^{at} \sin(bt + \varphi)$$

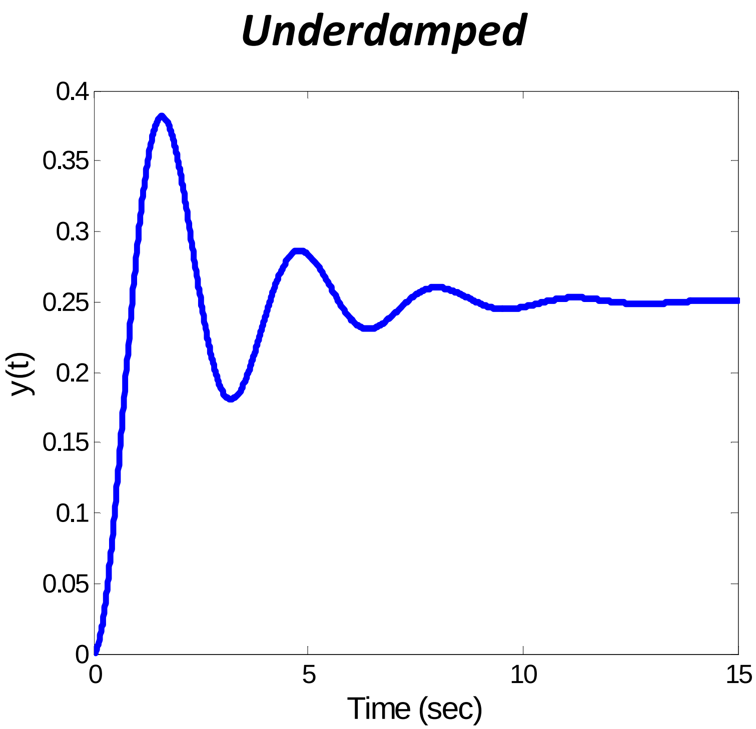
Sample Problems

Overdamped



$$\frac{d^2y}{dt^2}+5\frac{dy}{dt}+4y=1, \quad y(0)=0, \left.\frac{dy}{dt}\right|_{t=0}=0$$

Sample Problems



$$\frac{d^2y}{dt^2} + 0.8\frac{dy}{dt} + 4y = 1, \quad y(0) = 0, \left.\frac{dy}{dt}\right|_{t=0} = 0$$

Summary

- Examined generic 2nd order differential equation
  - Vibratory systems, RLC circuits
- Showed steady-state solution
- Showed generic transient solutions to underdamped and overdamped responses
- Showed characteristic plots of under damped and overdamped responses to a constant input applied at t=0

Next Lesson

- Demonstrate RLC circuit equations and responses