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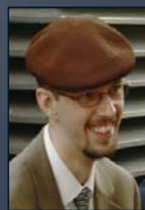

Linear Circuits

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An introduction to linear electric circuit elements and a study of circuits containing such devices.

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RL Circuits

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•Use differential equations to show the behavior of an RL circuit as the system changes.

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Previous Lesson

- Identified behavior of circuits with resistors and capacitors

Module 3: Reactive Circuits

- Capacitors
- Inductors
- First-order differential equations
- RC Circuits
- RL Circuits
- Second-order differential equations
- RLC Circuits

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Lesson Objectives

- Generate a differential equation from a circuit
- Identify initial and final conditions
- Solve the differential equation
- Graph the result

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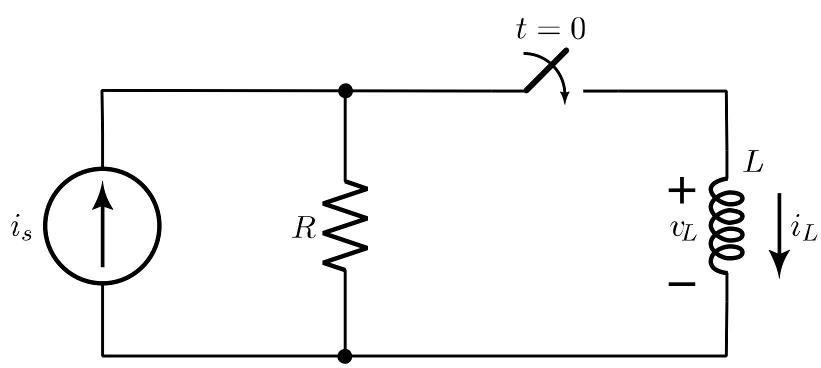
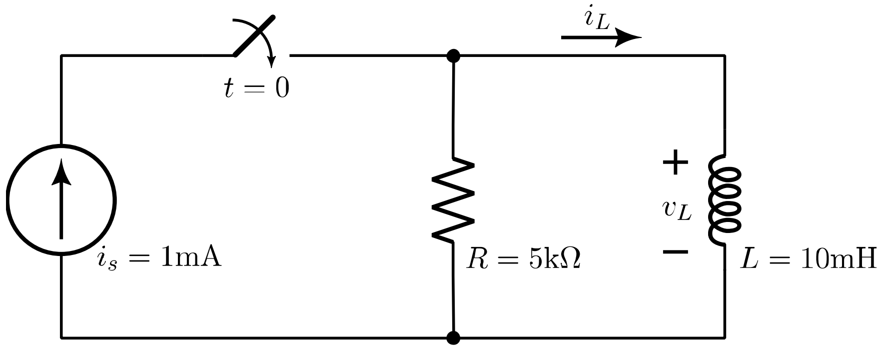
Behavior of RL circuits



Example 1: Initial and Final Conditions



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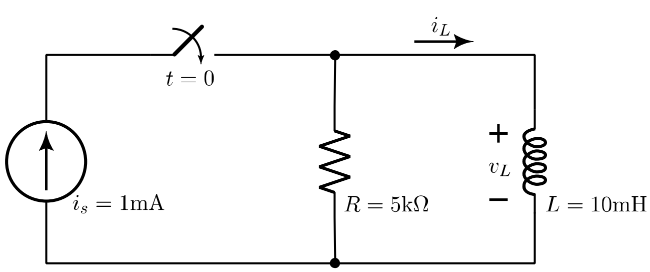
Example 1: Differential Equation



$$i_L = i_s\left[1 - e^{\frac{-t}{\tau}}\right]$$

$$v_L = i_s R e^{\frac{-t}{\tau}}$$

$v_L = L\frac{di_L}{dt}$ $\quad$ $i_R = \frac{v}{R}$ $\quad$ $i_s = i_R + i_L$

$i_s = \frac{L}{R}\frac{di_L}{dt} + i_L$

$\frac{di_L}{dt} + \frac{R}{L}i_L = \frac{R}{L}i_s$ $\quad$ Let $\tau = \frac{L}{R}$

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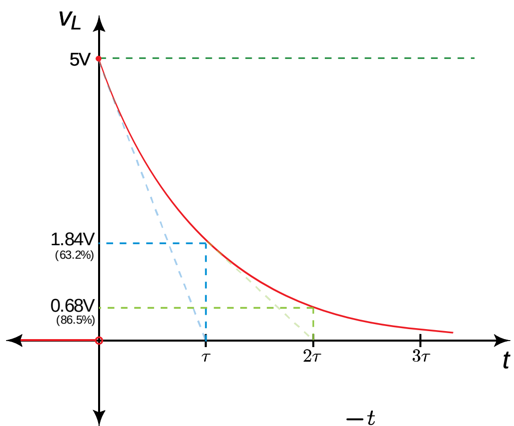
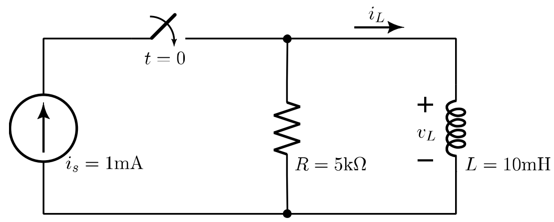
Example 1: Graph



$$v_L = 5e^{\frac{-t}{\tau}}$$



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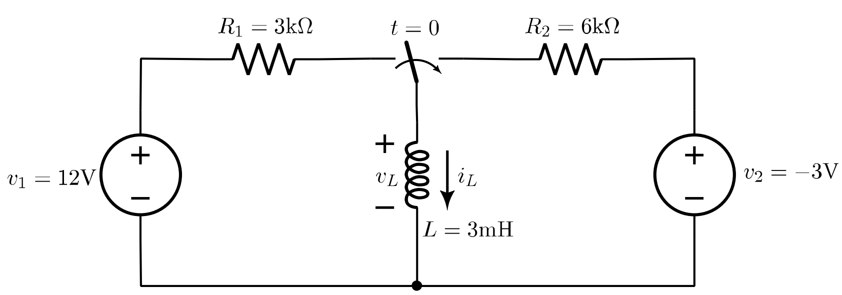
Example 2: Initial and Final Conditions



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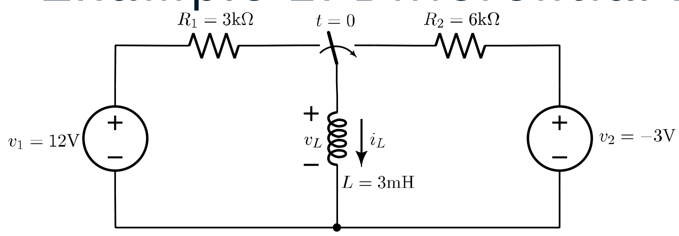
Example 2: Differential Equation



$$i_L = \frac{v_2}{R_2}\left[1 - e^{\frac{-t}{\tau}}\right] + \frac{v_1}{R_1}e^{\frac{-t}{\tau}}$$

$$v_L = L\frac{di_L}{dt} \qquad i_L = \frac{v_2 - v_L}{R_2}$$

$$i_L = \frac{v_2 - L\frac{di_L}{dt}}{R_2}$$

$$\frac{di_L}{dt} + \frac{R_2}{L}i_L = \frac{v_2}{L} \qquad \text{Let } \tau = \frac{L}{R_2}$$

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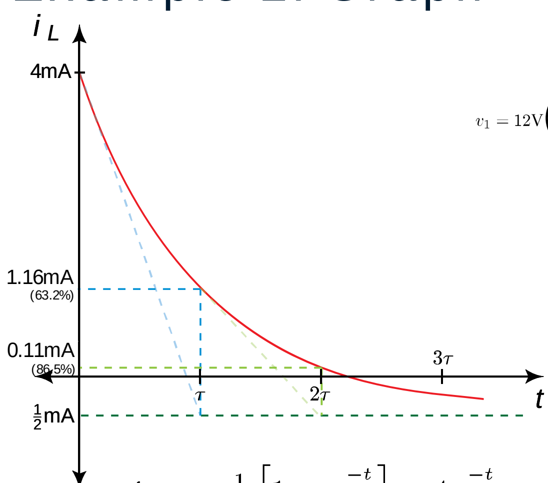
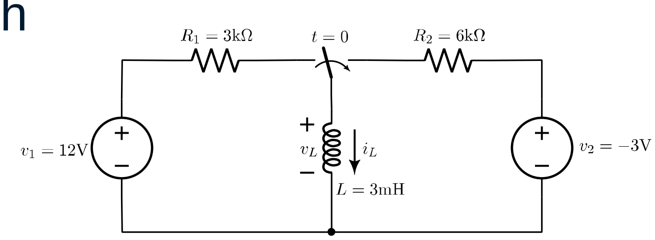
Example 2: Graph





$$i_L = -\frac{1}{2}\left[1 - e^{\frac{-t}{\tau}}\right] + 4e^{\frac{-t}{\tau}}$$

Summary

- Got some intuition about how RL circuits behave
- Identified initial and final conditions
- Found differential equations for the circuit and solved them
- Graphed the results

Next Lesson

- Differential equations applied to systems with resistors and inductors