Circuits & Electronics

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An introduction to electric circuit elements and electronic devices, and a study of circuits containing such devices. Both analog and digital systems are considered.

RC Circuits

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- *Generate a differential equation that describes the behavior of a circuit with resistors and capacitors*
- *Solve the differential equation for step inputs (or switching constant inputs)*
- *Graph the behavior*

Previous Lesson

- Solving some simple first-order differential equations

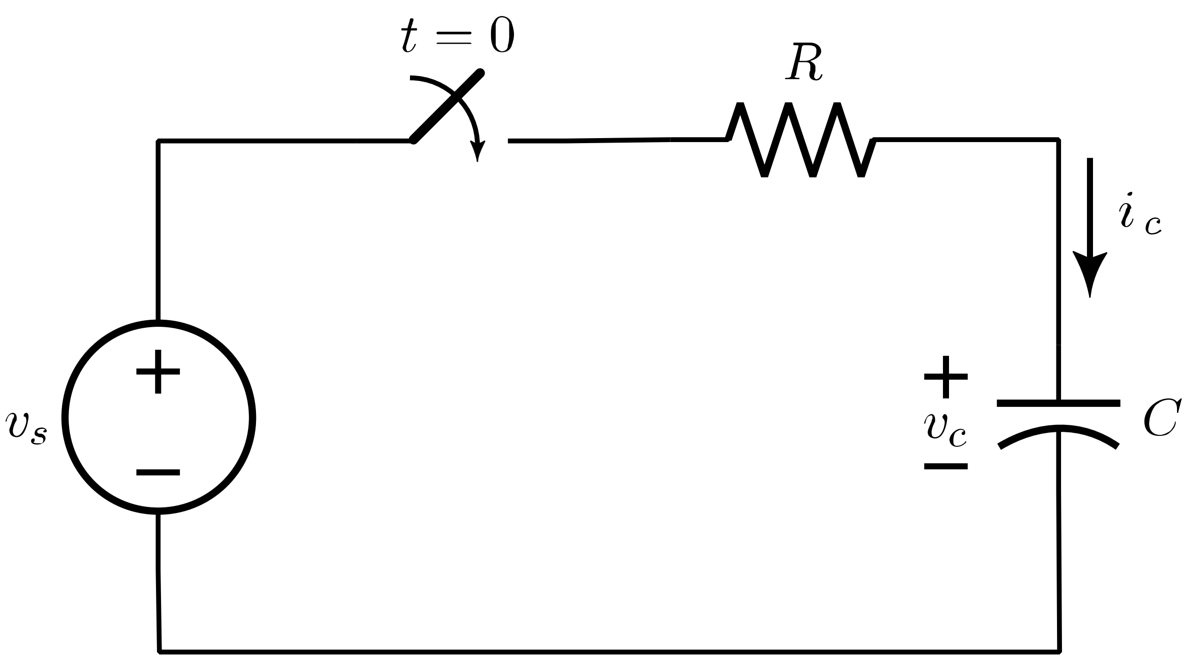
Module 3: Reactive Circuits

- Capacitors
- Inductors
- First-order differential equations
- RC Circuits
- RL Circuits
- Second-order differential equations
- RLC Circuits

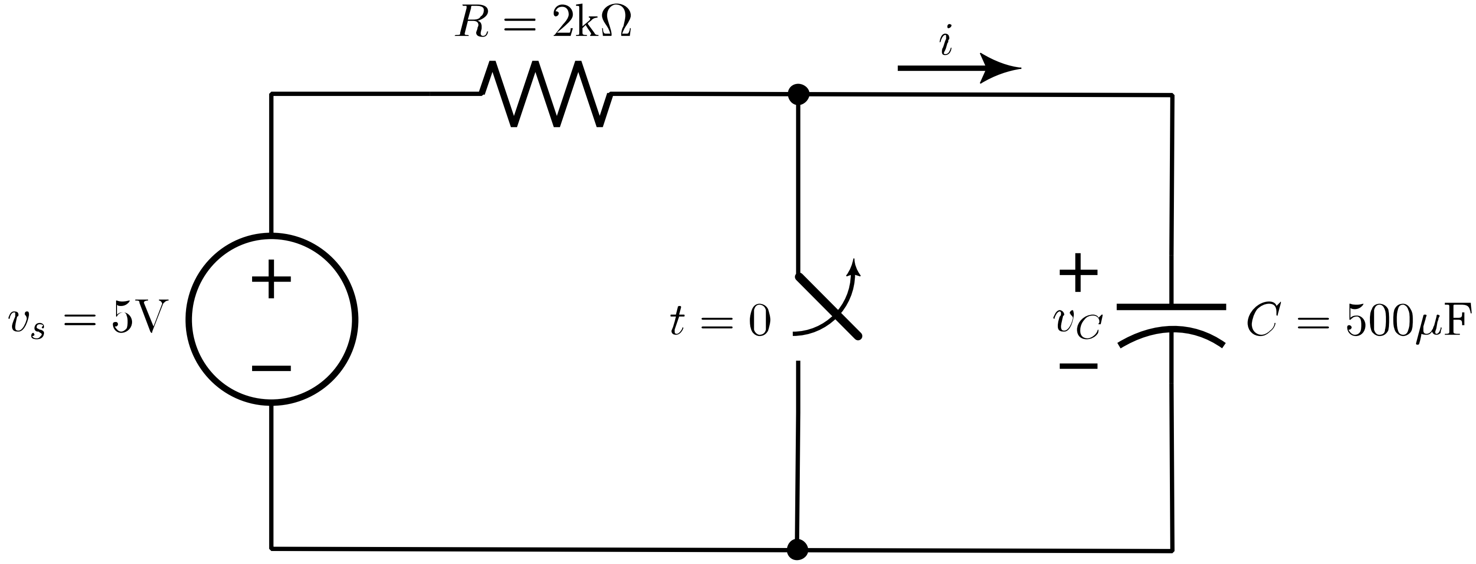
Lesson Objectives

- Generate a differential equation from a circuit
- Identify initial and final conditions
- Solve the differential equation
- Graph the result

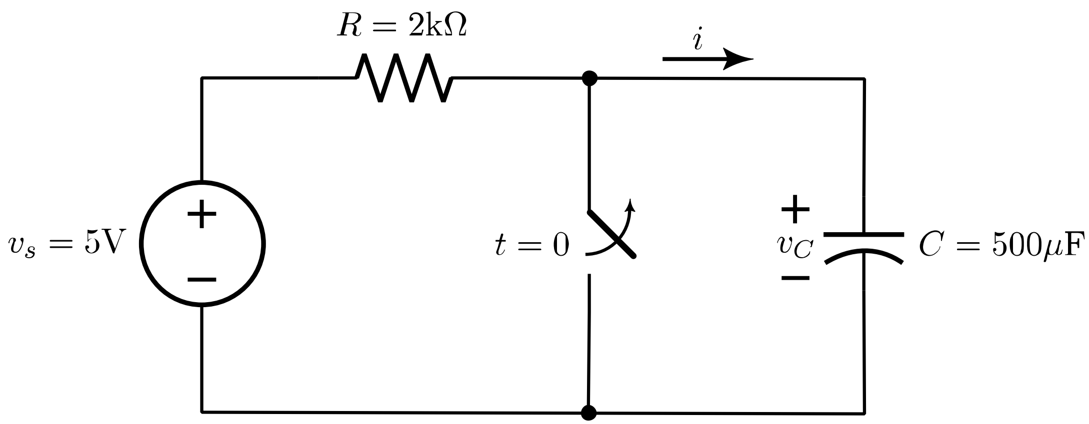
Behavior of RC circuits



Example 1: Initial and Final Conditions



Example 1: Differential Equation



$\frac{dy}{dt} + ay = K$	$y(0)$
$y(t) = \frac{K}{a}(1 - e^{-at}) + y(0)e^{-at}, \quad t \geq 0$	

$$v_c = v_s \left[1 - e^{\frac{-t}{\tau}}\right]$$

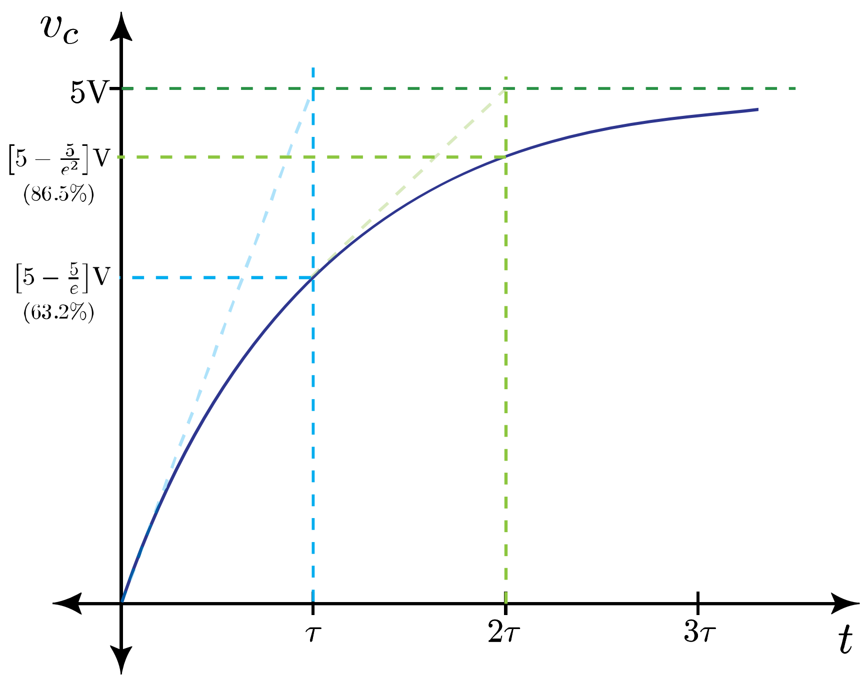
$$i = C\frac{dv_c}{dt} \qquad i = \frac{v_s - v_c}{R}$$

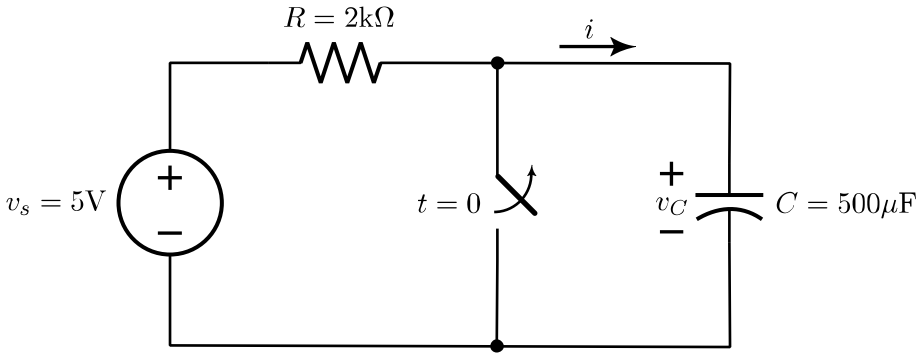
$$\frac{v_s - v_c}{R} = C\frac{dv_c}{dt}$$

$$\frac{dv_c}{dt} + \frac{1}{RC}v_c = \frac{v_s}{RC}$$

Let $\tau = RC$

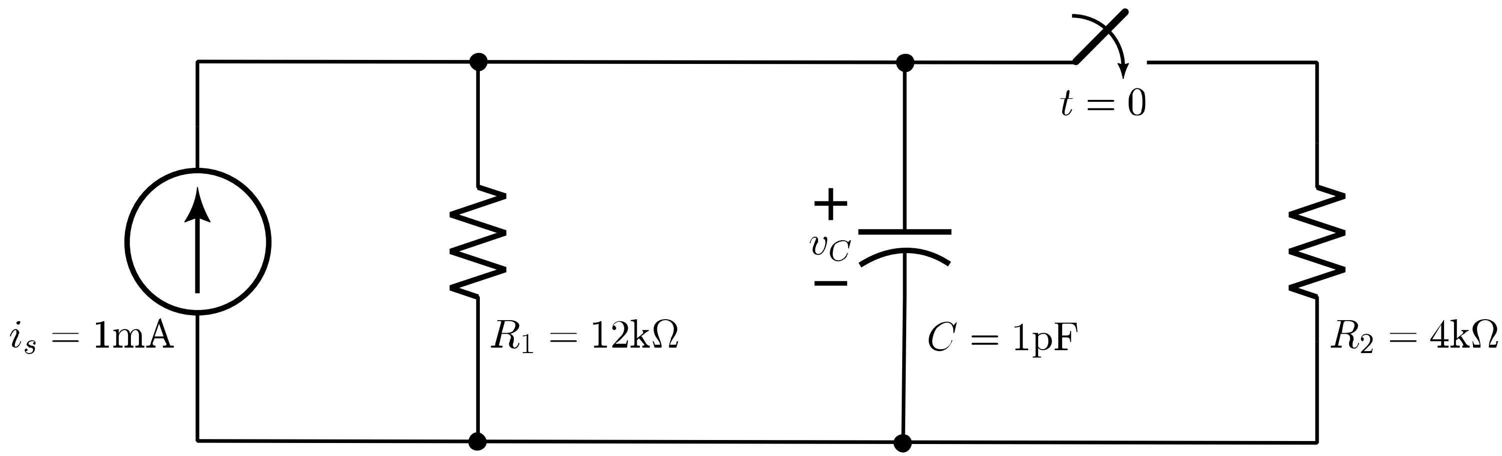
Example 1: Graph



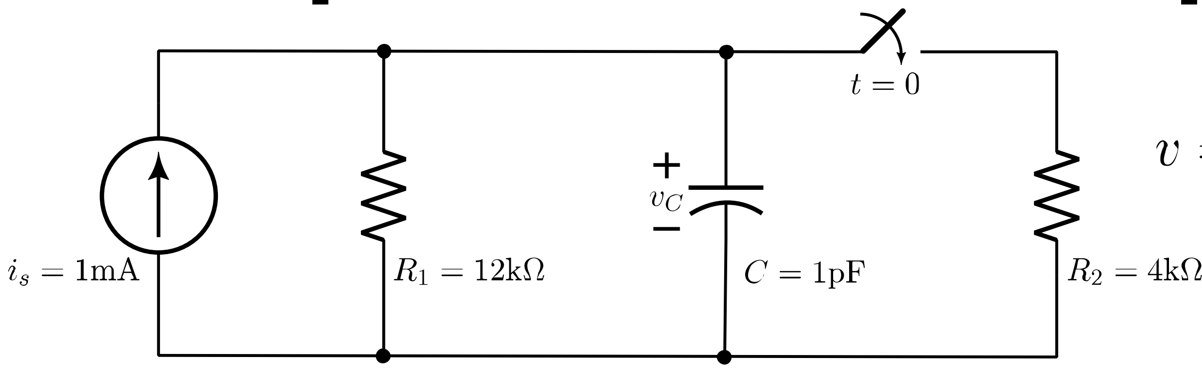


$$v_c = v_s\left[1 - e^{\frac{-t}{\tau}}\right]$$

Example 2: Initial and Final Conditions



Example 2: Differential Equation



$$v = i_s R_{\text{eq}}(1 + e^{\frac{-t}{\tau}}) + i_s R_1 e^{\frac{-t}{\tau}}$$

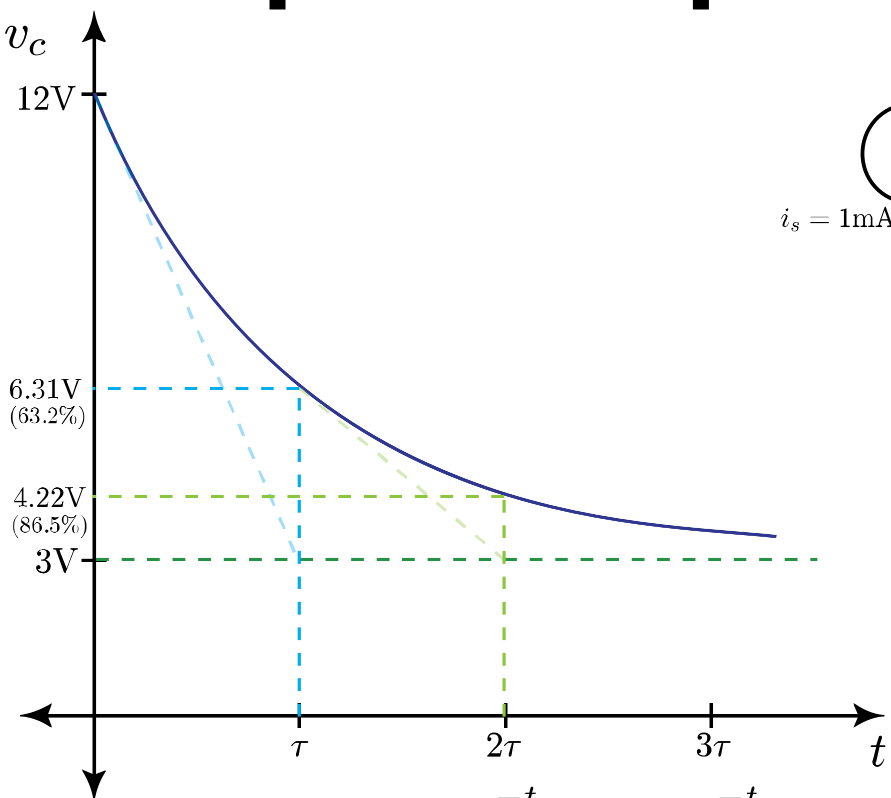
$$i_s = i_{R_1} + i_C + i_{R_2}$$

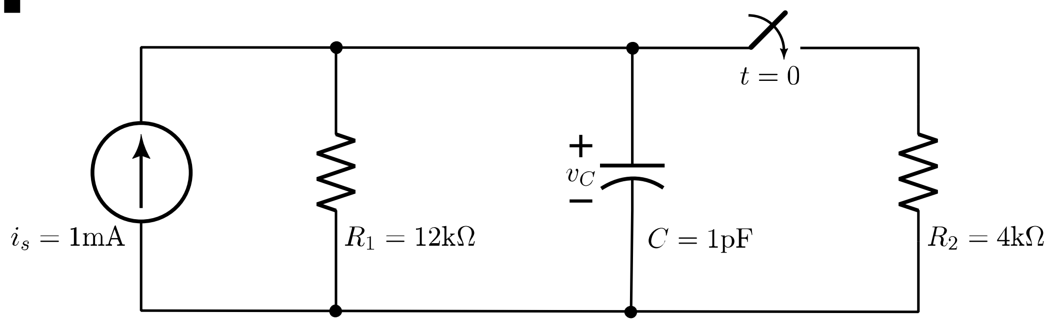
$$i_c = C\frac{dv}{dt} \qquad i_{R_1} = \frac{v}{R_1} \qquad i_{R_2} = \frac{v}{R_2}$$

$$i_s = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v + C\frac{dv}{dt} \qquad \text{Let } R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$\frac{i_s}{C} = \frac{dv}{dt} + \frac{1}{R_{\text{eq}}C}v \qquad \text{Let } \tau = R_{\text{eq}}C$$

Example 2: Graph





$$v = 3(1 + e^{\frac{-t}{\tau}}) + 12e^{\frac{-t}{\tau}}$$

Summary

- Got some intuition about how RC circuits behave
- Identified initial and final conditions
- Found differential equations for the circuit and solved them
- Graphed the results

Next Lesson

- Differential equations applied to systems with resistors and inductors