Georgia Institute of Technology

Linear Circuits

Dr. Bonnie H. Ferri
Professor and Associate Chair
School of Electrical and
Computer Engineering

An introduction to linear electric components and a study of circuits containing such devices.

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First-Order Differential Equations

Dr. Bonnie H. Ferri
Professor and Associate Chair
School of Electrical and
Computer Engineering

Solve and graph solutions to first-order differential equations

Previous Lessons

- Inductors and capacitors have *i*-*v* relationships that include derivatives

Module 3: Reactive Circuits

- Capacitance
- Inductance
- First-Order Differential Equations
- RC and RL Circuits
- Second-Order Differential Equations
- RLC Circuits
- Applications

Lesson Objectives

Examine first-order differential equations with a constant input

- Write the solution
- Sketch the solution

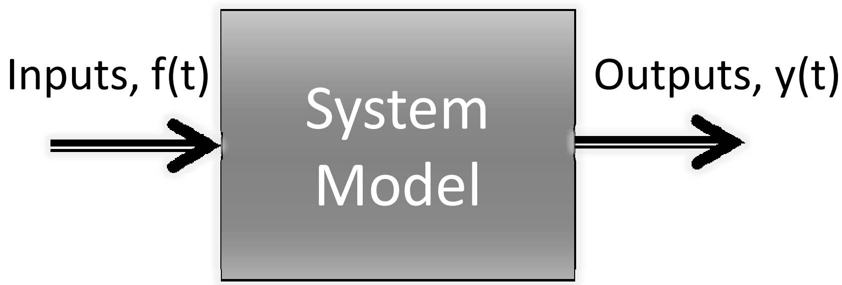
Ordinary Differential Equations

- ODE: Include functions of variables and their derivatives.

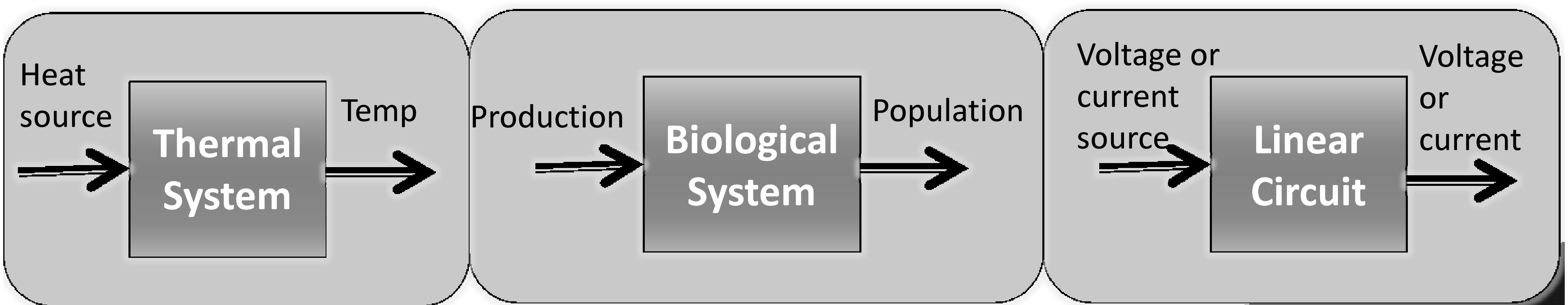
$$\frac{dy}{dt} + 2y = 4 \qquad \frac{dy}{dt} - 2y = 4\sin(\omega t)$$

$$\frac{d^2 y}{dt^2} + 2\frac{dy}{dt} + 4y = f(t) \qquad \frac{dv}{dt} + 2v = i(t)$$

Models of Physical Systems



$$\frac{dy}{dt} + ay = f(t)$$



Solution to First-Order Differential Equation

$$\frac{dy}{dt} + ay = K, \quad y(0)$$

Has solution:

$$y(t) = \frac{K}{a}(1 - e^{-at}) + y(0)e^{-at}, \quad t \geq 0$$

If $a \geq 0$, $e^{-at} \to 0$ ⟹ $y(t) \to \frac{K}{a}$ = steady-state

Graph of Response

$$y(t)=\frac{K}{a}(1-e^{-at})+y(0)e^{-at}, \quad t\geq 0$$

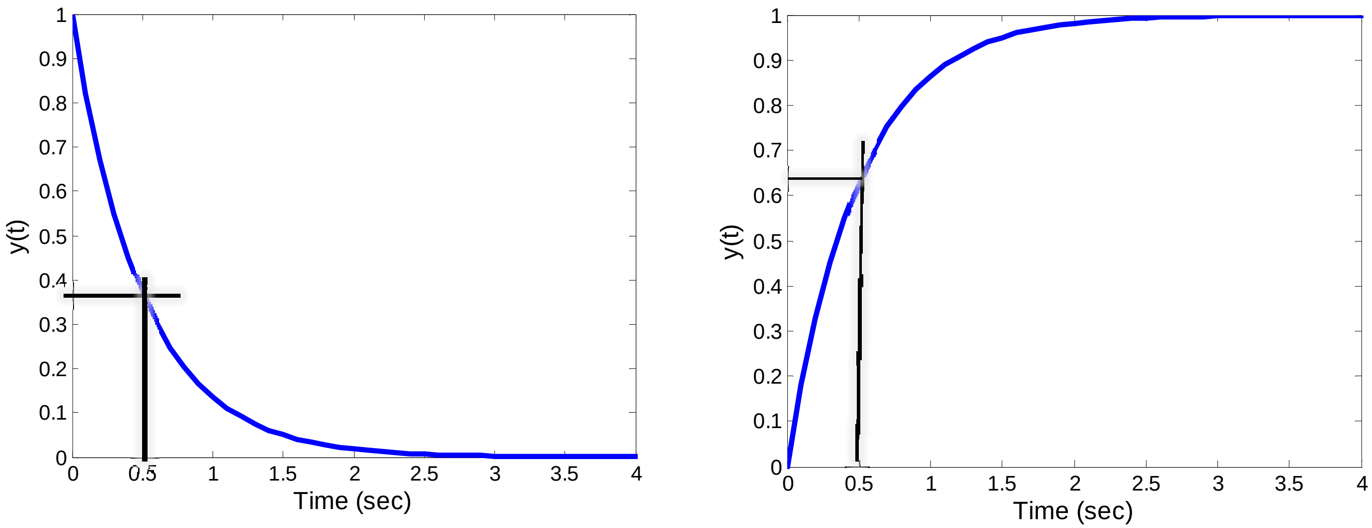
STEADY-STATE

$$\frac{K}{a}$$

TRANSIENT

$$\left(y(0)-\frac{K}{a}\right)e^{-at}, \quad t\geq 0$$

Time Constant



TIME CONSTANT – time, τ, for exponential transient to decay to $e^{-1} \approx 0.37$ of its initial value (or 63% to its final value)

Sample Problems

Pause

Summary

- Discussed how various physical phenomena are modeled by differential equations
- Showed the solution to a generic first-order differential equation with a constant input and initial condition
- Introduced the transient and steady-state responses
- Showed how to sketch the response and plot the time constant

Next Lesson

- Solve RC circuit equations and plot responses using this generic method