Linear Circuits

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An introduction to linear electric circuit elements and a study of circuits containing such devices.

Inductors

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- *Present how inductors work in a system*
- *Identify behavior in DC circuits*
- *Graphically represent the relationships between current, voltage, power, and energy*

Previous Class

- Behavior of individual inductors
- Meaning of inductance

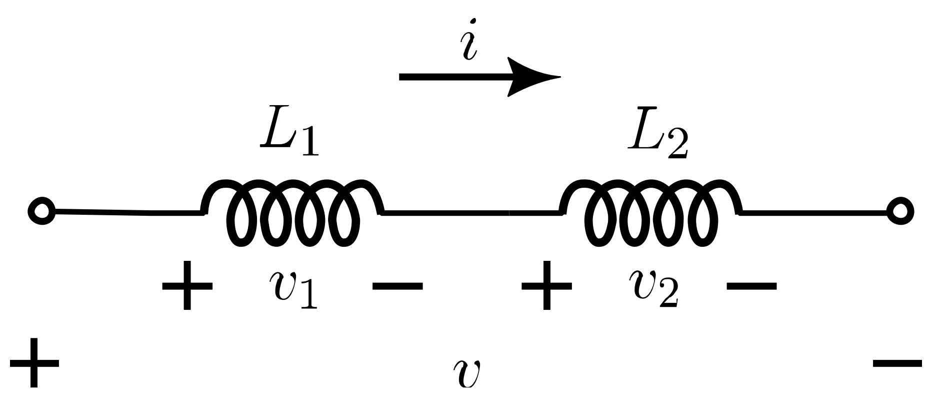
Module 3: Reactive Circuits

- Capacitors
- Inductors
- First-order differential equations
- RC Circuits
- RL Circuits
- Second-order differential equations
- RLC Circuits

Learning Objectives

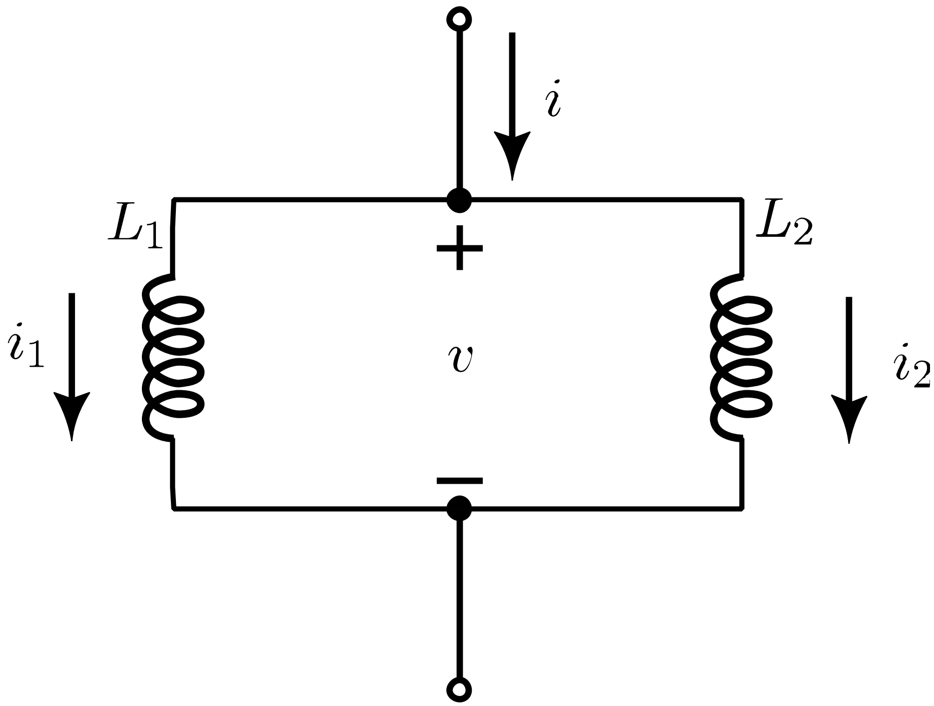
- Analyze inductors in series/parallel
- Analyze DC circuits with inductors
- Calculate the energy in an inductor
- Sketch current/voltage/power/energy curves

Inductors in Series



$$
\begin{aligned}
v &= v_1 + v_2 \\
&= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} \\
&= (L_1 + L_2) \frac{di}{dt}
\end{aligned}
$$

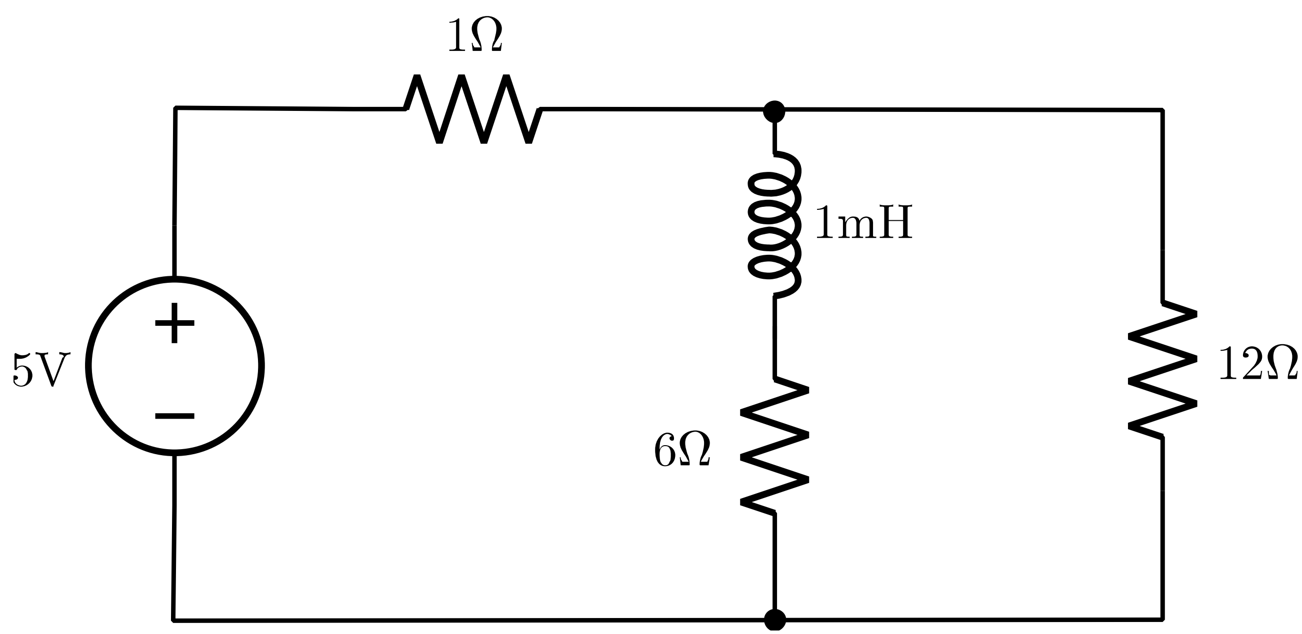
Inductors in Parallel



$$
\begin{aligned}
i &= i_1 + i_2 \\
&= \frac{1}{L_1}\int_{t_0}^{t} v(\tau)d\tau + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v(\tau)d\tau + i_2(t_0) \\
&= \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_{t_0}^{t} v(\tau)d\tau + i_1(t_0) + i_2(t_0)
\end{aligned}
$$

$$
v(t) = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1}\frac{di(t)}{dt}
$$

Behavior in DC Circuits



Stored Energy

$$p(t) = i(t)v(t)$$

$$w(t) = \int_{t_0}^{t} p(\tau)d\tau + w(t_0)$$

$$v = L\frac{di}{dt}$$

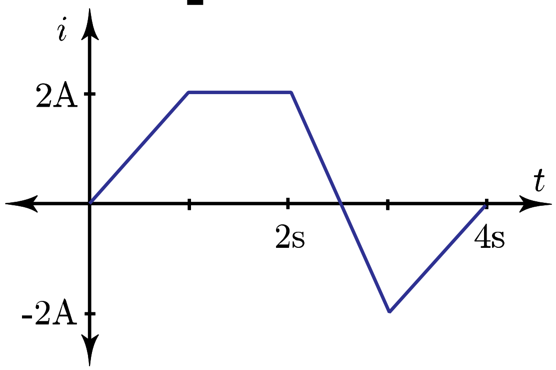
$$w = \int_{t_0}^{t} Li(\tau)\frac{di(\tau)}{d\tau}d\tau + w(t_0)$$

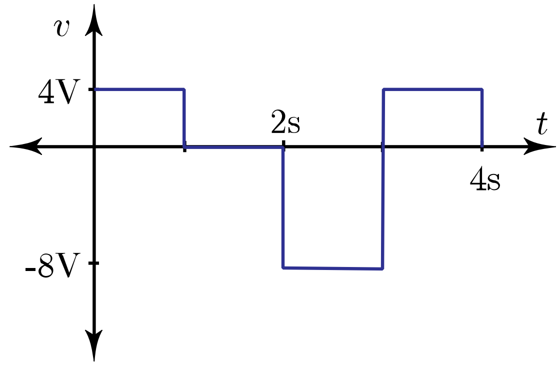
$$w = \int_{\mathrm{i}(t_0)}^{\mathrm{i}(t)} Lidi + w(t_0)$$

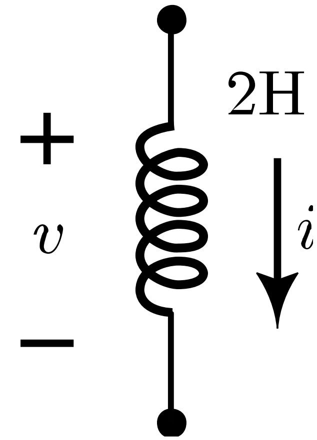
$$w = \frac{1}{2}Li^2(t)\Big|_{\mathrm{i}(t_0)}^{\mathrm{i}(t)} + w(t_0)$$

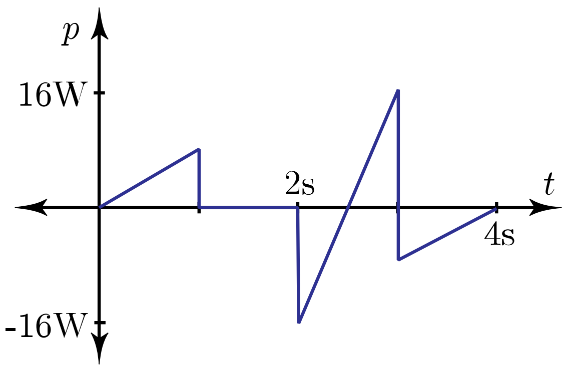
$$w = \frac{1}{2}Li^2(t)$$

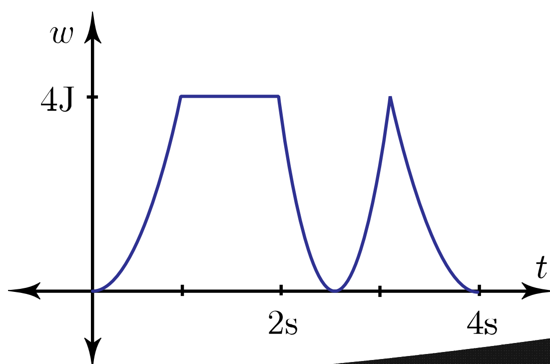
Graphs











Summary

- Calculated inductance for inductors in parallel/series configurations
- Identified how inductors in DC circuits behave like short circuits
- Derived an equation for the energy stored by an inductor as a magnetic field
- Showed graphically the relationships between voltage/current/power/energy in inductors

Next Class

- How to solve first-order linear differential equations
- Apply this math to see how systems with reactive elements adapt to system changes