Linear Circuits

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An introduction to linear electric circuit elements and a study of circuits containing such devices.

Capacitors

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- *Present how capacitors work in a system*
- *Identify behavior in DC circuits*
- *Graphically represent the relationships between current, voltage, power, and energy*

Previous Class

- Behavior of isolated capacitors
- Meaning of capacitance

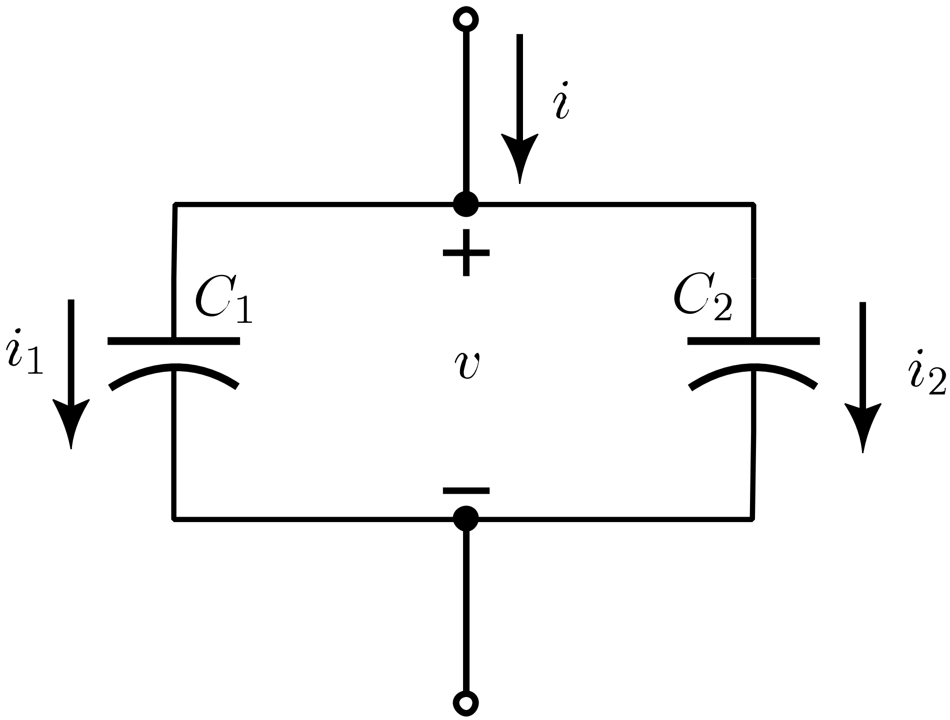
Module 3: Reactive Circuits

- Capacitors
- Inductors
- First-order differential equations
- RC Circuits
- RL Circuits
- Second-order differential equations
- RLC Circuits

Lesson Objectives

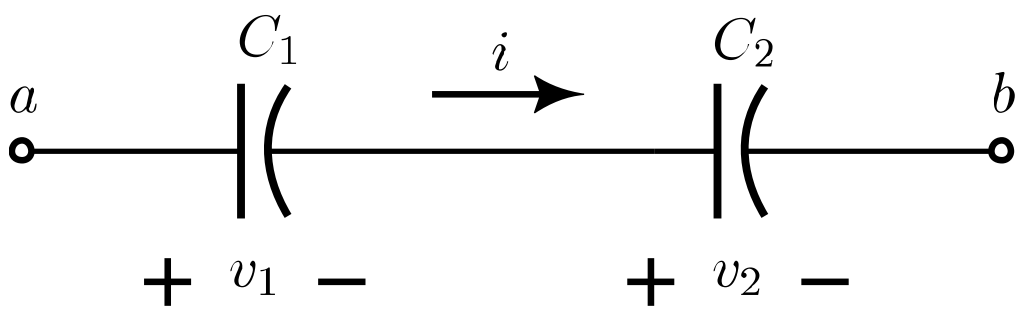
- Analyzing capacitors in series/parallel
- Analyze DC circuits with capacitors
- Calculate energy in a capacitor
- Sketch current/voltage/power/energy curves

Capacitors in Parallel



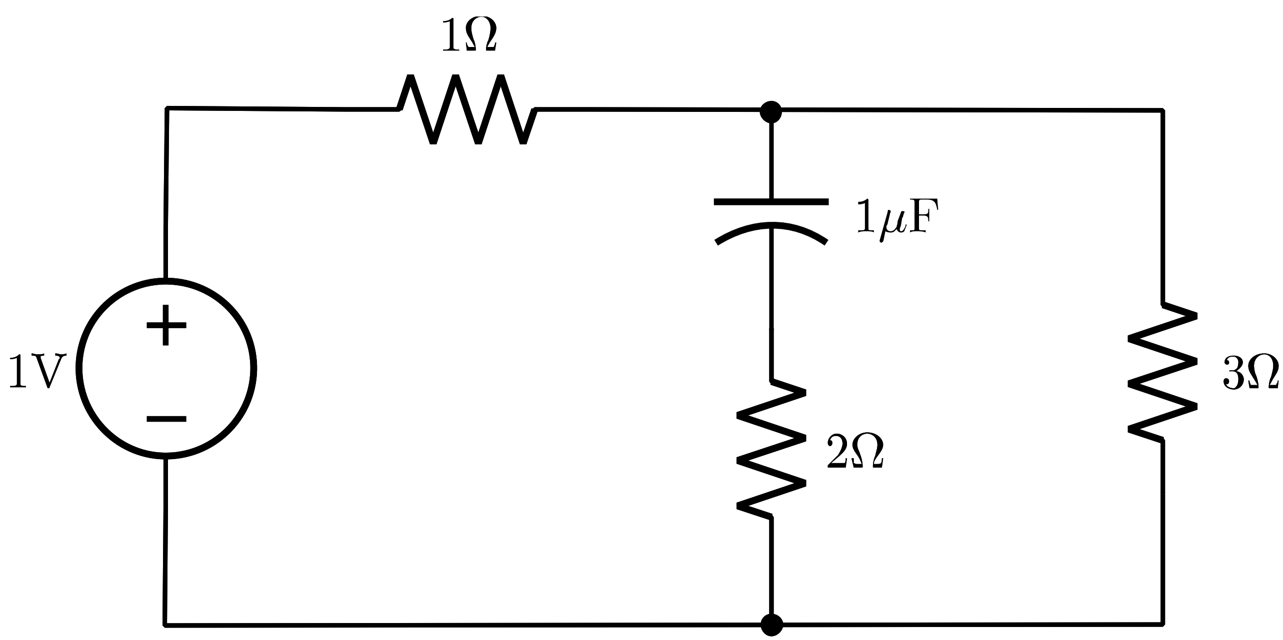
$$
\begin{aligned}
i &= i_1 + i_2 \\
&= C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} \\
&= (C_1 + C_2) \frac{dv}{dt}
\end{aligned}
$$

Capacitors in Series



$$\begin{aligned}
v_{ab} &= v_1 + v_2 \\
&= \frac{1}{C_1}\int_0^t i(\tau)d\tau + v_1(0) + \frac{1}{C_2}\int_0^t i(\tau)d\tau + v_2(0) \\
&= \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i(\tau)d\tau + v_1(0) + v_2(0) \\
\frac{dv_{ab}}{dt} &= \left(\frac{1}{C_1} + \frac{1}{C_2}\right) i(t) \\
i(t) &= \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} \frac{dv_{ab}}{dt}
\end{aligned}$$

Behavior in DC Circuits



Stored Energy

$$p(t) = i(t)v(t)$$

$$w(t) = \int_{t_0}^{t} p(\tau)d\tau + w(t_0)$$

$$i = C\frac{dv}{dt}$$

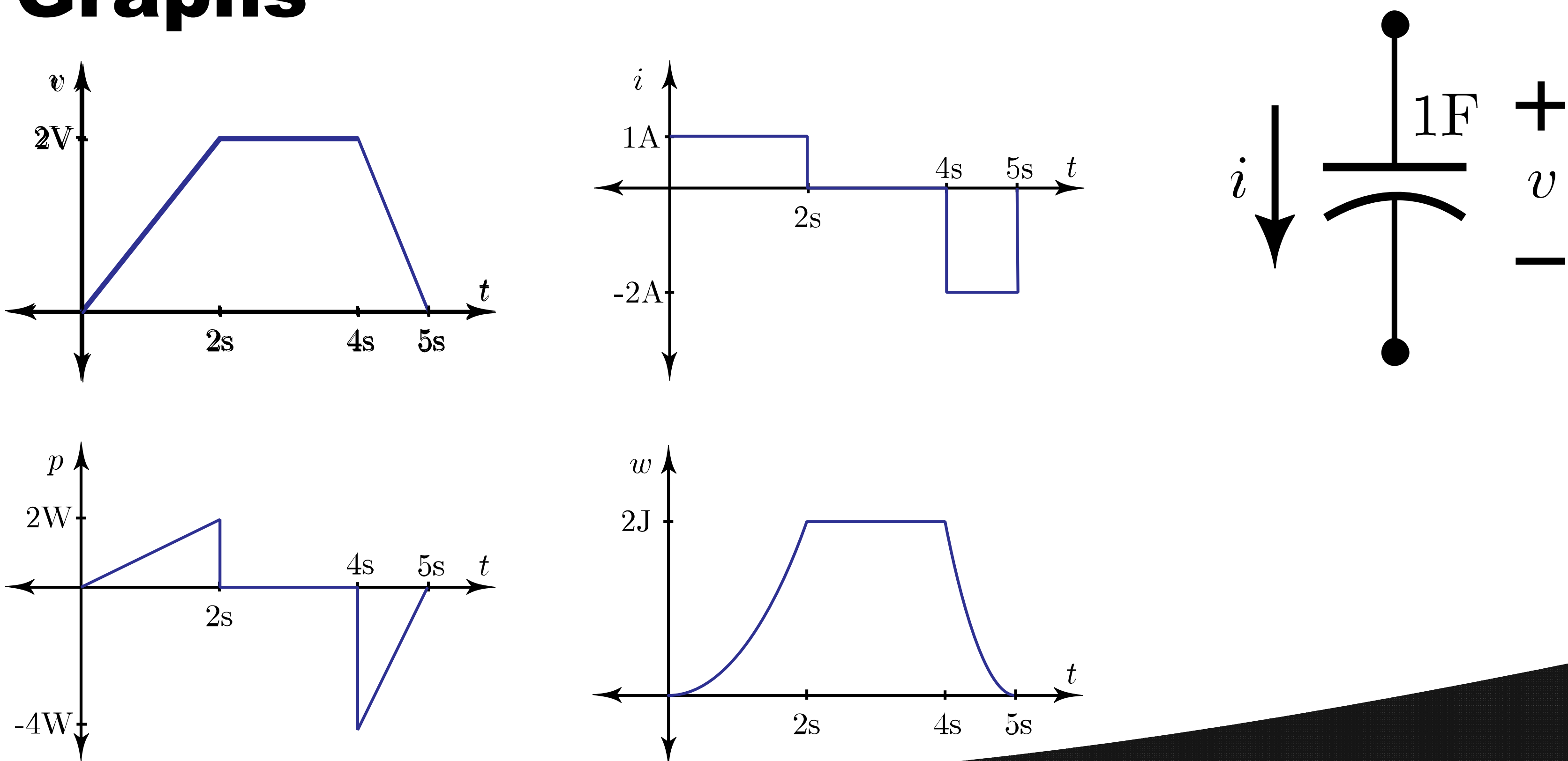
$$w = \int_{t_0}^{t} Cv(\tau)\frac{dv(\tau)}{d\tau}d\tau + w(t_0)$$

$$w = \int_{\mathrm{v}(t_0)}^{\mathrm{v}(t)} Cvdv + w(t_0)$$

$$w = \frac{1}{2}Cv^2(t)\Big|_{v=\mathrm{v}(t_0)}^{\mathrm{v}(t)} + w(t_0)$$

$$w = \frac{1}{2}Cv^2(t)$$

Graphs



Summary

- Calculated capacitance for capacitors in parallel/series configurations
- Identified how capacitors in DC circuits behave like open circuits
- Derived an equation for the energy stored by a capacitor as an electric field
- Showed graphically the relationships between voltage/current/power/energy in capacitors

Next Class

- Considerations for working with real capacitors
- Magnetic fields and current
- Inductance