Discussion of Common Problems – 2

Kleene Star

Paths in DFA

Challenge Problem 1

Kleene Star

- Example
  - **1***
  - Does NOT mean an infinite long string of 1's
  - L(**1***)={ε,1,11,111,1111,…}
  - Each element in L(**1***) has finite length

Infiniteness

- Infinite objects are important in mathematics (set of integers, a line containing infinite number of points)
- In a computational model, you can never read such object into a computer
  - Unless you can represent it in finite form
    - Regular expression for a regular language
    - Triple (a,b,c) for a line ax+by+c=0

What is "Last"

- A DFA which accepts all strings of 0's and 1's except those whose last character is 1
- What about the string **1**?
- The last character of any string $a_1a_2...a_n$ is just a_n ($n \geq 1$).
- So **1** is not accepted by this DFA
- ε has no "last" character, so it IS accepted by this DFA

Convert DFA into RE

- Review
  - k-Path Induction
  - Let R_{ij}^{k} be the regular expression for the set of labels of k-paths from state i to j

k-Path Inductive Case

- A k-path from i to j either:
  1. Never goes through state k, or
  2. Goes through k one or more times.

$$R_{ij}^{k} = R_{ij}^{k-1} + R_{ik}^{k-1}(R_{kk}^{k-1})^{*} R_{kj}^{k-1}.$$

Doesn't go through k

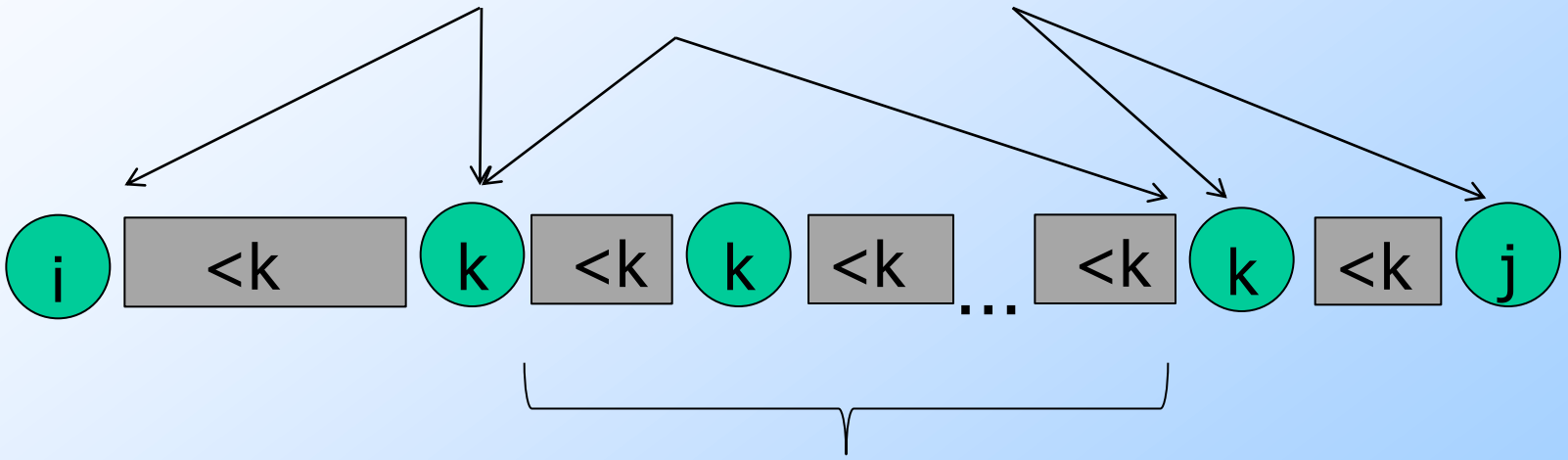
Goes from i to k the first time

Zero or more times from k to k

Then, from k to j

Zoom in

- What is $R_{ik}^{k-1}(R_{kk}^{k-1})^* R_{kj}^{k-1}$?



R_{kk}^{k}

$(R_{kk}^{k-1})^* = R_{kk}^{k}$

Challenge Problem 1

- L is a language with alphabet {0, 1, 2}
- L contains no strings that have any three consecutive 0's, any three consecutive 1's, or any three consecutive 2's.
  - e.g. 11000220 is not in L
- Prove that L is regular and give a DFA for L.

Challenge Problem 1

- The complement of L has a regular expression:
  (0+1+2)*(000+111+222)(0+1+2)*
  - All strings that DO contain three consecutive 0's or 1's or 2's.

- Regular languages are closed under complement

Challenge Problem 1

- DFA for L
  - The state represents the run of the same symbol that appears at the end of the string
    - Start state S
    - State a_0, a_{00}, a_1, a_{11}, a_2, a_{22}
    - Dead state D
- Example:
  - 012011 should get to a_{11}

Challenge Problem 1

	0	1	2
→*S	a_0	a_1	a_2
*a_0	a_{00}	a_1	a_2
*a_1	a_0	a_{11}	a_2
*a_2	a_0	a_1	a_{22}
*a_{00}	D	a_1	a_2
*a_{11}	a_0	D	a_2
*a_{22}	a_0	a_1	D
D	D	D	D