Algorithms: Design and Analysis, Part II

Exact Algorithms for NP-Complete Problems

The Vertex Cover Problem

The Vertex Cover Problem

Input: an undirected graph $G=(V,E)$.

Goal: compute a minimum-cardinality vertex cover – a subset $S \subseteq V$ that contains at least one endpoint of each edge of G.

Quiz

Question: what is the minimum size of a vertex cover of a star graph with n vertices and a clique with n vertices, respectively?

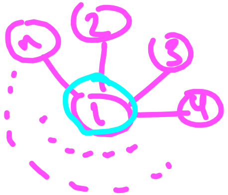
(A) 1 and $n-1$

(B) 1 and n

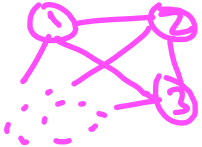
(C) 2 and $n-1$

(D) $n-1$ and n

Fact: in general, Vertex Cover is an NP-Complete problem.



star graph



clique

Strategies for NP-Complete Problems

1. identify computationally tractable special cases
   - trees [application of dynamic programming – try it!]
   - bipartite graphs [application of the maximum flow problem]
   - when the optimal solution is "small" ($\approx \log n$ or less)
2. heuristics (e.g., via suitable greedy algorithms)
3. exponential time but better than brute-force search
   [coming up next]