All-Pairs Shortest Paths (APSP)

Problem Definition

Algorithms: Design and Analysis, Part II

Problem Definition

Input: directed graph $G = (V, E)$ with edge costs c_e for each $e \in E$. [no distinguished source vertex]

Goal: either

(A) Compute the length of a shortest $u \to v$ path for all pairs of vertices $u, v \in V$

OR

(B) correctly report that G contains a negative cycle

Quiz

Question: How many invocations of a single-source shortest-path subroutine are needed to solve the all-pairs shortest path problem? [n = # of vertices]

(A) 1

(B) n-1

(C) n

(D) n^2

Running time (nonnegative edge costs)

n·Dijkstra = $O(nm \log n)$ → $O(n^2 \log n)$ if $m = O(n)$

→ $O(n^3 \log n)$ if $m = O(n^2)$

Running time (general edge costs)

n·Bellman-Ford = $O(n^2 m)$ → $O(n^3)$ if $m = O(n)$

→ $O(n^4)$ if $m = O(n^2)$