Minimum Spanning Trees

Correctness of Kruskal's Algorithm

Algorithms: Design and Analysis, Part II

Correctness of Kruskal (Part I)

Theorem: Kruskal's algorithm is correct.

Proof: Let T^* = output of Kruskal's algorithm on input graph G.

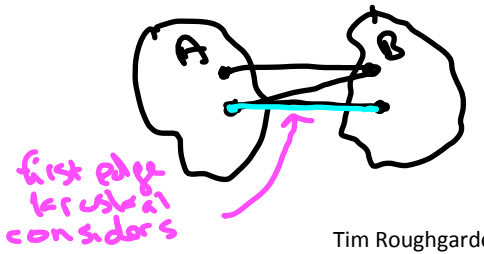
(1) Clearly T^* has no cycles

(2) T^* is connected. Why?

(2a) By Empty Cut Lemma, only need to show that T^* crosses every cut.

(2b) Fix a cut (A, B). Since G connected, at least one of its edges crosses (A, B).

Key point: Kruskal will include first edge crossing (A, B) that it sees [by Lonely Cut Corollary, cannot create cycle]



Correctness of Kruskal (Part II)

(3) every edge of T^* justified by the Cut Property (implies T^* is the MST)

Reason for (3):

Consider iteration where edge (u,v) added to current set T.

Since $T \cup \{(u,v)\}$ has no cycle, T has no u-v path.

$\Rightarrow$ Empty cut (A,B) separating u and v (as in Proof of Empty Cut Lemma)

$\Rightarrow$ by (2b), no edges crossing (A,B) were previously considered by Kruskal's algorithm

$\Rightarrow$ (u,v) is the first (+ hence the cheapest!) edge crossing (A,B)

$\Rightarrow$ (u,v) justified by the Cut Property

QED!