Greedy Algorithms

A Scheduling Application: Correctness Proof Part I

Algorithms: Design and Analysis, Part II

Correctness Claim

Claim: Algorithm #2 (order jobs according to decreasing ratios w_j/ℓ_j) is always correct.

Proof: by an Exchange Argument.

Plan: Fix arbitrary input of n jobs.
Will proceed by contradiction.
Let σ = greedy schedule, σ^* = optimal schedule. (with σ^* better than σ^*)
will produce schedule even better than σ^*, contradicting purported optimality of σ^*.

Correctness Proof

Assume: all w_j/ℓ_j's distinct.

Assume: [just by renaming jobs] $\frac{w_1}{\ell_1} > \frac{w_2}{\ell_2} > \cdots > \frac{w_n}{\ell_n}$

Thus: greedy schedule σ is just $1, 2, 3, \ldots, n$.

Thus: if optimal schedule $\sigma^* \neq \sigma$, then there are consecutive jobs i, j with $i > j$.

[only schedule where indices always go up is $1, 2, 3, \ldots, n$]

Correctness Proof (con'd)

So far: ① $\frac{w_1}{\ell_1} > \frac{w_2}{\ell_2} > \cdots > \frac{w_n}{\ell_n}$

② in optimal o^*, $\exists$ consecutive jobs i, j with $i > j$.

Thought experiment: suppose we exchange order of i, j in o^* (leaving other jobs unchanged):