Design and Analysis of Algorithms I

Data Structures

Heaps: Some Implementation Details

Heap: Supported Operations

- A container for objects that have keys
- Employer records, network edges, events, etc.

Insert: add a new object to a heap.

Running time : O(log(n))

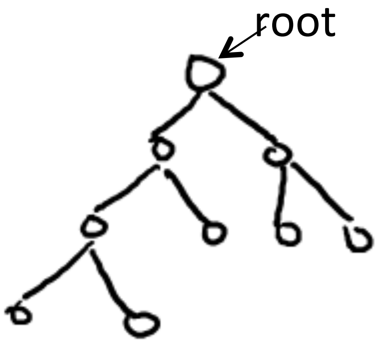
Equally well, EXTRACT MAX

Extract-Min: remove an object in heap with a minimum key value. [ties broken arbitrarily]

Running time : O(log n)   [n = # of objects in heap]

Also : HEAPIFY (n batched Inserts in O(n) time), DELETE(O(log(n)) time)

The Heap Property

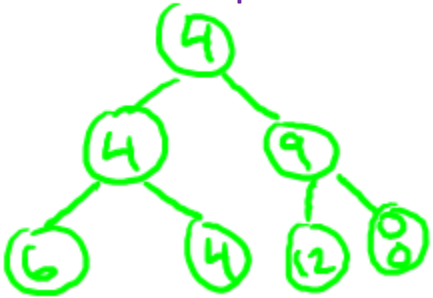


Conceptually : think of a heap as a tree.
-rooted, binary, as complete as possible

Heap Property: at every node x,
Key[x] <= all keys of x's children

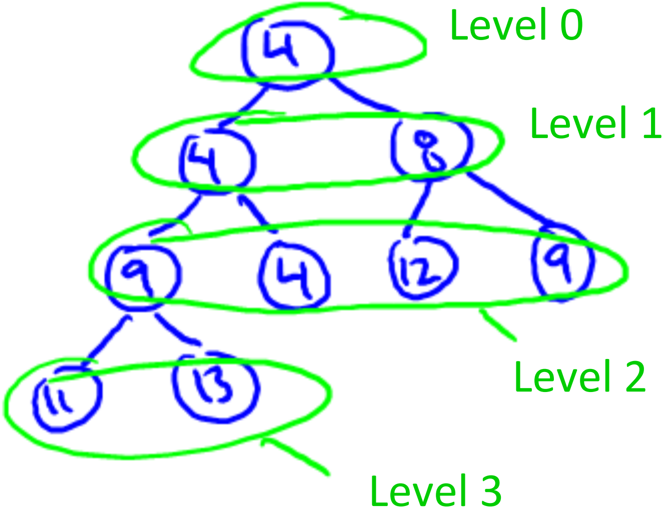
Consequence : object at root must have minimum key value

A heap



alternatively

Array Implementation



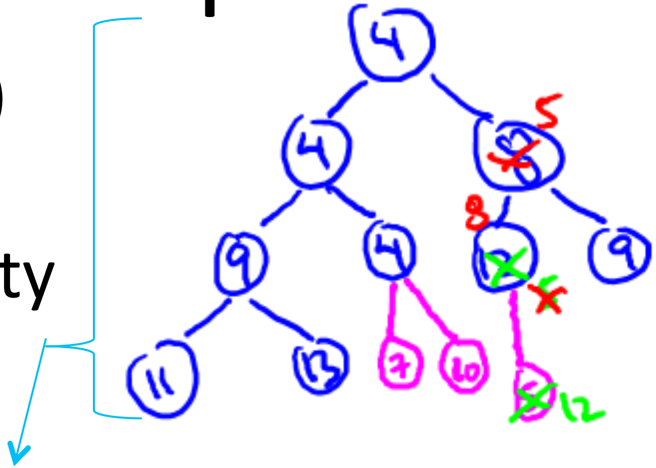
Note : parent(i) = i/2 if i even

= [i/2] if i odd

i.e., round down

and children of i are 2i, 2i+1

Insert and Bubble-Up



~ log2n levels (n = # of items in heap)

<u>Implementation of Insert</u> (given key k)

<u>Step 1</u>: stick k at end of last level.

<u>Step 2</u> : Bubble-Up k until heap property is restored (i.e., key of k's parent Is <= k)

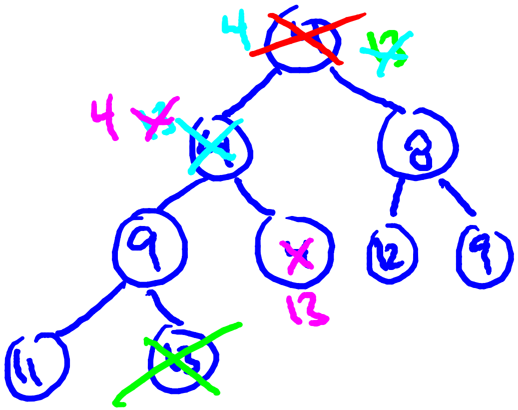
<u>Check</u> : 1.) bubbling up process must stop, with heap property restored

2.) runtime = O(log(n))

Extract-Min and Bubble-Down

Implementation of Extract-Min
1. Delete root
2. Move last leaf to be new root.
3. Iteratively Bubble-Down until heap property has been restored

   [always swap with smaller child!]



Check : 1.) only Bubble-Down once per level, halt with a heap

2.) run time = O(log(n))