Design and Analysis
of Algorithms I

Linear-Time Selection

Deterministic Selection (Analysis II)

Rough Recurrence (Revisited)

Let T(n) = maximum running time of Dselect on an input array of length n.

There is a constant c >= 1 such that :

1. T(1) = 1
2. T(n) <= c*n + T(n/5) + T(?)

<= 7n/10 by Key Lemma

sorting the groups partition

recursive call in line 3

recursive call in line 6 or 7

Rough Recurrence (Revisited)

$T(1) = 1$, $T(n) <= cn + T(n/5) + T(7n/10)$

Constant c>=1

Note : different-sized subproblems => can't use Master Method!

Strategy : "hope and check"

Hope : there is some constant a [independent of n]
Such that $T(n) <= an$ for all $n >=1$
[if true, then $T(n) = O(n)$ and algorithm is linear time]

Analysis of Rough Recurrence

T(1) = 1 ; T(n) <= cn + T(n/5) + T(7n/10)

Constant c>=1

Claim : Let a = 10c

Then T(n) <= an for all n >= 1

=> Dselect runs in O(n) time

Proof : by induction on n

Base case : T(1) = 1 <= a*1 (since a >= 1)

Inductive Step : [n > 1]

Inductive Hypothesis : $T(k) \leq ak \; \forall \; k < n$

We have $T(n) \leq cn + T(n/5) + T(7n/10)$

GIVEN

$\leq cn + a(n/5) + a(7n/10)$

IND HYP

$= n(c + 9a/10) = an$

Choice of a

Q.E.D.

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