



Design and Analysis  
of Algorithms I

# Master Method Intuition for the 3 Cases

# How To Think About (\*)

Our upper bound on the work at level j:

$$cn^d \times \left(\frac{a}{b^d}\right)^j$$




## Interpretation

a = rate of subproblem proliferation (RSP)

b<sup>d</sup> = rate of work shrinkage (RWS)

(per subproblem)

Which of the following statements are true?  
(Check all that apply.)

-  ☐ If  $RSP < RWS$ , then the amount of work is decreasing with the recursion level  $j$ .
-  ☐ If  $RSP > RWS$ , then the amount of work is increasing with the recursion level  $j$ .
- ☐ No conclusions can be drawn about how the amount of work varies with the recursion level  $j$  unless  $RSP$  and  $RWS$  are equal.
-  ☐ If  $RSP$  and  $RWS$  are equal, then the amount of work is the same at every recursion level  $j$ .

# Intuition for the 3 Cases

Upper bound for level  $j$ :  $cn^d \times (\frac{a}{b^d})^j$

1.  $RSP = RWS \Rightarrow$  Same amount of work each level (like Merge Sort)  
[expect  $O(n^d \log(n))$ ]
2.  $RSP < RWS \Rightarrow$  less work each level  $\Rightarrow$  most work at the root  
[might expect  $O(n^d)$ ]
3.  $RSP > RWS \Rightarrow$  more work each level  $\Rightarrow$  most work at the leaves  
[might expect  $O(\# \text{ leaves})$ ]