Design and Analysis
of Algorithms I

Master Method

Intuition for the 3 Cases

How To Think About (*)

Our upper bound on the work at level j:

$$cn^d \times \left(\frac{a}{b^d}\right)^j$$

Interpretation

a = rate of subproblem proliferation (RSP)

b^d = rate of work shrinkage (RWS) (per subproblem)

Which of the following statements are true?
(Check all that apply.)

- [ ] If RSP < RWS, then the amount of work is decreasing with the recursion level j.
- [ ] If RSP > RWS, then the amount of work is increasing with the recursion level j.
- [ ] No conclusions can be drawn about how the amount of work varies with the recursion level j unless RSP and RWS are equal.
- [ ] If RSP and RWS are equal, then the amount of work is the same at every recursion level j.

Intuition for the 3 Cases

Upper bound for level j: $cn^d \times (\frac{a}{b^d})^j$

(1) RSP = RWS => Same amount of work each level (like Merge Sort) [expect $O(n^d \log n)$]

(2) RSP < RWS => less work each level => most work at the root [might expect $O(n^d)$]

(3) RSP > RWS => more work each level => most work at the leaves [might expect $O(\#\text{leaves})$]

Tim Roughgarden

The Master Method

Our upper bound on the work at level j:

$$cn^d \times \left(\frac{a}{b^d}\right)^j$$