



Design and Analysis
of Algorithms I

Master Method

The Precise Statement

The Master Method

Cool feature: a "black box" for solving recurrences.

Assumption: all subproblems have equal size.

Recurrence Format

① Base case: $T(n) \leq$ a constant
for all sufficiently small n .

② For all larger n :

$$T(n) \leq aT\left(\frac{n}{b}\right) + O(n^d)$$

where

a = number of recursive calls (≥ 1)

b = input size shrinkage factor (> 1)

d = exponent in running time of "combine step" (≥ 0)

[a, b, d independent of n]

The Master Method

$$T(n) = \begin{cases} O(n^d \log n) & \text{if } a = b^d \\ O(n^d) & \text{if } a < b^d \\ O(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

The Master Method

$$T(n) = \begin{cases} O(n^d \log n) & \text{if } a = b^d & \text{(Case 1)} \\ O(n^d) & \text{if } a < b^d & \text{(Case 2)} \\ O(n^{\log_b a}) & \text{if } a > b^d & \text{(Case 3)} \end{cases}$$

base doesn't matter (only changes leading constant)

base matters!