



Design and Analysis  
of Algorithms I

# Master Method

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## The Precise Statement

# The Master Method

Cool feature: a "black box" for solving recurrences.

Assumption: all subproblems have equal size.

# Recurrence Format

① Base case:  $T(n) \leq \text{a constant}$   
for all sufficiently small  $n$ .

② For all larger  $n$ :

$$T(n) \leq aT\left(\frac{n}{b}\right) + O(n^d)$$

where

$a$  = number of recursive calls ( $\geq 1$ )

$b$  = input size shrinkage factor ( $> 1$ )

$d$  = exponent in running time of "combine step" ( $\geq 0$ )

$[a, b, d \text{ independent of } n]$

# The Master Method

$$T(n) = \begin{cases} O(n^d \log n) & \text{if } a = b^d \\ O(n^d) & \text{if } a < b^d \\ O(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

# The Master Method

$$T(n) = \begin{cases} O(n^d \log n) & \text{if } a = b^d \quad (\text{Case 1}) \\ O(n^d) & \text{if } a < b^d \quad (\text{Case 2}) \\ O(n^{\log_b a}) & \text{if } a > b^d \quad (\text{Case 3}) \end{cases}$$

*base doesn't matter (only changes leading constant)*

*base matters!*