Design and Analysis of Algorithms I

Asymptotic Analysis

Additional Examples

Example #1

<u>Claim</u>: $2^{n+10} = O(2^n)$.

<u>Proof</u>: need to pick constants c, n_0 such that

$$(*) \quad 2^{n+10} \le c \cdot 2^n \quad \forall n \ge n_0$$

<u>Note</u>: $2^{n+10} = 2^{10} \cdot 2^n = (1024) \cdot 2^n$

<u>So</u>: if we choose $c = 1024$, $n_0 = 1$, then $(*)$ holds.

QED!

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Example #2

Claim: 2^{10n} is not $O(2^n)$.

Proof: by contradiction. If $2^{10n} = O(2^n)$, then $\exists$ constants $c, n_0 > 0$ such that

$$2^{10n} \le c \cdot 2^n \quad \forall n \ge n_0.$$

But then [cancelling 2^n]:

$$2^{9n} \le c \quad \forall n \ge n_0$$

which is certainly false. QED!

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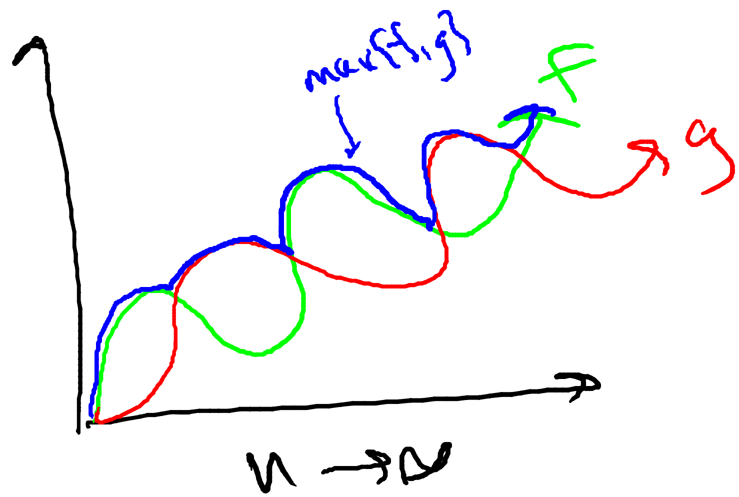
Example #3

Claim: for every pair of (positive) functions $f(n), g(n)$, $\max\{f,g\} = \Theta(f(n)+g(n))$.



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Example #3 (continued)

Proof: $[\max\{f,g\} = \Theta(f(n)+g(n))]$

For every n, we have

$$\max\{f(n),g(n)\} \le f(n)+g(n)$$

and

$$\cancel{2}\max\{f(n),g(n)\} \ge \tfrac{1}{2}(f(n)+g(n))$$

Thus: $\frac{1}{2}(f(n)+g(n)) \le \max\{f(n),g(n)\} \le f(n)+g(n)$.

for all $n \ge 1$.

$\Rightarrow \max\{f,g\} = \Theta(f(n)+g(n))$. [where $n_0 = 1$, $c_1 = 1/2$, $c_2 = 1$]

QED!

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