Design and Analysis of Algorithms I

Asymptotic Analysis

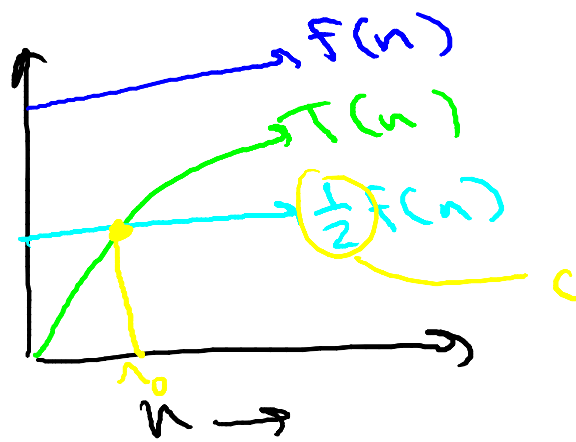
Big-Oh: Relatives (Omega & Theta)

Omega Notation

<u>Definition</u> : $T(n) = \Omega(f(n))$
If and only if there exist constants c, n_0 such that

$$T(n) \geq c \cdot f(n) \quad \forall n \geq n_0 .$$

Picture



$$T(n) = \Omega(f(n))$$

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Theta Notation

<u>Definition</u> : $T(n) = \theta(f(n))$ if and only if
$T(n) = O(f(n))$ and $T(n) = \Omega(f(n))$

<u>Equivalent</u> : there exist constants c_1, c_2, n_0 such that

$$c_1 f(n) \leq T(n) \leq c_2 f(n)$$

$$\forall n \geq n_0$$

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Let $T(n) = \frac{1}{2}n^2 + 3n$. Which of the following statements are true ? (Check all that apply.)

- ☐ $T(n) = O(n)$.
- ☐ $T(n) = \Omega(n)$. $[n_0 = 1, c = 1/2]$
- ☐ $T(n) = \Theta(n^2)$. $[n_0 = 1, c_1 = 1/2, c_2 = 4]$
- ☐ $T(n) = O(n^3)$. $[n_0 = 1, c = 4]$

Little-Oh Notation

Definition : $T(n) = o(f(n))$ if and only if for all constants c>0, there exists a constant n_0 such that

$$T(n) \leq c \cdot f(n) \quad \forall n \geq n_0$$

Exercise : $\forall k \geq 1, n^{k-1} = o(n^k)$

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Where Does Notation Come From?

- "On the basis of the issues discussed here, I propose that members of SIGACT, and editors of compter science and mathematics journals, adopt the O, Ω, and Θ notations as defined above, unless a better alternative can be found reasonably soon".

-D. E. Knuth, "Big Omicron and Big Omega and Big Theta", SIGACT News, 1976. Reprinted in "Selected Papers on Analysis of Algorithms."