Asymptotic Analysis

Big-Oh: Basic Examples

Design and Analysis of Algorithms I

Example #1

Claim: if $T(n) = a_k n^k + \cdots + a_1 n + a_0$ then $T(n) = O(n^k)$.

Proof: Choose $n_0 = 1$ and $c = |a_k| + |a_{k-1}| + \cdots + |a_1| + |a_0|$.

Need to show that $\forall n \geq 1$, $T(n) \leq c \cdot n^k$.

We have, for every $n \geq 1$,

$$T(n) \leq |a_k| n^k + \cdots + |a_1| n + |a_0|$$
$$\leq |a_k| n^k + \cdots + |a_1| n^k + |a_0| n^k$$
$$= c \cdot n^k,$$

Q.E.D.!

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Example #2

Claim: for every $k \geq 1$, n^k is not $O(n^{k-1})$.

Proof: by contradiction. Suppose $n^k = O(n^{k-1})$.
Then $\exists$ constants $c, n_0 > 0$ such that

$$n^k \leq c \cdot n^{k-1} \quad \forall n \geq n_0.$$

But then [cancelling n^{k-1} from both sides]:

$$n \leq c \quad \forall n \geq n_0$$

which is clearly false [contradiction]. QED!