

Design and Analysis of Algorithms I

Introduction Why Study Algorithms?

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 - "Everyone knows Moore's Law a prediction made in 1965 by Intel cofounder Gordon Moore that the density of transistors in integrated circuits would continue to double every 1 to 2 years....in many areas, performance gains due to improvements in algorithms have vastly exceeded even the dramatic performance gains due to increased processor speed."
 - Excerpt from Report to the President and Congress: Designing a Digital Future, December 2010 (page 71).

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 - quantum mechanics, economic markets, evolution

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- fun

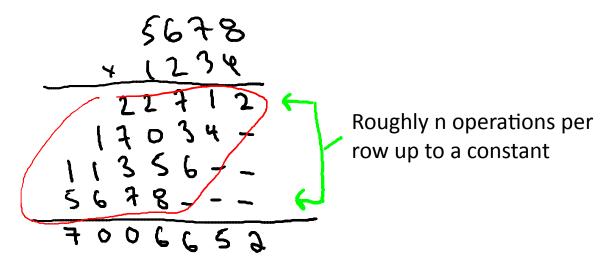
Integer Multiplication

Input: 2 n-digit numbers x and y

Output: product x*y

"Primitive Operation" - add or multiply 2 single-digit numbers

The Grade-School Algorithm



of operations overall ~ constant* $\,n^2\,$

The Algorithm Designer's Mantra

"Perhaps the most important principle for the good algorithm designer is to refuse to be content."

-Aho, Hopcroft, and Ullman, *The Design and Analysis of Computer Algorithms*, 1974

CAN WE DO BETTER?
[than the "obvious" method]

A Recursive Algorithm

Write
$$x = 10^{n/2}a + b$$
 and $y = 10^{n/2}c + d$

Where a,b,c,d are n/2-digit numbers.

Then
$$x.y = (10^{n/2}a + b)(10^{n/2}c + d)$$

= $(10^n ac + 10^{n/2}(ad + bc) + bd$ (*)

Idea: recursively compute ac, ad, bc, bd, then compute (*) in the obvious way

Karatsuba Multiplication

$$x.y = (10^n ac + 10^{n/2} (ad + bc) + bd)$$

- 1. Recursively compute ac
- 2. Recursively compute bd
- 3. Recursively compute (a+b)(c+d) = ac+bd+ad+bc

Gauss' Trick:
$$(3) - (1) - (2) = ad + bc$$

Upshot: Only need 3 recursive multiplications (and some additions)

Q : which is the fastest algorithm?