Quantum Mechanics & Quantum Computation

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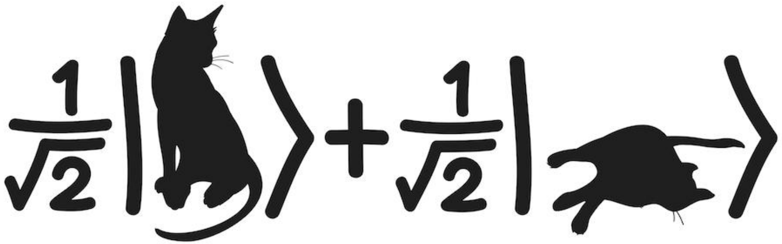
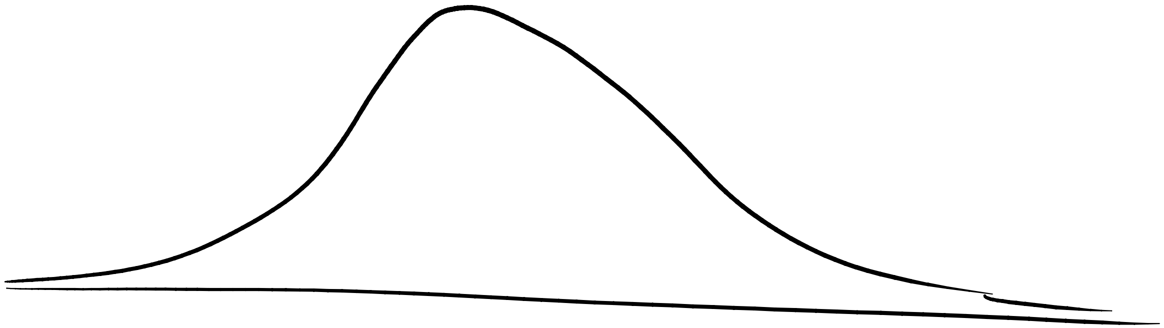
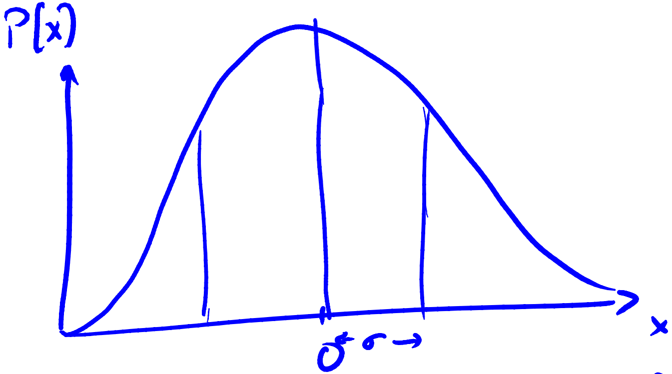


Lecture 9: Continuous quantum states, Schrödinger's equation, uncertainty principle

Uncertainty principle



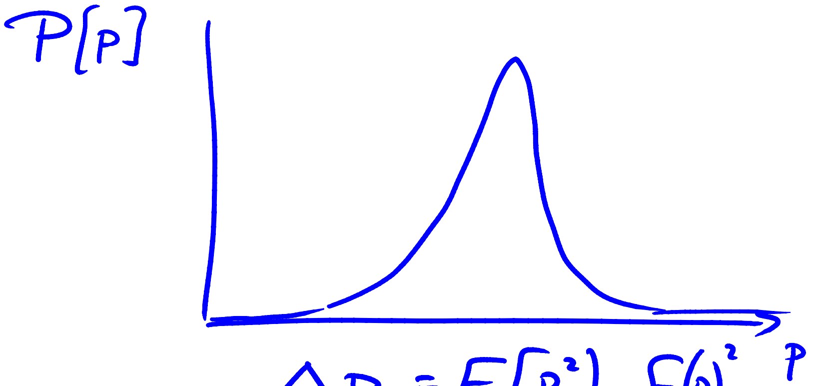
Measure its position:



$E(x) = 0$

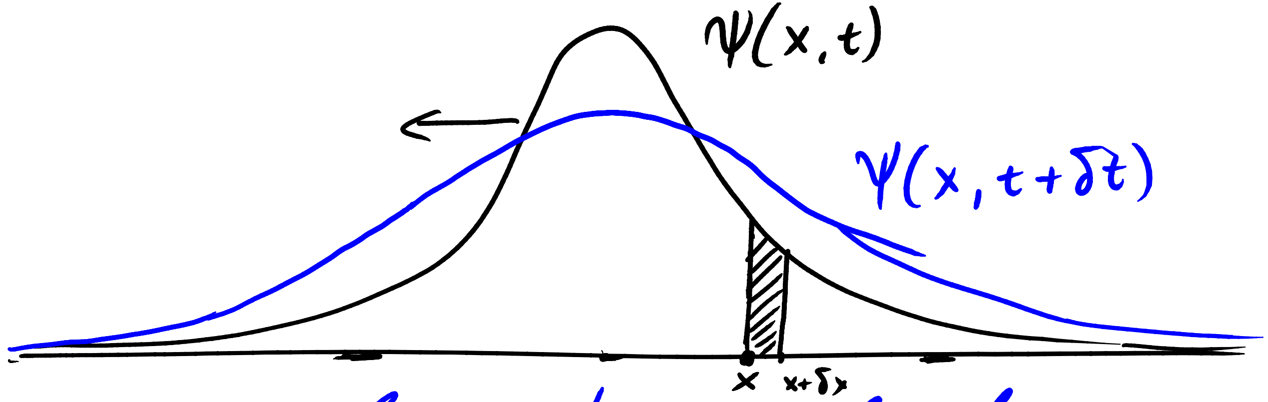
$$\Delta x = E[x^2] - E(x)^2 = \sigma^2$$

Measure its momentum



$$\Delta p = E[p^2] - E(p)^2$$

$$\Delta x \, \Delta p \geq \hbar/2$$



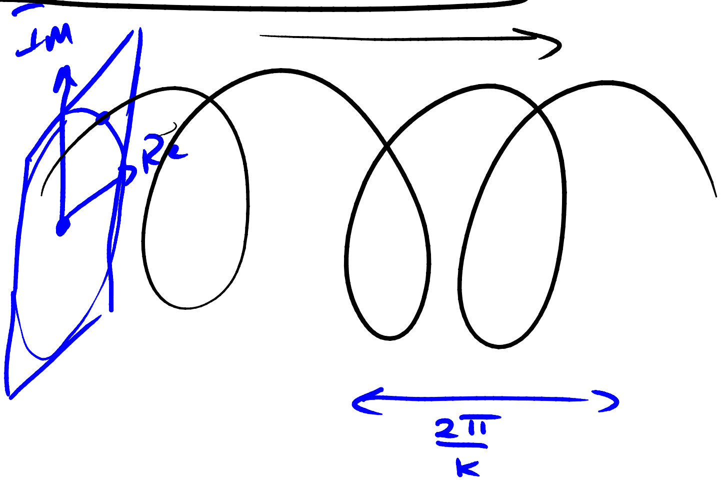
What is the velocity of the particle at time t?

$$\int_{x}^{x+\delta x} |\psi(x,t)|^2 \, dx$$

In a superposition of velocities.

$$\boxed{\psi(x,t=0) = e^{ikx}}$$

$$\psi(x) = \psi\left(x + \frac{2\pi}{k}\right)$$



velocity $= k$

period $= \frac{2\pi}{k}$

time $= 2\pi/k^2$

velocity $= \dfrac{2\pi/k}{2\pi/k^2} = k$

$$\psi(x,t) = e^{i(kx+\omega t)}$$

$$i\,i\omega\, \cancel{e^{i(kx+\omega t)}} = (ik)^2\, \cancel{e^{i(kx+\omega t)}}$$

$$\omega = k^2$$

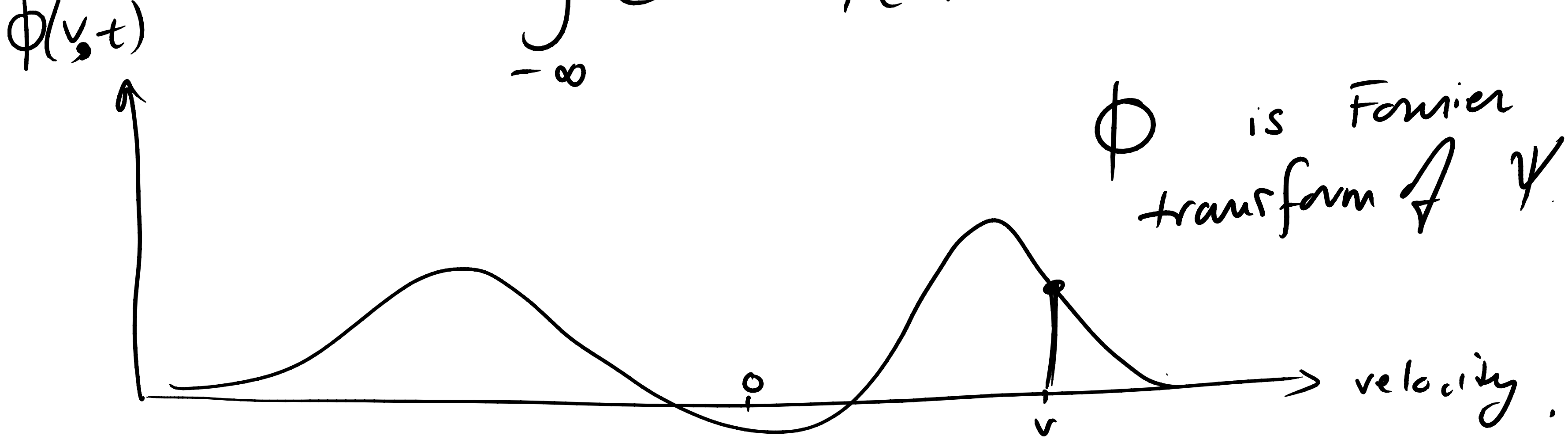
$$\psi(x,t) = e^{ik(x+kt)}$$

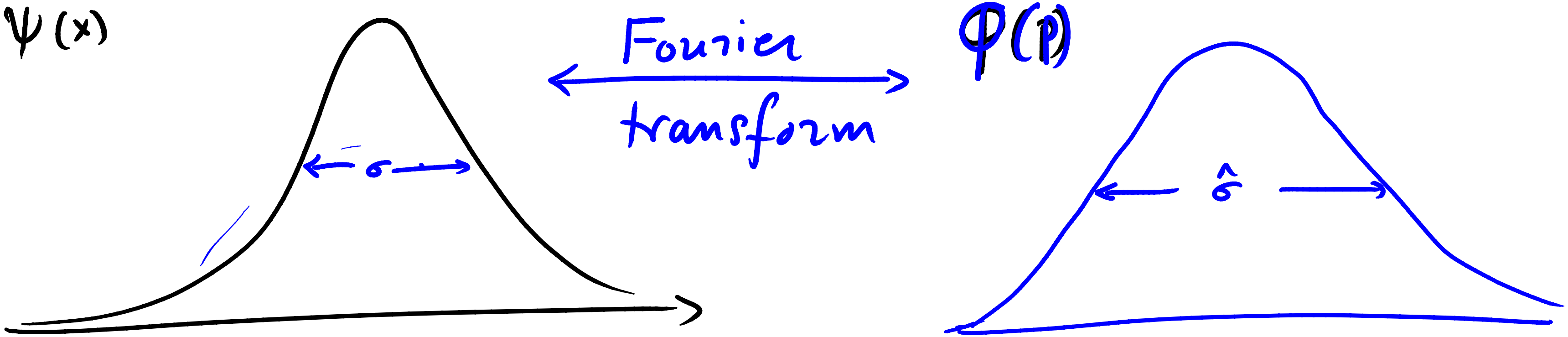
$\psi(x,t)$ $\qquad$ $\underline{\underline{e^{ikx}}}$ $\qquad$ $\underline{\underline{k}}$

$$\phi(v,t) = \langle e^{ikx}, \psi(x,t) \rangle$$

$$= \int_{-\infty}^{\infty} e^{-ikx} \cdot \psi(x,t)\, dx$$

ϕ is Fourier transform of ψ.





$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$