Quantum Mechanics & Quantum Computation

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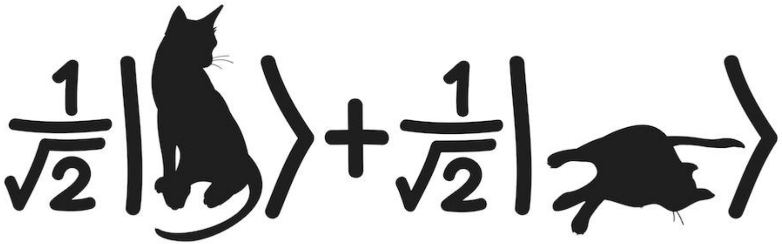


Lecture 7: Observables and Schrödinger's equation

Symmetry and Conservation Laws

Schrödinger's equation

- Energy observable H, called the Hamiltonian of the system.
  - Its eigenvectors $|\phi_i\rangle$'s are the states with definite energy.
  - The eigenvalues λ_i's are the energy of the corresponding state.

- Schrödinger's equation:

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$$

Emmy Noether 1882-1935

Why is H special?

Unitary evolution $\Rightarrow$ $U = e^{-iMt}$ for some hermitian M.

Why is $M = H$?

If A is any observable $\equiv$ conserved physical quantity

then A commutes with M

$$A \cdot M = M \cdot A$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$XZ = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$ZX = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Why is H special?

$|\psi\rangle \qquad |\psi'\rangle = U|\psi\rangle = e^{-iMt}|\psi\rangle$ at time t.

A conserved quantity means:

$$\langle\psi|\underline{\underline{A}}|\psi\rangle = \langle\psi'|A|\psi'\rangle = \langle\psi|\underline{\underline{U^+AU}}|\psi\rangle$$

$$\Rightarrow \quad A = U^+AU = e^{iMt}Ae^{-iMt}$$

$$\approx (1+iMt)A(1-iMt)$$

$$\approx A + \underbrace{it[MA-AM]}_{\overset{\shortparallel}{0}} + O(t^2)$$

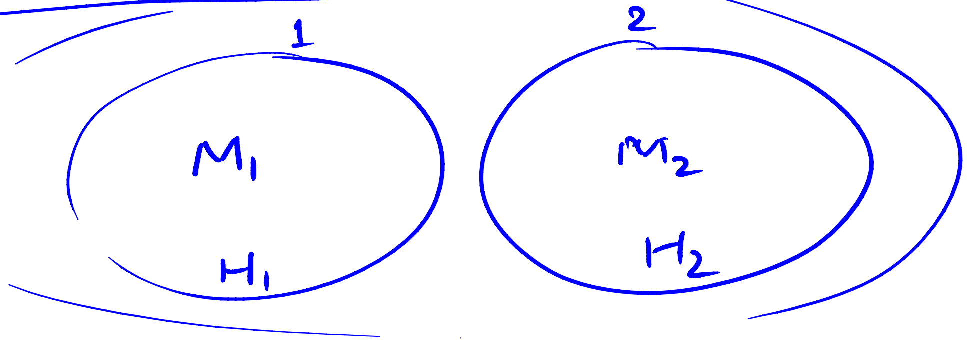
$$MA = AM$$

Why is H special?

* $U = e^{-iMt}$ M hermitian.
* A conserved $\Rightarrow$ $AM = MA$
* Intrinsic reason why M & H commute.

$H = \hbar M$

$H = f(M)$ $\Rightarrow$ f must necessarily be linear.



$H = H_1 + H_2$
"$f(M_1)$" "$f(M_2)$"

$H = f(M_1 + M_2)$

$f(M_1 + M_2) = f(M_1) + f(M_2)$