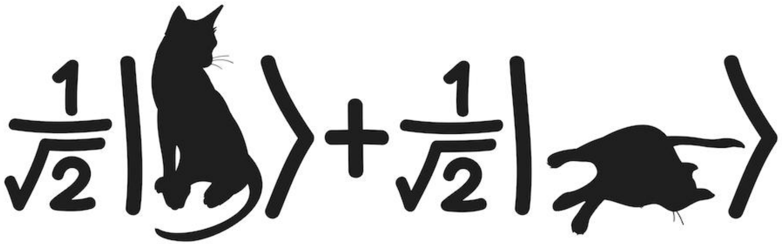
Quantum Mechanics & Quantum Computation



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Lecture 7: Observables and Schrödinger's equation

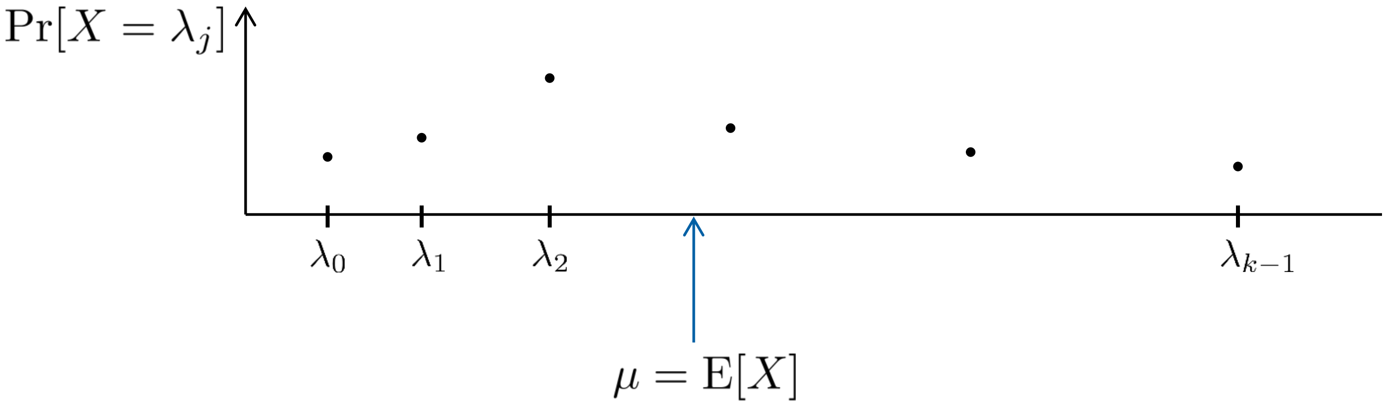
Expectation value and Variance

$M = M^\dagger$

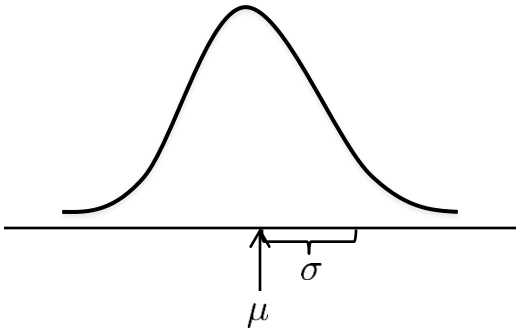
- An observable M for a k-level quantum system is a kxk Hermitian matrix.
  Random variable X denotes outcome of measurement of state $|\psi\rangle = \sum \alpha_i |\phi_i\rangle$

- Distribution of X:

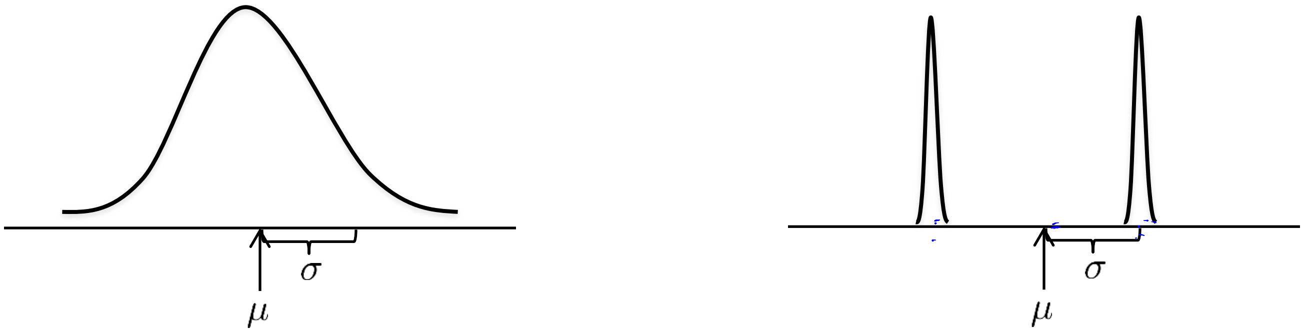
$P[X = \lambda_j] = |\alpha_j|^2$



$$\sigma^2 = \mathrm{Var}[X] = \mathrm{E}[(X - \mu)^2]$$



σ
μ



σ
μ
σ
μ

- Observable M on state $|\psi\rangle$

$M \quad |\phi_j\rangle \quad \lambda_j$

$|\psi\rangle = \sum \alpha_j |\phi_j\rangle$

$$\mu = \mathrm{E}[X] = \langle\psi|M|\psi\rangle$$

$\mu = E[x] = \sum |\alpha_j|^2 \lambda_j = (\alpha_0^* \ \alpha_1^* \cdots \alpha_{k-1}^*) \begin{pmatrix} \lambda_0 & & & 0 \\ & \lambda_1 & & \\ & & \ddots & \\ 0 & & & \lambda_{k-1} \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{k-1} \end{pmatrix}$

$= \sum \alpha_j^* \lambda_j \alpha_j = \sum \alpha_j^* \alpha_j \lambda_j$

$$\sigma^2 = \mathrm{Var}[X] = \mathrm{E}[X^2] - (\mathrm{E}[X])^2 = E[x^2] - \mu^2$$

$$= \langle\psi|M^2|\psi\rangle - \langle\psi|M|\psi\rangle^2$$

$E[x^2] = \sum |\alpha_j|^2 \lambda_j^2 = (\alpha_0^* \ \alpha_1^* \cdots \alpha_{k-1}^*) \begin{pmatrix} \lambda_0^2 & & & \\ & \lambda_1^2 & & \\ & & \ddots & \\ & & & \lambda_{k-1}^2 \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \vdots \\ \alpha_{k-1} \end{pmatrix}$