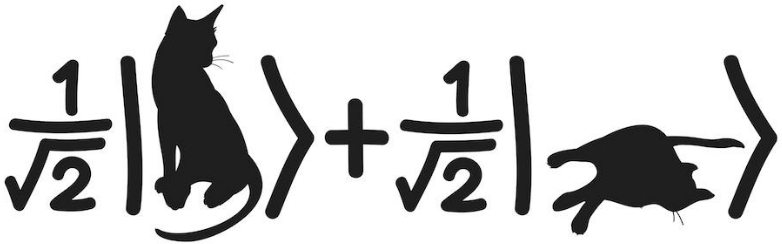
Quantum Mechanics & Quantum Computation

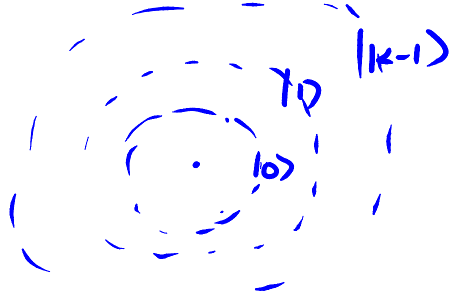
Umesh V. Vazirani
University of California, Berkeley



Lecture 7: Observables and Schrödinger's equation

Observables (part 2)

Observable



- Suppose we have a k-level system: $|\psi\rangle \in \mathbb{C}^k$

- An observable A for this system is an operator: a kxk Hermitian matrix.

$$\begin{bmatrix} E_0 & & & 0 \\ & E_1 & & \\ & & \ddots & \\ 0 & & & E_{k-1} \end{bmatrix}$$

$$A = A^\dagger$$

e.g. $\begin{pmatrix} 1 & 1+i \\ 1-i & -2 \end{pmatrix}$

<u>What's special about it?</u> **Spectral theorem!**

A has orthonormal eigenvectors $|\phi_1\rangle, \ldots, |\phi_k\rangle$ with real eigenvalues $\lambda_1, \ldots, \lambda_k$

$$A|\phi_i\rangle = \lambda_i|\phi_i\rangle$$

<u>How do we measure with it?</u>

Let $|\psi\rangle = \sum \alpha_i|\phi_i\rangle$. Measurement outcome is λ_i with probability $|\alpha_i|^2$

new state $|\psi_{new}\rangle = |\phi_i\rangle$

Observable

- Suppose we have a k-level system: $|\psi\rangle \in \mathbb{C}^k$

- An observable A for this system is an operator: a kxk Hermitian matrix.

$$A = A^\dagger \qquad \text{e.g.} \begin{pmatrix} 1 & 1+i \\ 1-i & -2 \end{pmatrix}$$

- How general is this?

- Suppose we wish to measure in an arbitrary basis $|\phi_1\rangle, \ldots, |\phi_k\rangle$
  and want arbitrary real outcomes $\lambda_1, \ldots, \lambda_k$
  is there an observable A with corresponding eigenvectors and eigenvalues?

- Example: $|+\rangle, |-\rangle$ with $2, -3$

$$|+\rangle\langle+| = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$(2|+\rangle\langle+|)|\psi\rangle = 2|+\rangle(\langle+|\psi\rangle) = 2\langle+|\psi\rangle|+\rangle \qquad \begin{pmatrix} 5/2 & -1/2 \\ -1/2 & 5/2 \end{pmatrix}$$

$$\mathbb{C}$$

If $|\psi\rangle = |+\rangle$

$$|-\rangle\langle-| = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\left(2|+\rangle\langle+| + (-3)|-\rangle\langle-|\right)|+\rangle = 2|+\rangle + 0 = 2|+\rangle$$

- In general: Given $|\phi_i\rangle, \lambda_i$ corresponding observable is:

$$A = \sum \lambda_i |\phi_i\rangle\langle\phi_i|$$

$A|\phi_j\rangle = \lambda_j$

- Therefore equivalent to our previous notion of measurement

observable $\equiv$ Pick an orthonormal

$A = A^{+}$

basis $|\phi_i\rangle$'s

λ_i's