Quantum Mechanics & Quantum Computation

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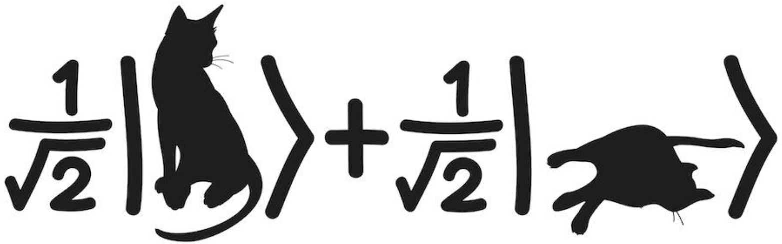
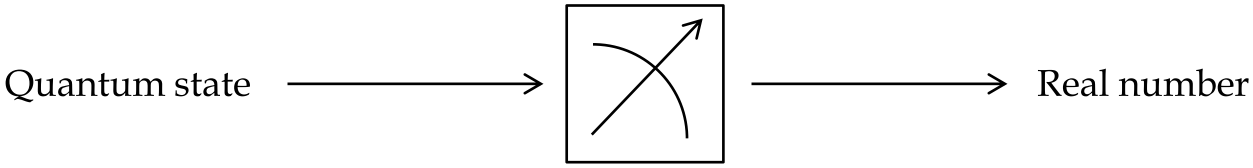


Lecture 7: Observables and Schrödinger's equation

Observables (part 1)

Observable

- An **observable** is a quantity like energy, position, momentum



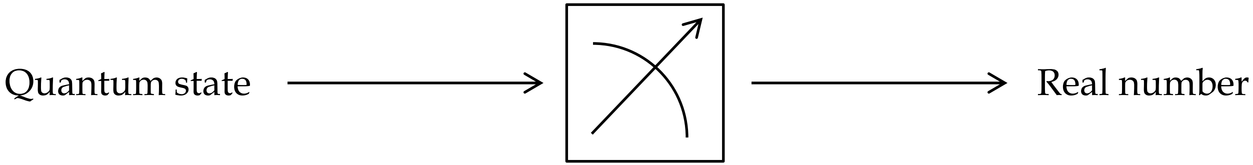
Observable

- Suppose we have a k-level system: $|\psi\rangle \in \mathbb{C}^k$

- An observable A for this system is an operator: a kxk Hermitian matrix.

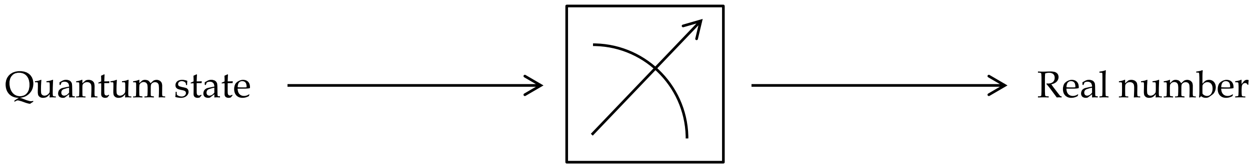
$$A = A^\dagger$$

e.g. $\begin{pmatrix} 1 & 1+i \\ 1-i & -2 \end{pmatrix}$



Measurement?

- An **observable** is a quantity like energy, position, momentum



- **Measure** in orthonormal basis $|\phi_1\rangle, \ldots, |\phi_k\rangle$ with corresponding outcomes 1 , . . . , k

Observable

- Suppose we have a k-level system: $|\psi\rangle \in \mathbb{C}^k$
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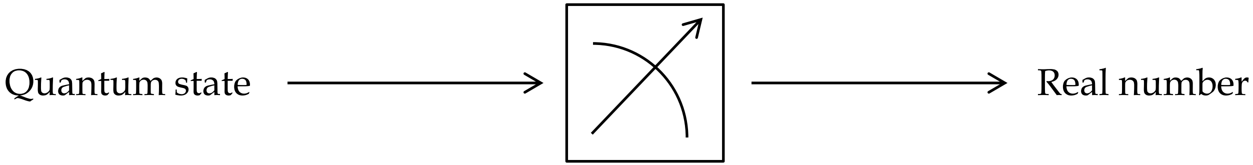
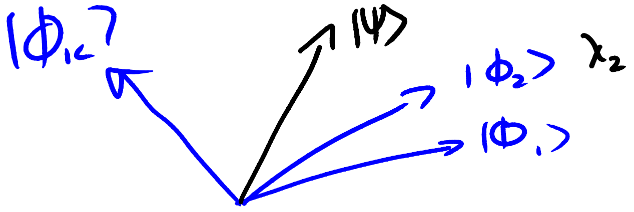


Observable



- Suppose we have a k-level system: $|\psi\rangle \in \mathbb{C}^k$

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What's special about it? **Spectral theorem!**

A has orthonormal eigenvectors $|\phi_1\rangle, \ldots, |\phi_k\rangle$ with real eigenvalues $\lambda_1, \ldots, \lambda_k$

$$A|\phi_i\rangle = \lambda_i|\phi_i\rangle$$

How do we measure with it?

Let $|\psi\rangle = \sum \alpha_i|\phi_i\rangle$. Measurement outcome is λ_i with probability $|\alpha_i|^2$

new state $|\psi_{new}\rangle = |\phi_i\rangle$

- Example $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.
- Observable $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

eigenvectors $|\phi_1\rangle = |+\rangle$ $\lambda_1 = 1$

$|\phi_2\rangle = |-\rangle$ $\lambda_2 = -1$.

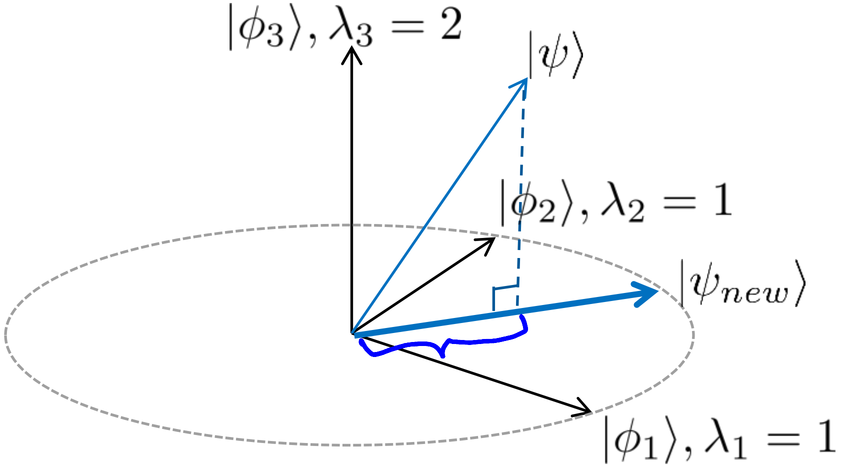
$$|\psi\rangle = \frac{\alpha+\beta}{\sqrt{2}}|+\rangle + \frac{\alpha-\beta}{\sqrt{2}}|-\rangle$$

Outcome: $+1$ wp $\left|\frac{\alpha+\beta}{\sqrt{2}}\right|^2$ New state $|+\rangle$

-1 wp $\left|\frac{\alpha-\beta}{\sqrt{2}}\right|^2$ New state $|-\rangle$

Expected value $= 1\left|\frac{\alpha+\beta}{\sqrt{2}}\right|^2 + (-1)\left|\frac{\alpha-\beta}{\sqrt{2}}\right|^2$

- Repeated eigenvalues?



What happens if the measurement outcome is 1?

Does it collapse to $|\phi_1\rangle$ or $|\phi_2\rangle$?

It gets projected into the eigenspace.

$$A\left(\frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle\right)$$
$$= \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle$$