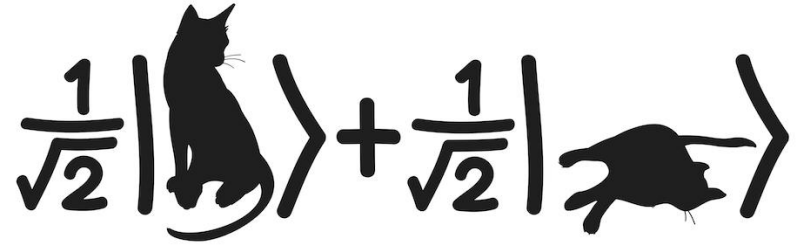


Quantum Mechanics & Quantum Computation

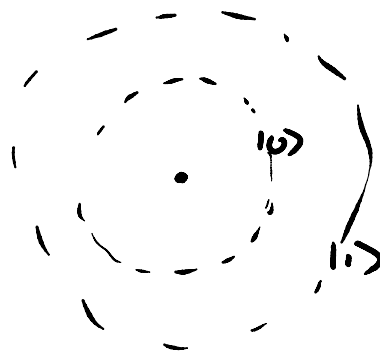
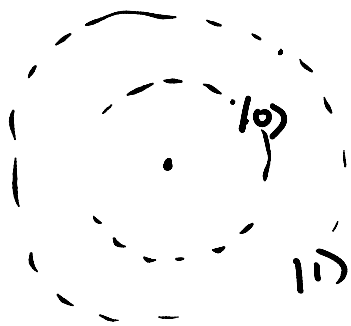
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Lecture 7: Bra-ket notation, Eigenvectors, Tensor products

Tensor Products



$|00\rangle$
 $|01\rangle$
 $|10\rangle$
 $|11\rangle$

$(\mathbb{C}^2$
 $\mathcal{H}_1 \otimes$

\mathbb{C}^2
 \mathcal{H}_2

\mathbb{C}^4
 \mathcal{H}

$$|0\rangle \otimes |0\rangle = |00\rangle = |0\rangle|0\rangle$$

$$|0\rangle \otimes |1\rangle = |01\rangle = |0\rangle|1\rangle$$

$$\begin{aligned}
 |\phi_1\rangle \otimes |\phi_2\rangle &= \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \\
 &= \frac{1}{2}|0\rangle \otimes |0\rangle - \frac{1}{2}|0\rangle \otimes |1\rangle + \frac{1}{2}|1\rangle \otimes |0\rangle - \frac{1}{2}|1\rangle \otimes |1\rangle
 \end{aligned}$$

$$|\phi_1\rangle \otimes |\phi_2\rangle$$

$$|\psi_1\rangle \otimes |\psi_2\rangle$$

inner product :

$$(\langle \phi_1 | \psi_1 \rangle) (\langle \phi_2 | \psi_2 \rangle)$$

K-state quantum system.

l-state system.

$$\mathcal{H}_1 = \mathbb{C}^K$$

$$\mathcal{H}_2 = \mathbb{C}^l$$

$$\mathcal{H} = \mathbb{C}^{K \cdot l}$$

$$|0\rangle, \dots, |K-1\rangle$$

$$|0\rangle, \dots, |l-1\rangle.$$

$$\mathcal{H}_1 \otimes \mathcal{H}_2 = \mathcal{H}.$$

$$|0\rangle |l-1\rangle = |0 \quad l-1\rangle$$

$$\begin{aligned} &|0\rangle \otimes |0\rangle, |0\rangle \otimes |1\rangle, \dots \\ &|1\rangle \otimes |0\rangle, \dots \end{aligned}$$

$$\begin{aligned} &|0\rangle \otimes |l-1\rangle \\ &|1\rangle \otimes |l-1\rangle \end{aligned}$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} K \cdot l.$$

$$\vdots \\ |K-1\rangle \otimes |0\rangle, \dots$$

$$|K-1\rangle \otimes |l-1\rangle.$$

K complex numbers.
in \mathcal{H}_1

l complex numbers
in \mathcal{H}_2

K.l complex #s
in $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$

\mathcal{H}_1 \mathcal{H}_2 k l $|\Phi_1\rangle$ \otimes $|\Phi_2\rangle$ $k+l$ parameters.

Entangled state.

 $k \cdot l$ parameters.

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$$