Quantum Mechanics & Quantum Computation

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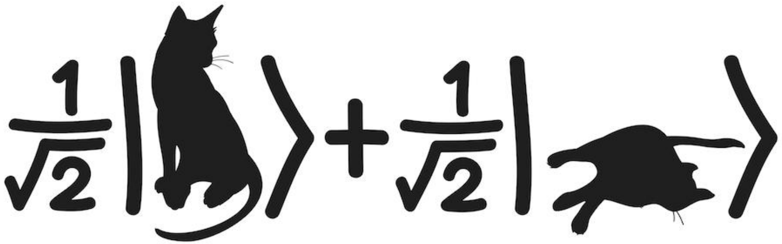


Lecture 7: Bra-ket notation, Eigenvectors, Tensor products

Bra-ket notation

Ket $\quad |\psi\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \\ \alpha_{k-1} \end{pmatrix} = \alpha_0|0\rangle + \alpha_1|1\rangle + \cdots + \alpha_{k-1}|k-1\rangle$

$\alpha_0|0\rangle + \alpha_1|1\rangle \qquad \alpha_0|\uparrow\rangle + \alpha_1|\downarrow\rangle \qquad \alpha_0|\text{blue}\rangle + \alpha_1|\text{red}\rangle$

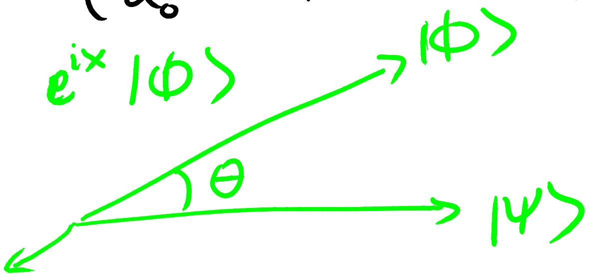
$$\boxed{\begin{gathered} \cos\theta = \frac{|\langle\psi|\phi\rangle|}{\||\psi\rangle\| \, \||\phi\rangle\|} \\ 0 \le \theta \le \pi/2 \end{gathered}}$$

Bra $\quad (\alpha_0^* \ \ \alpha_1^* \ \cdots \ \alpha_{k-1}^*) = \langle\psi|$

$|\phi\rangle = \beta_0|0\rangle + \beta_1|1\rangle + \cdots + \beta_{k-1}|k-1\rangle$

Inner product of $|\psi\rangle$ & $|\phi\rangle$

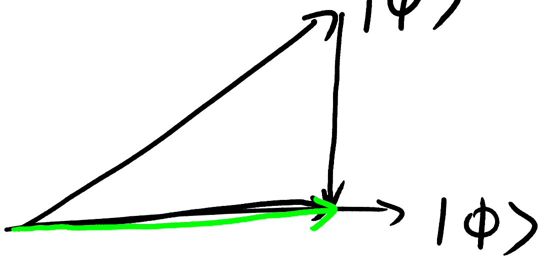
$$(\alpha_0^* \ \ \alpha_1^* \ \cdots \ \alpha_{k-1}^*)\begin{pmatrix} \beta_0 \\ \beta_1 \\ \\ \beta_{k-1} \end{pmatrix} = \sum_j \alpha_j^* \beta_j = \langle\psi|\phi\rangle$$



$$\||\psi\rangle\| = \sqrt{\langle\psi|\psi\rangle} = \sqrt{\sum \alpha_j^* \alpha_j} = \sqrt{\sum |\alpha_j|^2}$$

Projection Matrix P onto $|\phi\rangle = \begin{pmatrix} \beta_0 \\ \vdots \\ \beta_{k-1} \end{pmatrix}$

$P|\psi\rangle$



$$P = \begin{pmatrix} \beta_0 \\ \\ \\ \beta_{k-1} \end{pmatrix} \begin{pmatrix} \beta_0^* & \cdots & \beta_{k-1}^* \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

$P = |\phi\rangle\langle\phi|$ $\quad |\phi\rangle\langle\phi|$

$$P|\psi\rangle = \left(|\phi\rangle\langle\phi|\right)|\psi\rangle = |\phi\rangle\underbrace{\left(\langle\phi|\psi\rangle\right)}$$

$P^2 = P$

$$|\phi\rangle\underbrace{\langle\phi|\phi\rangle}_{1}\langle\phi| = |\phi\rangle\langle\phi|$$

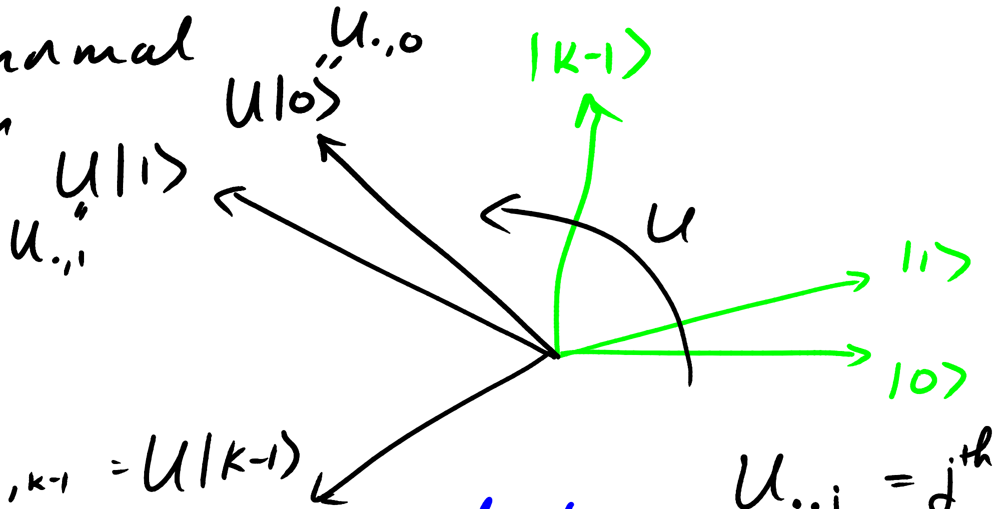
$= P$

$$= \underbrace{\langle\phi|\psi\rangle}_{\in \mathbb{C}}\ |\phi\rangle$$

U unitary $\iff$ $UU^{\dagger} = U^{\dagger}U = I$

rows are orthonormal

columns " "



$U_{\cdot,j}$ = j^{th} column of U

U preserves angles or inner products:

$\langle \phi , \psi \rangle$ $\quad U|\phi\rangle = |\phi'\rangle \quad U|\psi\rangle = |\psi'\rangle$

$\langle \phi' | = \langle \phi | U^{\dagger}$

$$\langle \phi , \psi \rangle = \langle \phi', \psi' \rangle$$

$$= \langle \phi \underbrace{U^{\dagger} | U}_{} \psi \rangle$$

$U^{\dagger}U = I$