Quantum Mechanics & Quantum Computation

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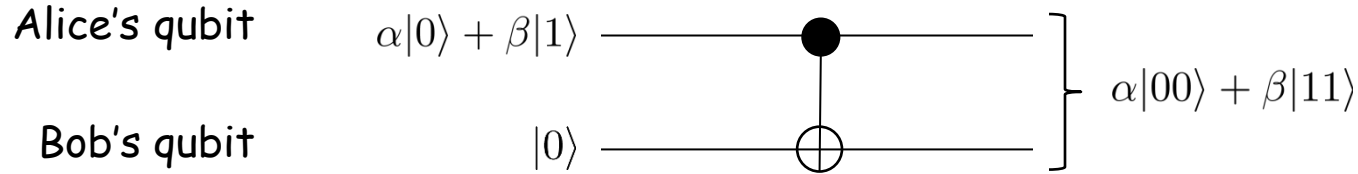**University of California, Berkeley**

$$\frac{1}{\sqrt{2}}\lvert\text{🐱}\rangle + \frac{1}{\sqrt{2}}\lvert\text{🐭}\rangle$$
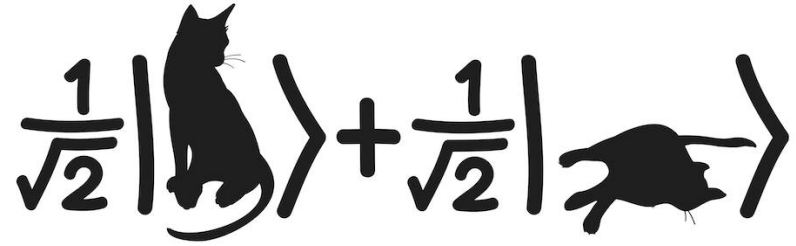
Lecture 6: Quantum Circuits and Teleportation

Teleportation (part 2)

Teleportation using CNOT

Alice's qubit $\qquad \alpha|0\rangle + \beta|1\rangle$

Bob's qubit $\qquad |0\rangle$

$$\alpha|00\rangle + \beta|11\rangle$$

Alice measures in the sign basis: $\quad + \quad \longrightarrow \quad |+\rangle \otimes (\alpha|0\rangle + \beta|1\rangle)$

$\qquad\qquad\qquad\qquad\qquad\qquad\quad - \quad \longrightarrow \quad |-\rangle \otimes (\alpha|0\rangle - \beta|1\rangle)$

If measurement result is -, Alice calls Bob and tells him to flip the phase.
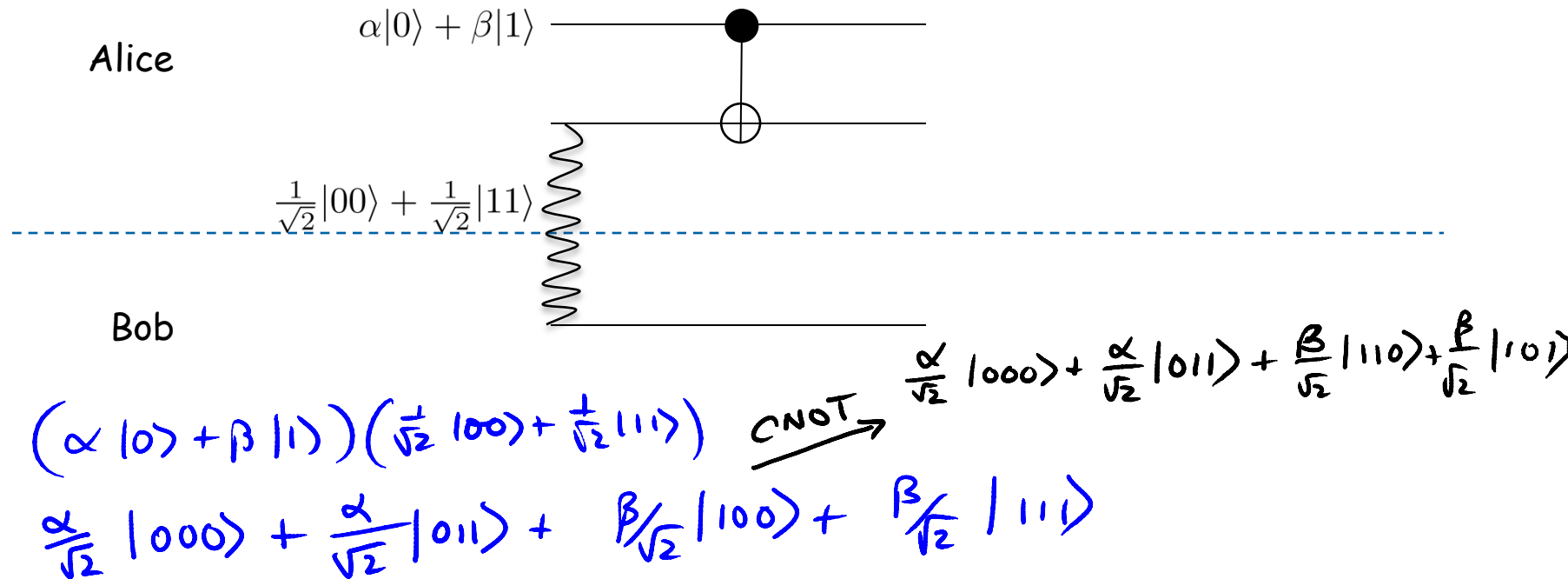
But they are far apart...

- Suppose they share a Bell state.
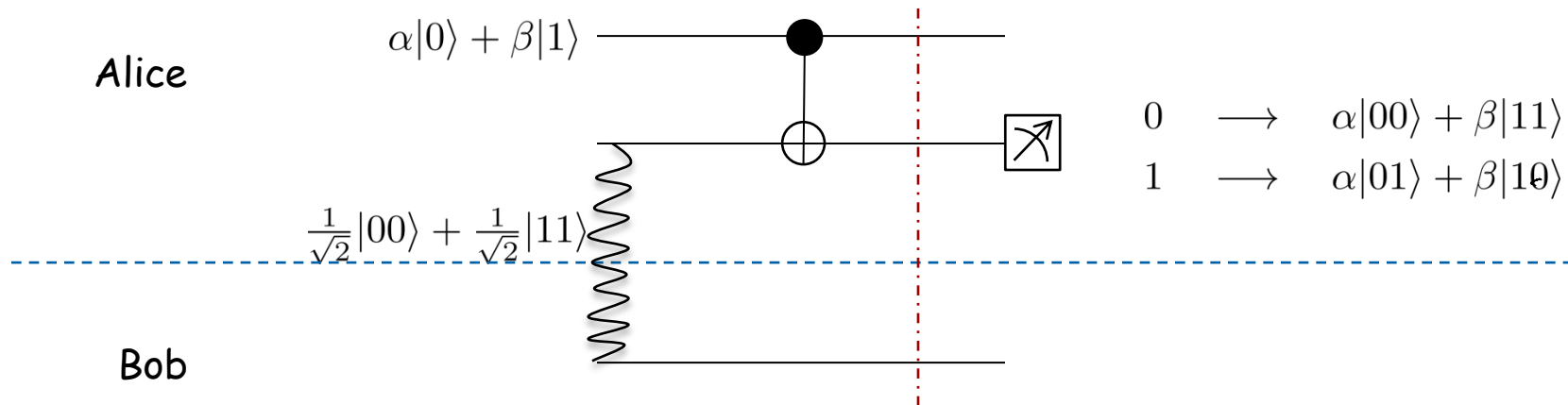- Can we use it to effectively apply CNOT remotely?

$\alpha|0\rangle + \beta|1\rangle$ ⎯⎯⎯⎯⎯⎯⎯

Alice

$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

Bob

But they are far apart…

- Suppose they share a Bell state.
- Can we use it to effectively apply CNOT remotely?

Alice $\quad \alpha|0\rangle + \beta|1\rangle$

$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

Bob

$$\left(\alpha|0\rangle + \beta|1\rangle\right)\left(\tfrac{1}{\sqrt{2}}|00\rangle + \tfrac{1}{\sqrt{2}}|11\rangle\right) \xrightarrow{\text{CNOT}} \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|110\rangle + \frac{\beta}{\sqrt{2}}|101\rangle$$

$$= \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|100\rangle + \frac{\beta}{\sqrt{2}}|111\rangle$$

But they are far apart...

Alice

$\alpha|0\rangle + \beta|1\rangle$

$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

Bob

$0 \quad \longrightarrow \quad \alpha|00\rangle + \beta|11\rangle$

$1 \quad \longrightarrow \quad \alpha|01\rangle + \beta|10\rangle$
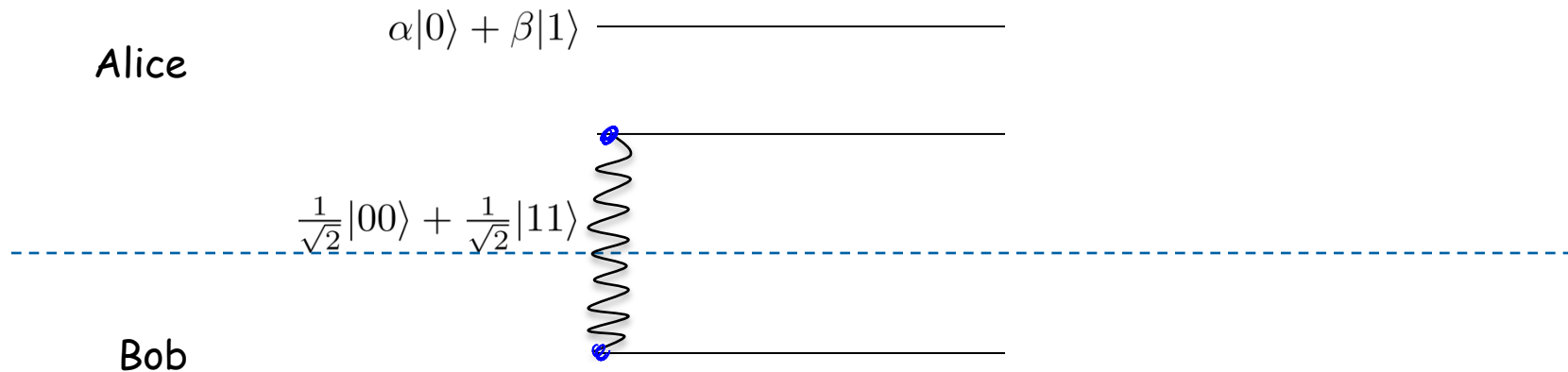
$$\frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\beta}{\sqrt{2}}|110\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|101\rangle$$

Measure second qubit.

Complete quantum teleportation protocol