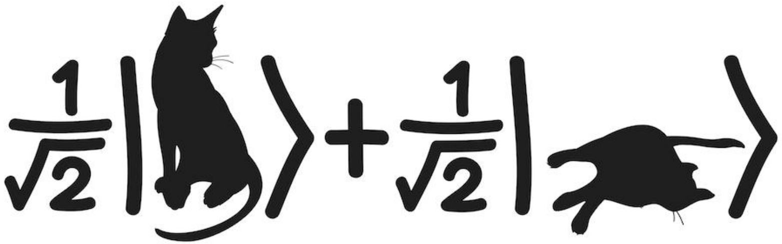
Quantum Mechanics & Quantum Computation



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Lecture 6: Quantum Circuits and Teleportation

CNOT gate + Circuits

- One-qubit gates:

$U = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$UU^\dagger = U^\dagger U = I$

X, Z, H

$\alpha_0|0\rangle + \alpha_1|1\rangle \longrightarrow \boxed{U} \longrightarrow \alpha_0'|0\rangle + \alpha_1'|1\rangle$

$|0\rangle \longrightarrow a|0\rangle + b|1\rangle$

$|1\rangle \longrightarrow c|0\rangle + d|1\rangle$

- What is the dimension of two-qubit gates?

$$\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \;\; \boxed{U} \;\; \alpha'_{00}|00\rangle + \alpha'_{01}|01\rangle + \alpha'_{10}|10\rangle + \alpha'_{11}|11\rangle$$

$|00\rangle \longrightarrow a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$

$|01\rangle \longrightarrow$

$|10\rangle \longrightarrow$

$|11\rangle \longrightarrow$

$$\begin{pmatrix} \alpha'_{00} \\ \alpha'_{01} \\ \alpha'_{10} \\ \alpha'_{11} \end{pmatrix} = U \begin{pmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{pmatrix}$$

U is 4-dimensional

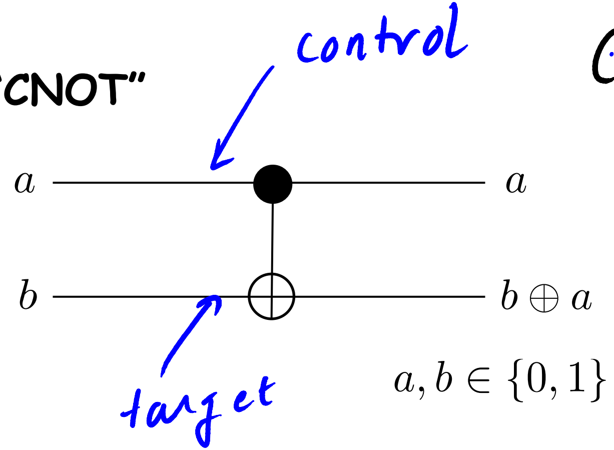
$$\begin{pmatrix} a & \cdot & \cdot & \cdot \\ b & \cdot & \cdot & \cdot \\ c & \cdot & \cdot & \cdot \\ d & \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

U is a 4x4 unitary matrix.

$$UU^\dagger = U^\dagger U = I$$

Controlled-NOT

"CNOT"



$a, b \in \{0,1\}$

$|00\rangle \longrightarrow |00\rangle$

$|01\rangle \longrightarrow |01\rangle$

$|10\rangle \longrightarrow |11\rangle$

$|11\rangle \longrightarrow |10\rangle$

$$\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

$$\Big\downarrow \text{CNOT}$$

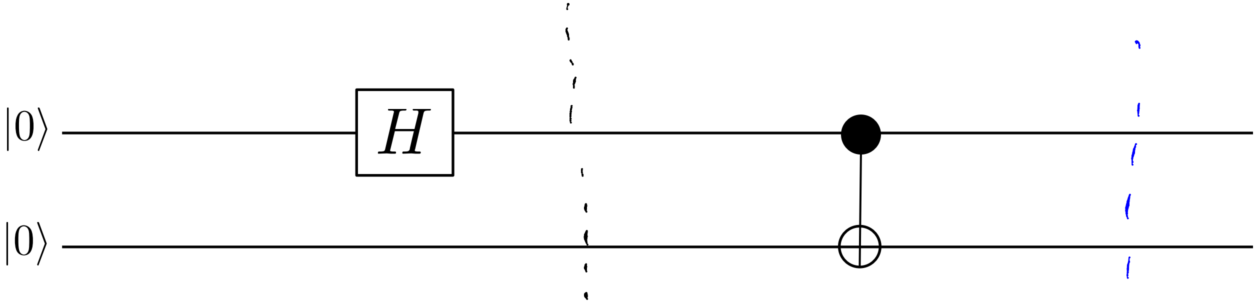
$$\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{11}|10\rangle + \alpha_{10}|11\rangle$$

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$CNOT \cdot CNOT^{\dagger} = I$$

$H|0\rangle = |+\rangle$

$H|1\rangle = |-\rangle$

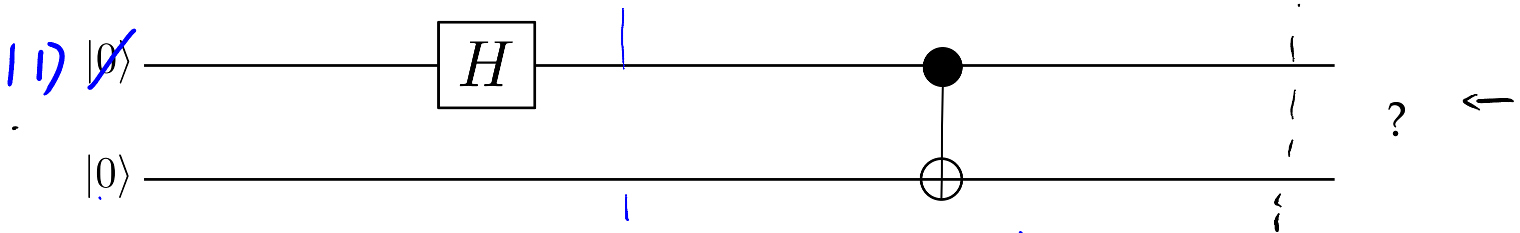


?

$$\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\left(|0\rangle\right) = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Bell State



$$\left(\tfrac{1}{\sqrt{2}}|0\rangle - \tfrac{1}{\sqrt{2}}|1\rangle\right)\left(|0\rangle\right) = \tfrac{1}{\sqrt{2}}|00\rangle - \tfrac{1}{\sqrt{2}}|10\rangle \qquad \tfrac{1}{\sqrt{2}}|00\rangle - \tfrac{1}{\sqrt{2}}|11\rangle$$

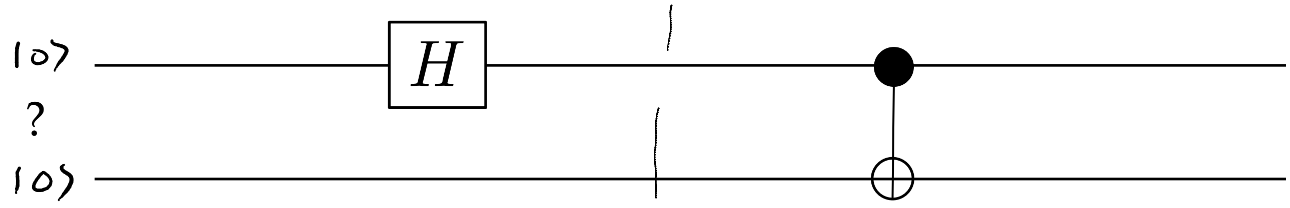
By varying the input state, we can get all four Bell states.

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\checkmark \quad |\phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$



$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Bell states measurement!

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$