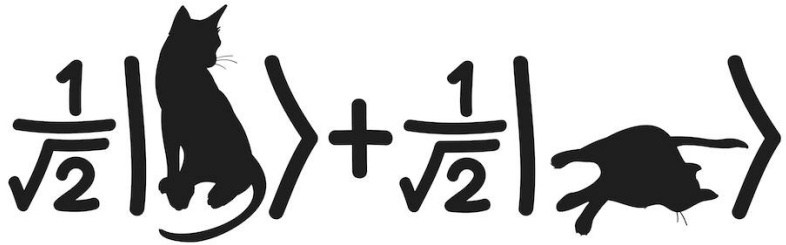
Quantum Mechanics & Quantum Computation



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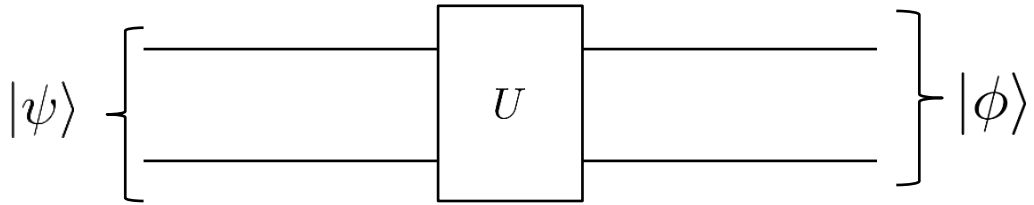
Lecture 5: Quantum Gates

No cloning Theorem

- Two qubit gate



$$U = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$U|\psi_1\rangle = |\phi_1\rangle$$

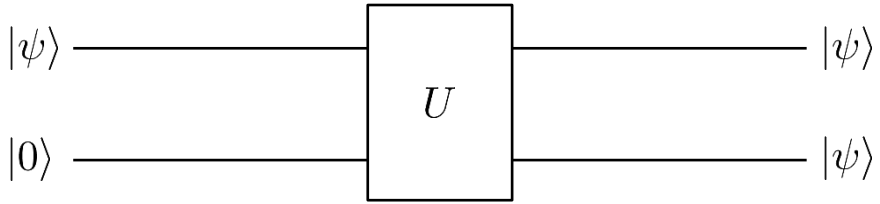
$$U|\psi_2\rangle = |\phi_2\rangle$$

$$|\psi\rangle = |\psi_1\rangle + |\psi_2\rangle$$

$$U U^+ = U^+ U = I$$

$$U|\psi\rangle = |\phi_1\rangle + |\phi_2\rangle$$

- Construct a quantum circuit for copying a quantum bit.



$|\psi\rangle = a|0\rangle + b|1\rangle$

$(a|0\rangle + b|1\rangle)(a|0\rangle + b|1\rangle)$

$= a^2|00\rangle + ab|01\rangle + ab|10\rangle + b^2|11\rangle$

output.

$|\psi\rangle = |0\rangle$ output $|00\rangle$

$|\psi\rangle = |1\rangle$ output $|11\rangle$

$|\psi\rangle = a|0\rangle + b|1\rangle$ output $a|00\rangle + b|11\rangle$

$ab = 0$

$a = 0$ or $b = 0$

No cloning theorem

- It is **impossible** to clone an unknown quantum state.