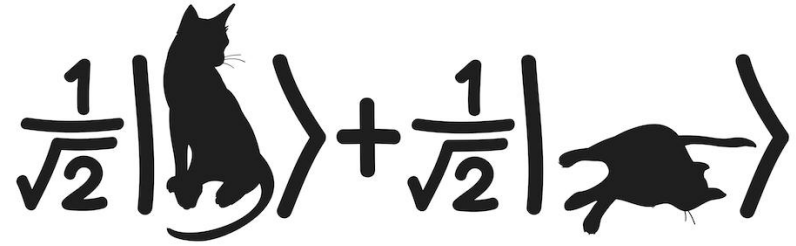


Quantum Mechanics & Quantum Computation

Umesh V. Vazirani

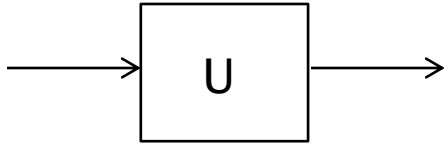
University of California, Berkeley



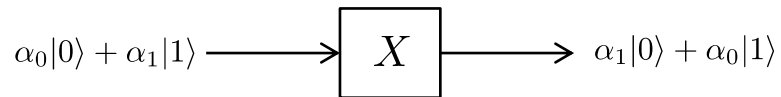
Lecture 5: Quantum Gates

Single qubit gates

Quantum gates



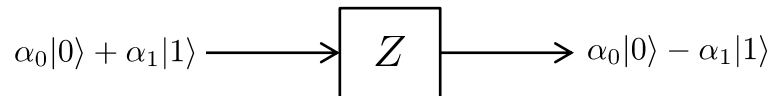
"Bit flip"



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{aligned} X|0\rangle &= |1\rangle \\ X|1\rangle &= |0\rangle \end{aligned}$$

$$X^\dagger = X \quad X^2 = I$$

"Phase flip"

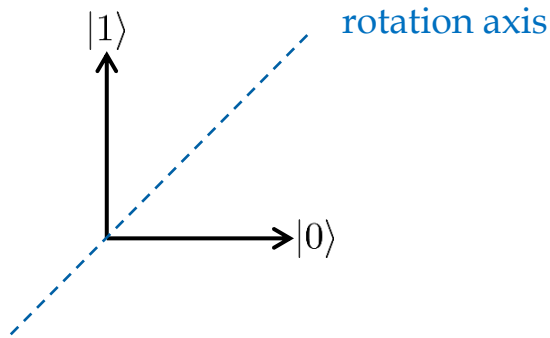


$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \begin{aligned} Z|0\rangle &= |0\rangle \\ Z|1\rangle &= -|1\rangle \end{aligned}$$

$$Z^\dagger = Z \quad Z^2 = I$$

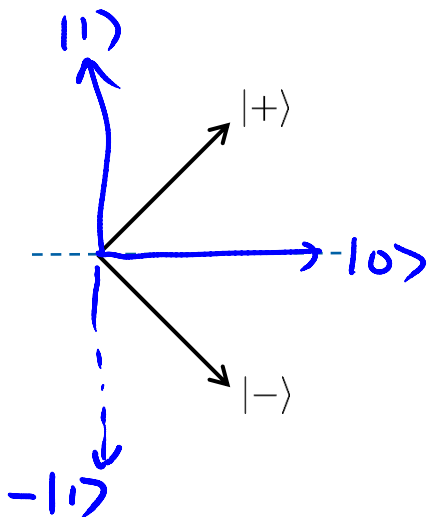
Bit flip

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



Phase flip

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



$$|0\rangle \rightarrow |0\rangle$$

$$|1\rangle \rightarrow -|1\rangle$$

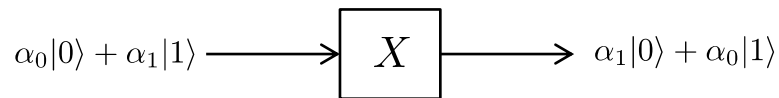
$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

→

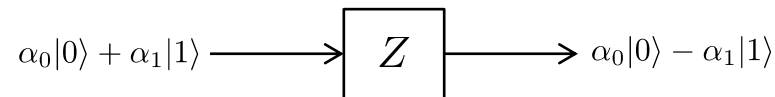
$$|-\rangle \rightarrow |+\rangle$$

"Bit flip"



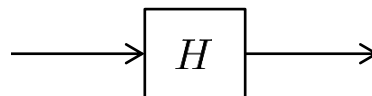
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{aligned} X|0\rangle &= |1\rangle \\ X|1\rangle &= |0\rangle \end{aligned}$$

"Phase flip"



$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \begin{aligned} Z|0\rangle &= |0\rangle \\ Z|1\rangle &= -|1\rangle \end{aligned}$$

"Hadamard transform"



$$H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{aligned} H^\dagger &= H \\ H^2 &= I \end{aligned}$$

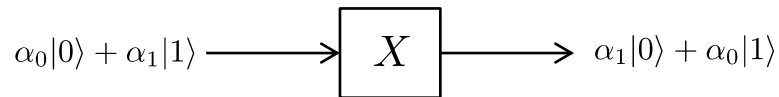
$$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$H|1\rangle = |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$H|+\rangle = |0\rangle$$

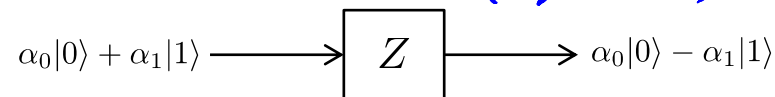
$$H|-\rangle = |1\rangle$$

"Bit flip"



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{aligned} X|0\rangle &= |1\rangle \\ X|1\rangle &= |0\rangle \end{aligned}$$

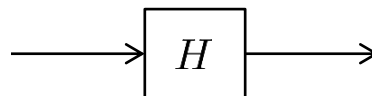
"Phase flip"



$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \begin{aligned} Z|0\rangle &= |0\rangle \\ Z|1\rangle &= -|1\rangle \end{aligned}$$

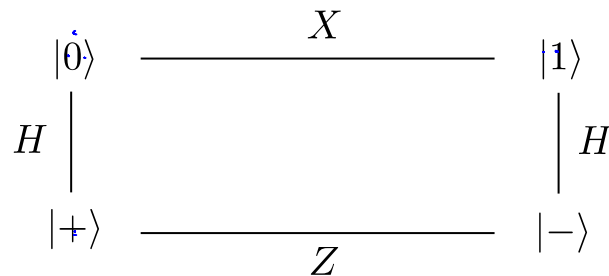
$Z|+\rangle = |-\rangle$
 $Z|-\rangle = |+\rangle$

"Hadamard transform"



$$H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$|0\rangle \xrightarrow{H} |+\rangle \xrightarrow{Z} |-\rangle \xrightarrow{H} |1\rangle$

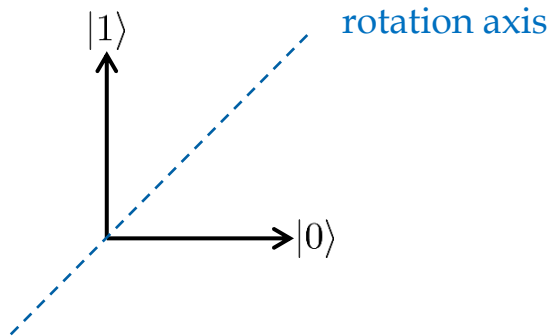


$Z = H X H$
 $X = H^{-1} Z H$

$H = H^{-1}$
 $H^2 = I$

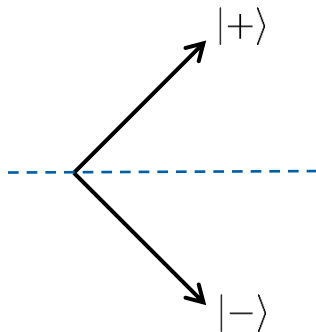
Bit flip

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



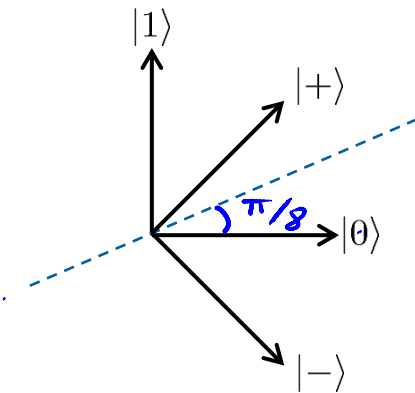
Phase flip

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



Hadamard

$$H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$



$$\underline{|0\rangle} \rightarrow \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle = |+\rangle$$

$$\underline{|1\rangle} \rightarrow \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle = |-\rangle$$