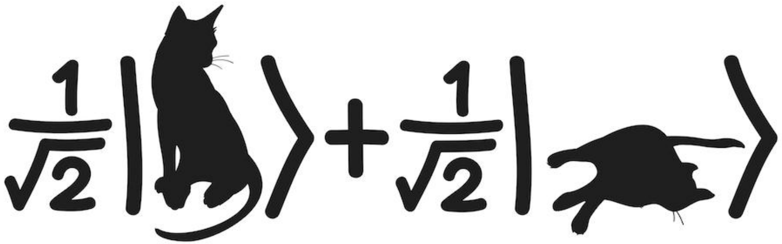
Quantum Mechanics & Quantum Computation



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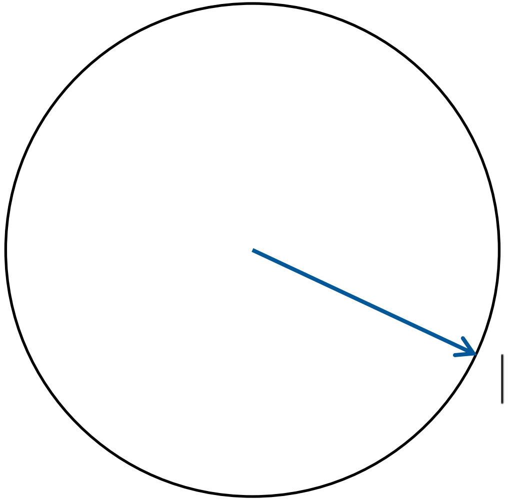
Lecture 5: Quantum Gates

Evolution of a qubit

Superposition

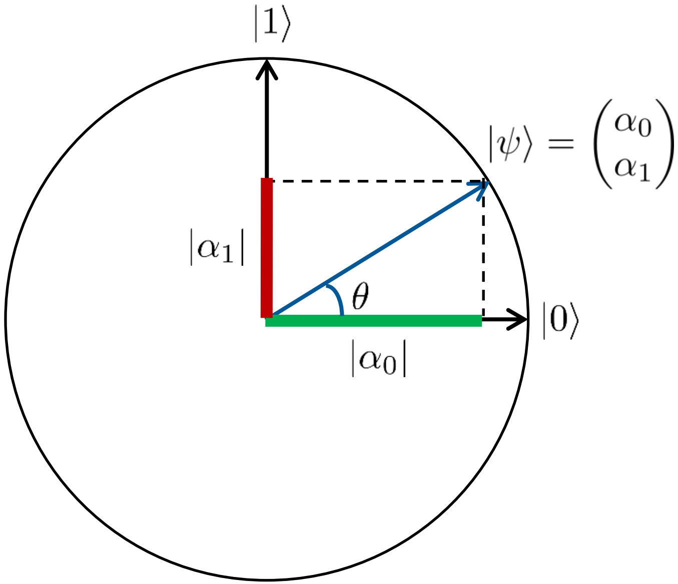
- Allowable states of k-level system: unit vector in a k-dimensional complex vector space (called a Hilbert space).



$$|\psi\rangle = \alpha_0|0\rangle + \cdots + \alpha_{k-1}|k-1\rangle = \begin{pmatrix} \alpha_0 \\ \vdots \\ \alpha_{k-1} \end{pmatrix} \in \mathbb{C}^k$$

Measurement

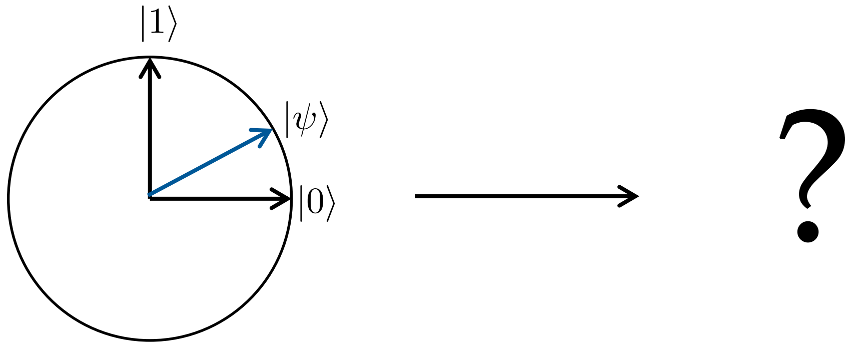
- A measurement is specified by choosing an orthonormal basis.
- The probability of each outcome is the square of the length of the projection onto the corresponding basis vector.
- The state collapses to the observed basis vector.



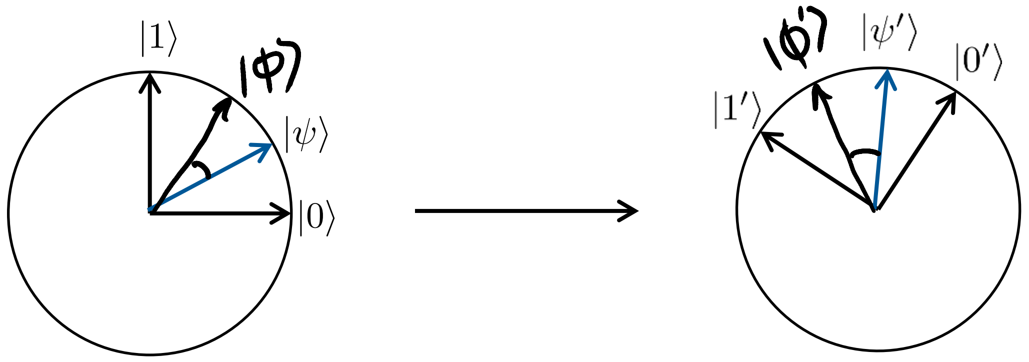
If $|u_0\rangle, |u_1\rangle, \ldots, |u_{k-1}\rangle$ was the chosen basis and $|\psi\rangle = \alpha_0|u_0\rangle + \alpha_1|u_1\rangle + \cdots + \alpha_{k-1}|u_{k-1}\rangle$

u_i with probability $|\alpha_i|^2 = |(|\psi\rangle, |u_i\rangle)|^2$

New state is $|u_i\rangle$



How does a qubit evolve?



How does a qubit evolve?

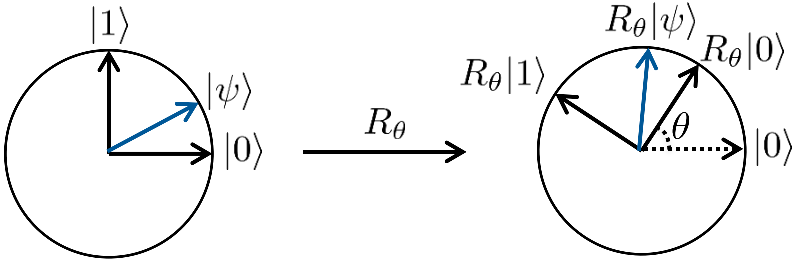
Qubits evolve by rotating the Hilbert space.

It is a rigid body rotation, meaning that the angles between vectors are preserved.

Rotation Matrix

- Rotation of the space is a linear transformation. Represent by a matrix:

- Example



$$|0\rangle \longrightarrow \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$$

$$|1\rangle \longrightarrow -\sin\theta\,|0\rangle + \cos\theta\,|1\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha(\cos\theta\,|0\rangle + \sin\theta\,|1\rangle) + \beta(-\sin\theta\,|0\rangle + \cos\theta\,|1\rangle)$$

$$= (\alpha\cos\theta - \beta\sin\theta)\,|0\rangle + (\alpha\sin\theta + \beta\cos\theta)\,|1\rangle$$

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \; \end{pmatrix}$$

$$R_{-\theta} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = R_\theta^T$$

$$R_\theta R_{-\theta} = I$$

$$R_\theta R_\theta^T = R_\theta^T R^\theta = I$$