Quantum Mechanics and Quantum Computation

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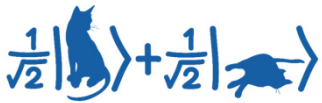

Lecture 2: Qubits and the axioms of quantum mechanics

Uncertainty principle

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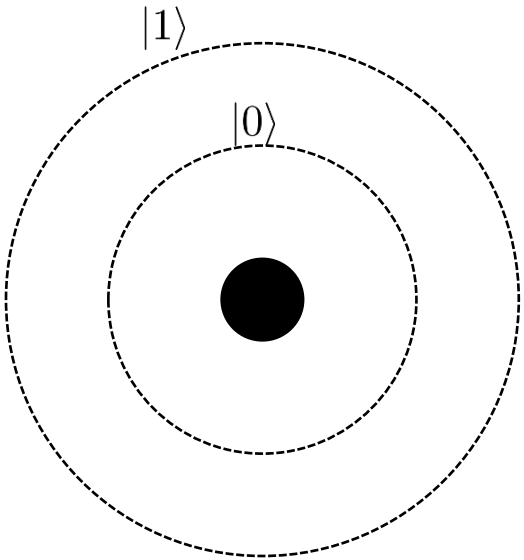
One can never know with perfect accuracy both of those two important factors which determine the movement of one of the smallest particles – its position and its velocity.

- Werner Heisenberg



Bit: $|0\rangle$ or $|1\rangle$

Sign: $|+\rangle$ or $|-\rangle$

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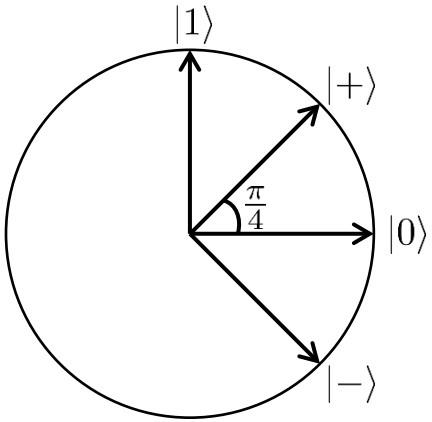

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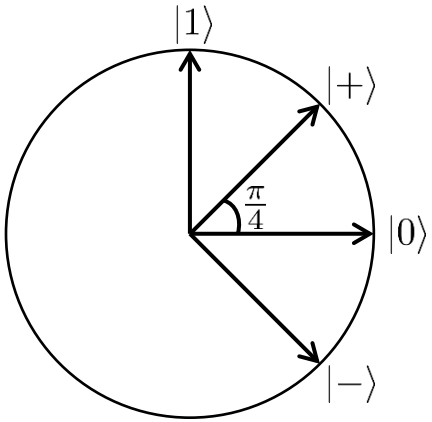
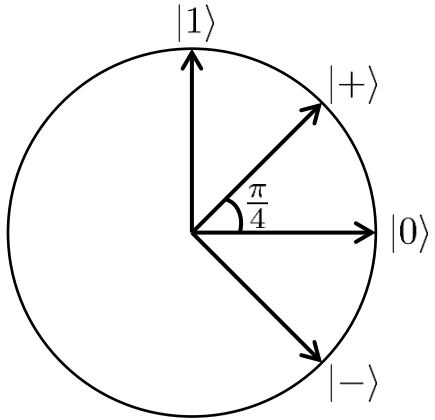


- To quantify this, we define **spread** of a quantum state.

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle = \beta_0|+\rangle + \beta_1|-\rangle$$



- The spread in standard basis := $S(|\psi\rangle) = |\alpha_0| + |\alpha_1|$
- The spread in sign basis := $\hat{S}(|\psi\rangle) = |\beta_0| + |\beta_1|$

$$S(|0\rangle) = 1 + 0 = 1 \qquad \hat{S}(|0\rangle) = \tfrac{1}{\sqrt{2}} + \tfrac{1}{\sqrt{2}} = \sqrt{2}$$

$$S(|+\rangle) = \tfrac{1}{\sqrt{2}} + \tfrac{1}{\sqrt{2}} = \sqrt{2} \qquad \hat{S}(|+\rangle) = 1 + 0 = 1$$

- **Uncertainty principle for bit and sign**: $S(|\psi\rangle)\hat{S}(|\psi\rangle) \geq \sqrt{2}$ for any $|\psi\rangle$.