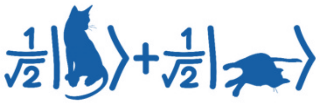
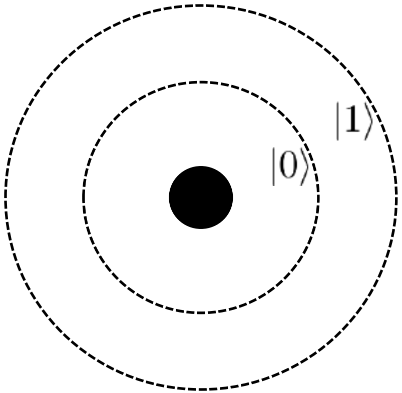
Quantum Mechanics and Quantum Computation

Umesh Vazirani, UC Berkeley



Lecture 2: Qubits and the axioms of quantum mechanics

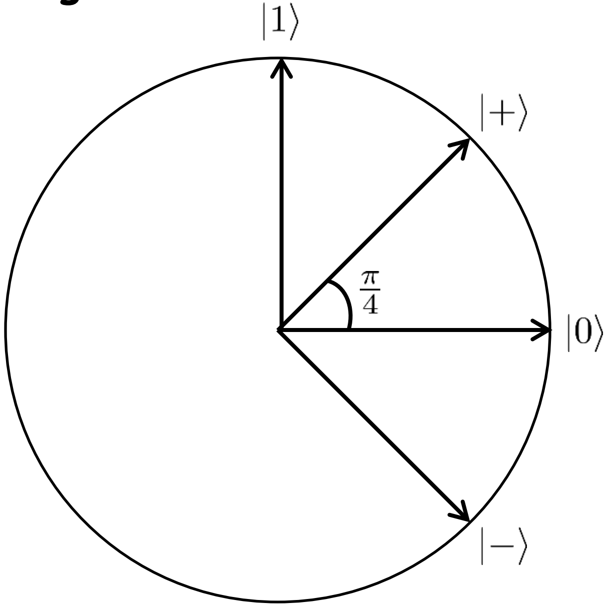
Measurement in an arbitrary basis


$|1\rangle$
$|0\rangle$

Example: sign basis (Hadamard basis)

- For qubits, there is a particularly important basis called "**sign basis**."

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

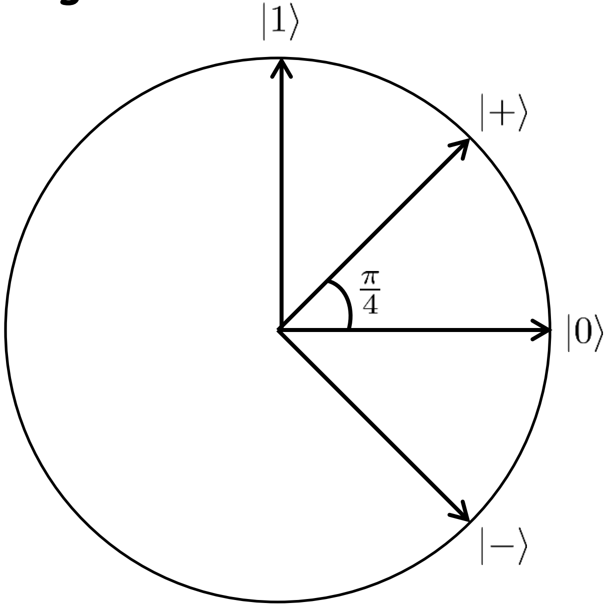


- Can we distinguish $|+\rangle$ from $|-\rangle$?

Example: sign basis (Hadamard basis)

- For qubits, there is a particularly important basis called "**sign basis**."

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$



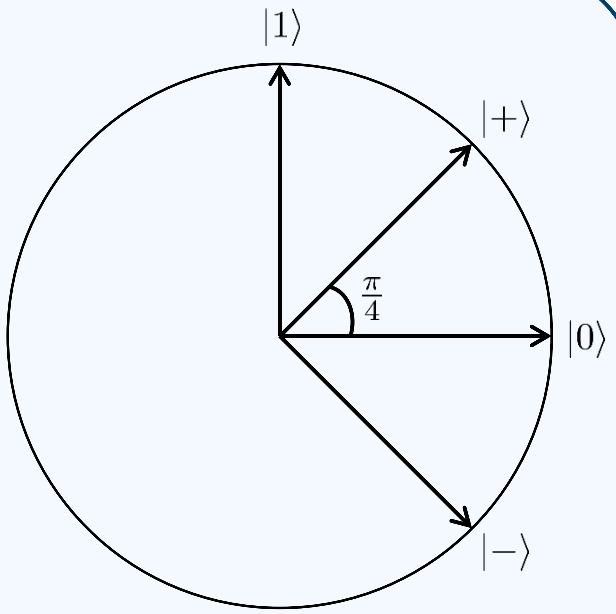
- Can we distinguish $|+\rangle$ from $|-\rangle$?

Measure $|\psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$ in $|+\rangle/|-\rangle$ basis:

Method 1: compute inner product

$$\begin{aligned}
\Pr[+] &= |(|\psi\rangle, |+\rangle)|^2 \\
&= |(\tfrac{1}{2}|0\rangle + \tfrac{\sqrt{3}}{2}|1\rangle, \tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle)|^2 \\
&= |\tfrac{1}{2\sqrt{2}} + \tfrac{\sqrt{3}}{2\sqrt{2}}|^2 \\
&= \tfrac{(1+\sqrt{3})^2}{8} = \tfrac{1+2\sqrt{3}+3}{8} = \tfrac{4+2\sqrt{3}}{8}
\end{aligned}$$

$$\Pr[-] = |(|\psi\rangle, |-\rangle)|^2$$



$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

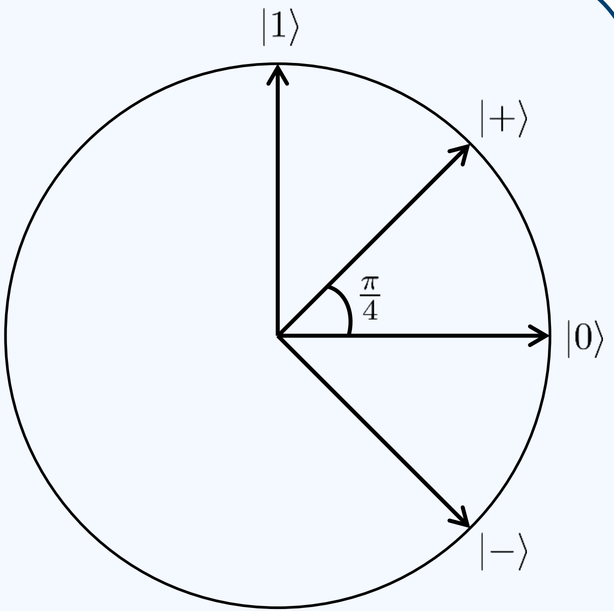
$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Measure $|\psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$ in $|+\rangle/|-\rangle$ basis:

Method 2: change of basis.

If we write it in the form $|\psi\rangle = \beta_0|+\rangle + \beta_1|-\rangle$,

$$\Pr[+] = |\beta_0|^2,\ \Pr[-] = |\beta_1|^2$$



$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Measure $|\psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$ in $|+\rangle/|-\rangle$ basis:

Method 2: change of basis.

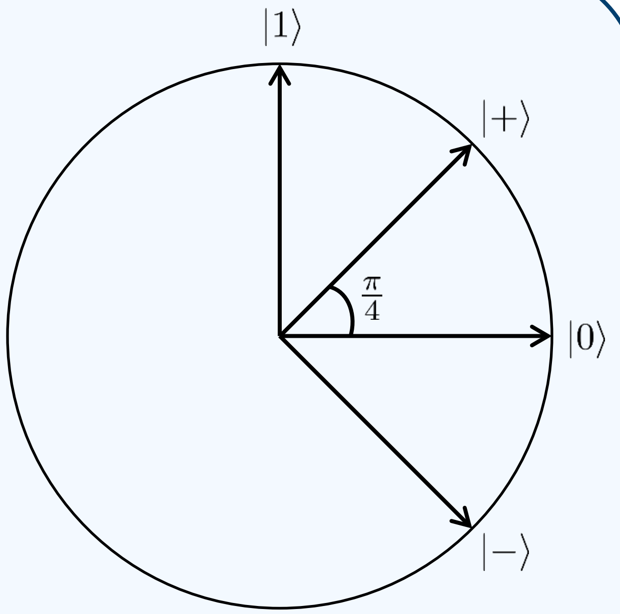
Check that $|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle$

$$|1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

$$\begin{aligned}
|\psi\rangle &= \tfrac{1}{2}|0\rangle + \tfrac{\sqrt{3}}{2}|1\rangle \\
&= \tfrac{1}{2}\left(\tfrac{1}{\sqrt{2}}|+\rangle + \tfrac{1}{\sqrt{2}}|-\rangle\right) + \tfrac{\sqrt{3}}{2}\left(\tfrac{1}{\sqrt{2}}|+\rangle - \tfrac{1}{\sqrt{2}}|-\rangle\right) \\
&= \left(\tfrac{1}{2\sqrt{2}} + \tfrac{\sqrt{3}}{2\sqrt{2}}\right)|+\rangle + \left(\tfrac{1}{2\sqrt{2}} - \tfrac{\sqrt{3}}{2\sqrt{2}}\right)|-\rangle
\end{aligned}$$

$$\Pr[+] = \left|\tfrac{1}{2\sqrt{2}} + \tfrac{\sqrt{3}i}{2\sqrt{2}}\right|^2 = \tfrac{4+2\sqrt{3}}{8}$$

$$\Pr[-] = \left|\tfrac{1}{2\sqrt{2}} - \tfrac{\sqrt{3}i}{2\sqrt{2}}\right|^2 = \tfrac{4-2\sqrt{3}}{8}$$



$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$