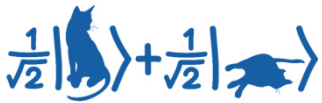
Quantum Mechanics and Quantum Computation

Umesh Vazirani, UC Berkeley



Lecture 2: Qubits and the axioms of quantum mechanics

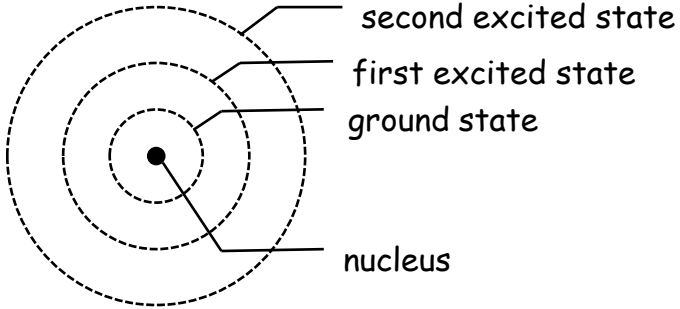
Superposition Principle

Today's Lecture

- Qubits
- Superposition principle, measurement
- Simple uncertainty principle for qubits.

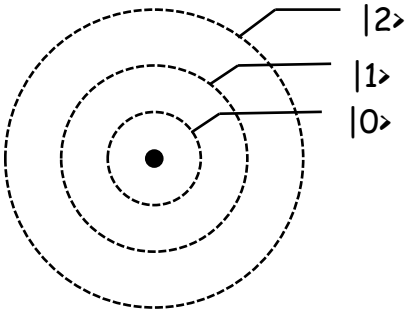
- **Quantization**: certain quantities are only allowed to take on a discrete set of values.

Energy of an electron in an atom



- Quantization occurs when the particle is confined.
- Will explain how this occurs later in the course.

Superposition axiom



- Suppose we have a k-level quantum system.
  - k distinguishable or classical states for the system.
  - Possible classical states: $|0\rangle, |1\rangle, \ldots, |k-1\rangle$.
- **Superposition principle**: if a system can be in one of k states, it can also be in any linear superposition of those k states.

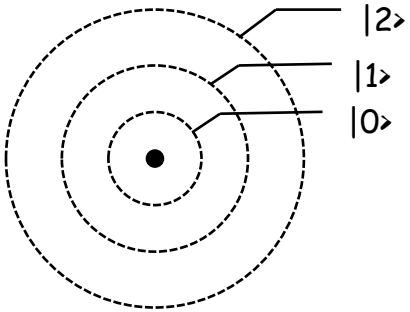
$$\alpha_0|0\rangle + \alpha_1|1\rangle + \cdots + \alpha_{k-1}|k-1\rangle$$

$a_i \in \mathbb{C}$

"probability amplitude"

$$\sum_{i=0}^{k-1} |a_i|^2 = 1$$

Measurement axiom



- Suppose that our system is in state

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \cdots + \alpha_{k-1}|k-1\rangle$$

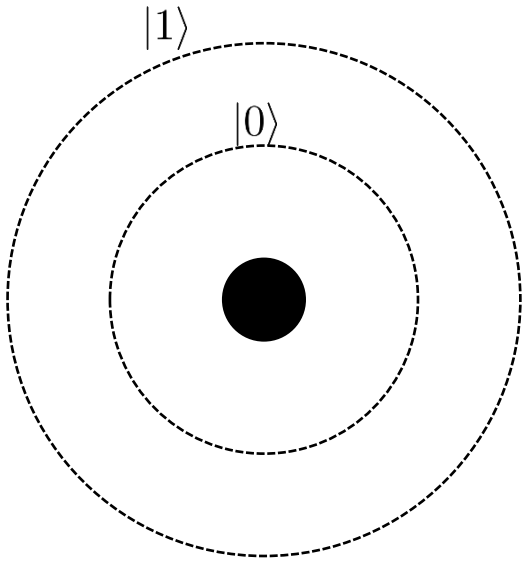
.

- Measure: outcome is one of the k classical states.
  - Observe j with probability $|a_j|^2$.
  - New state = $|j\rangle$.

Qubit

- Two-level systems are called **qubits**. (k=2)

$$\frac{1}{\sqrt{2}}|0\rangle + \left(\frac{1}{2} + \frac{i}{2}\right)|1\rangle$$



Quantization

- The state of a system can only take on a discrete set of values.
- We will have to wait a few weeks to really understand this phenomenon.
- For the next few lectures, simply assume that the systems we deal with are quantized.

Axioms of quantum mechanics

- Quantum mechanics is very simple, with only three axioms.
  - Superposition axiom
  - Measurement axiom
  - Unitary evolution axiom