Quantum Mechanics & Quantum Computation

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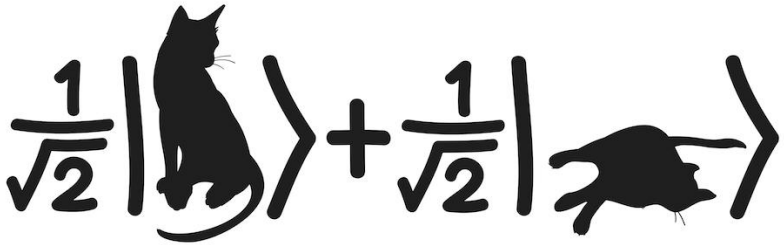
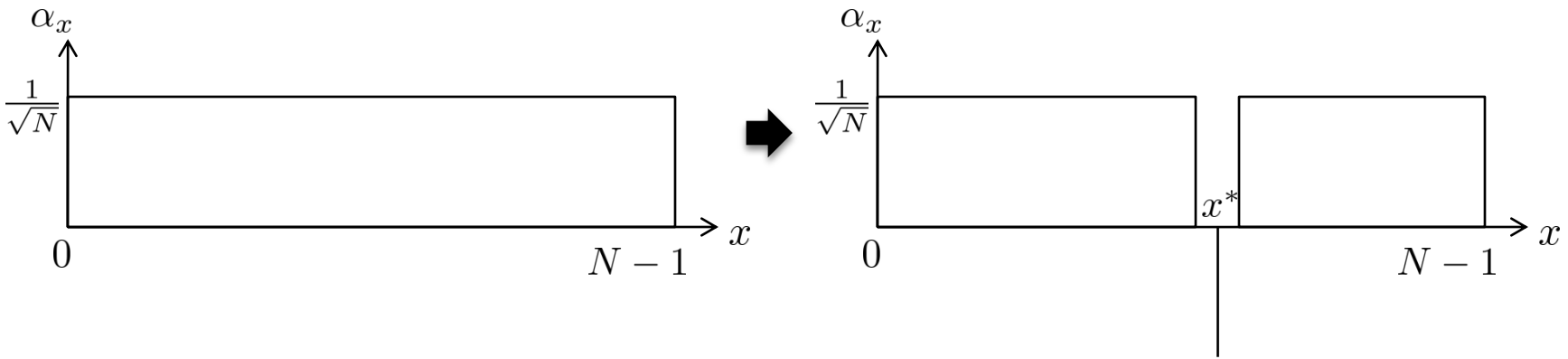


Lecture 15: Quantum Search

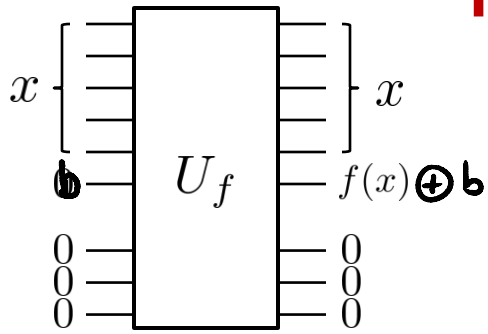
Implementing Grover's Algorithm

Phase Inversion

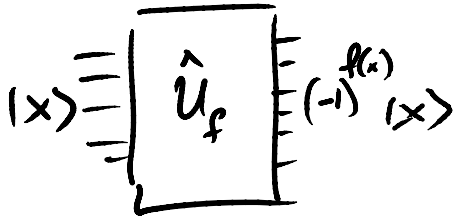
Problem. Given $f : \{0, \ldots, N-1\} \to \{0, 1\}$ such that $f(x) = 1$ for exactly one x, find x.



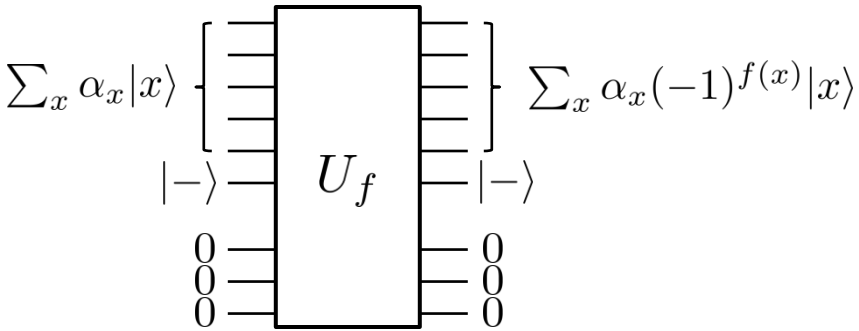
Phase Inversion

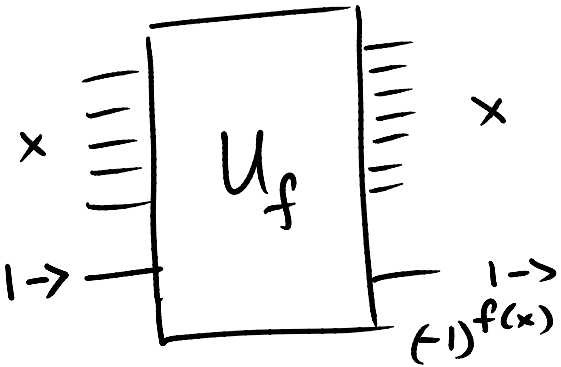


How do we send f(x) to the phase?



$$\sum_x \alpha_x |x\rangle \rightarrow \sum_x (-1)^{f(x)} \alpha_x |x\rangle$$





$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

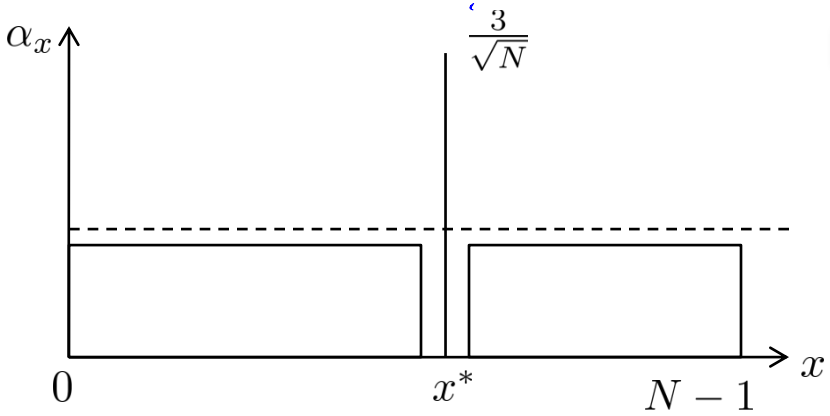
$|-\rangle \rightarrow \frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|0\rangle$

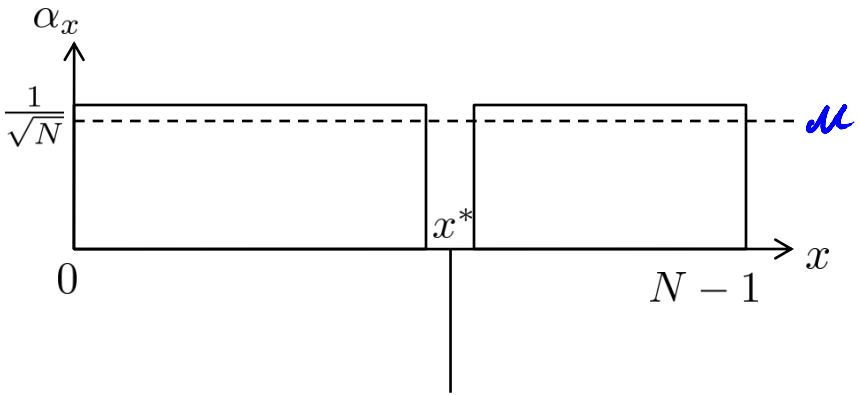
Case 1: $f(x)=0$ $\quad |-\rangle \rightarrow |-\rangle$ $\quad$ Case 2: $f(x)=1$ $\quad = -|-\rangle$

Reflection About Mean

$$\sum_x \alpha_x |x\rangle$$

$$\mu = \frac{\sum \alpha_x}{N}$$

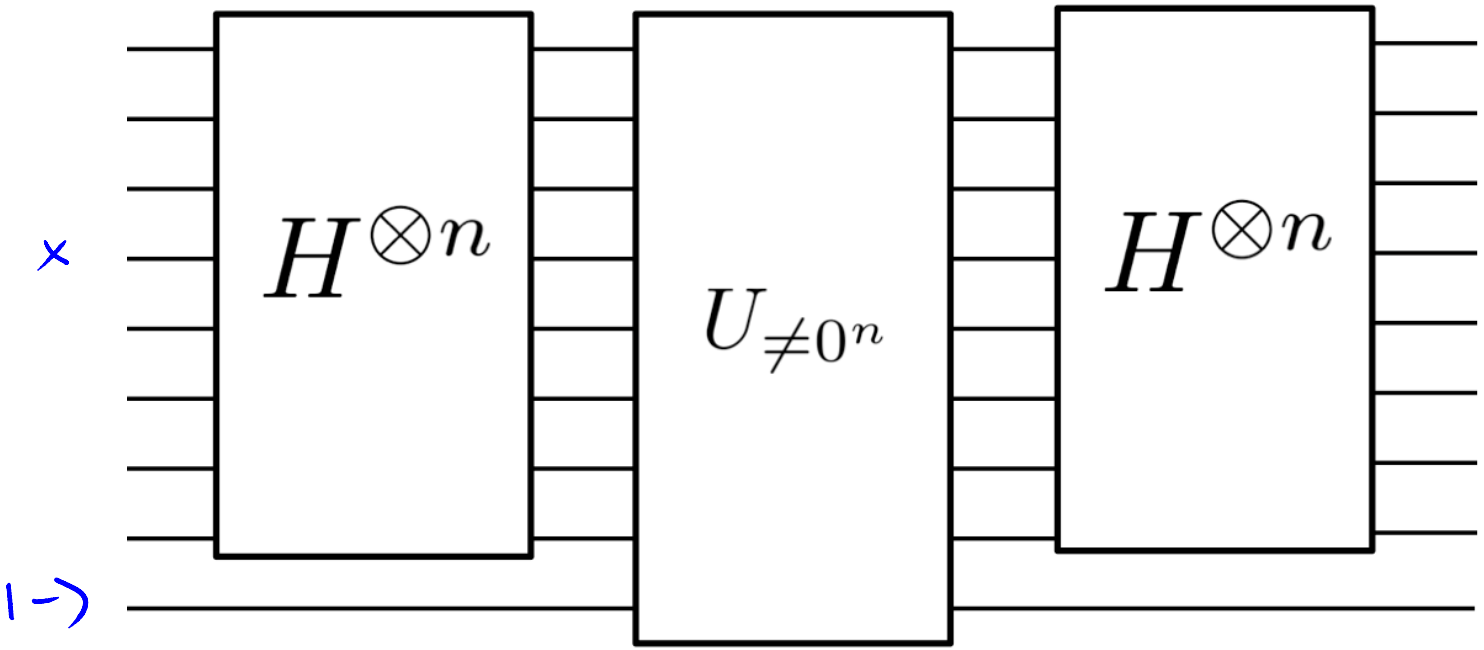




$$\sum_x (2\mu - \alpha_x)|x\rangle$$

Reflection About Mean

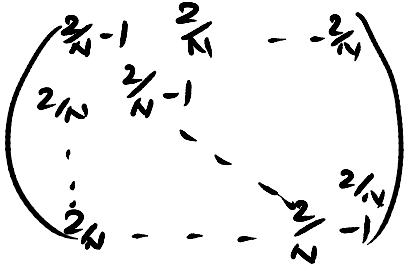
$$g(x) = \begin{cases} 0 & \text{if } x = 0\cdots0 \\ 1 & \text{o.w.} \end{cases}$$



Reflection about the mean is the same as doing reflection about $|u\rangle = \frac{1}{\sqrt{N}} \sum_x |x\rangle$

$$H^{\otimes n} \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & \ddots & \\ & & & -1 \end{pmatrix} H^{\otimes n}$$

$|0\cdots0\rangle$



$$= H^{\otimes n} \left[\begin{pmatrix} 2 & & \\ & \ddots & \\ & & 0 \end{pmatrix} - \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix} \right] H^{\otimes n}$$

$$= H^{\otimes n} \begin{pmatrix} 2 & & \\ & \ddots & \\ & & 0 \end{pmatrix} H^{\otimes n} - \underbrace{H^{\otimes n} I H^{\otimes n}}_{I} = \begin{pmatrix} \frac{2}{N} & \frac{2}{N} & \cdots & \frac{2}{N} \\ \frac{2}{N} & & & \\ \vdots & & & \\ \frac{2}{N} & \cdots & \cdots & \frac{2}{N} \end{pmatrix} - I$$

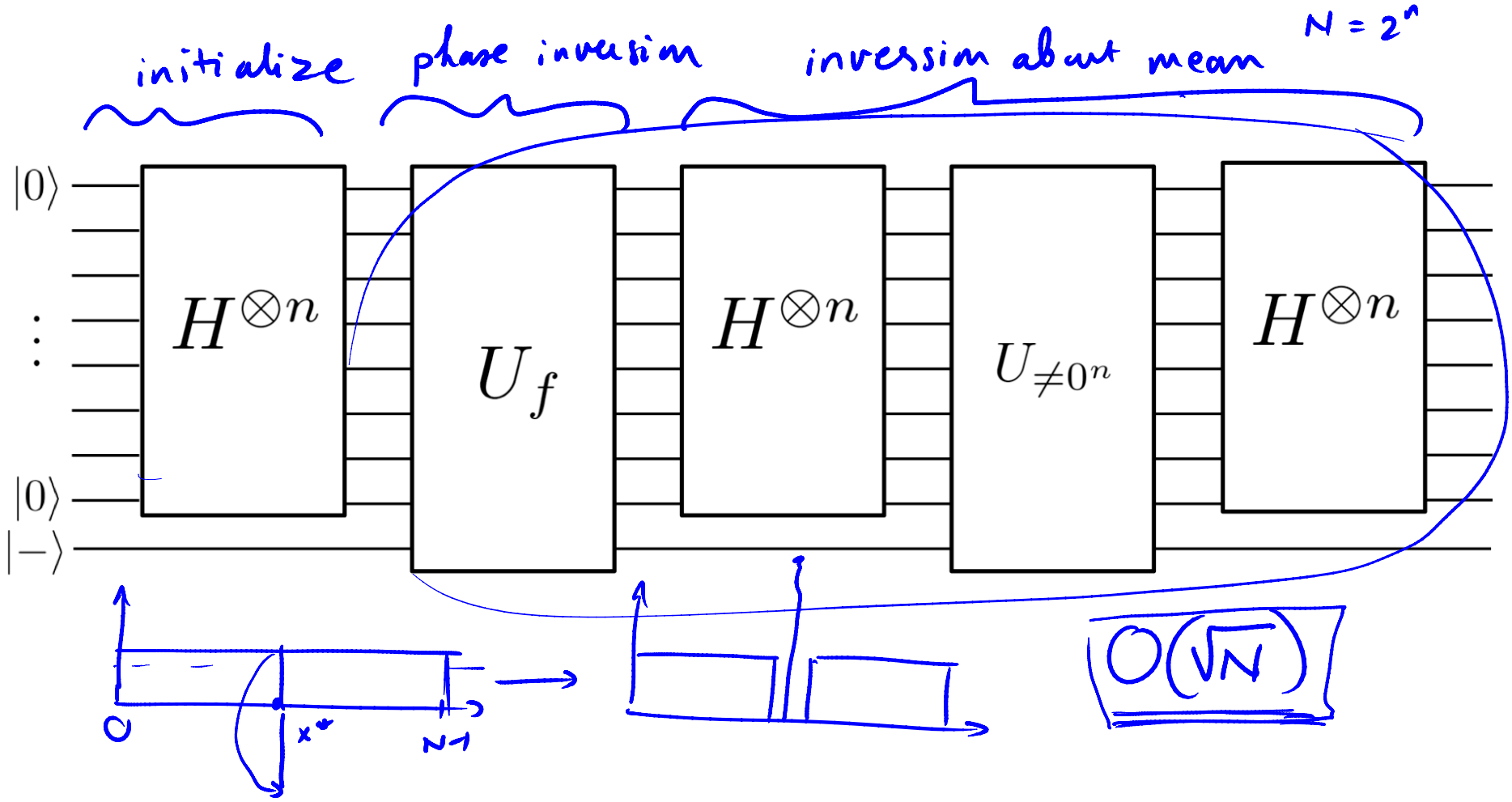
Reflection about the mean is the same as doing reflection about $|u\rangle = \frac{1}{\sqrt{N}} \sum_x |x\rangle$

$$H^{\otimes n} \begin{pmatrix} 1 & & 0 \\ & -1 & \\ & & \ddots \\ 0 & & -1 \end{pmatrix} H^{\otimes n} = \begin{pmatrix} \frac{2}{N}-1 & \frac{2}{N} & \cdots & \frac{2}{N} \\ \frac{2}{N} & \ddots & & \\ \vdots & & \ddots & \frac{2}{N} \\ \frac{2}{N} & & \frac{2}{N} & \frac{2}{N}-1 \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \vdots \\ \alpha_{N-1} \end{pmatrix}$$

$$\sum_x \alpha_x |x\rangle \longrightarrow \sum_x (2\mu - \alpha_x)|x\rangle = \begin{pmatrix} 2\mu - \alpha_0 \\ 2\mu - \alpha_1 \\ \vdots \\ 2\mu - \alpha_{N-1} \end{pmatrix}$$

$$\frac{2}{N}(\alpha_0 + \alpha_1 + \cdots + \alpha_{N-1})$$

$$= 2\mu$$


initialize
phase inversion
inversion about mean
$N = 2^n$
$|0\rangle$
$|0\rangle$
$|-\rangle$
$H^{\otimes n}$
U_f
$H^{\otimes n}$
$U_{\neq 0^n}$
$H^{\otimes n}$
0
x^*
N-1
$O(\sqrt{N})$