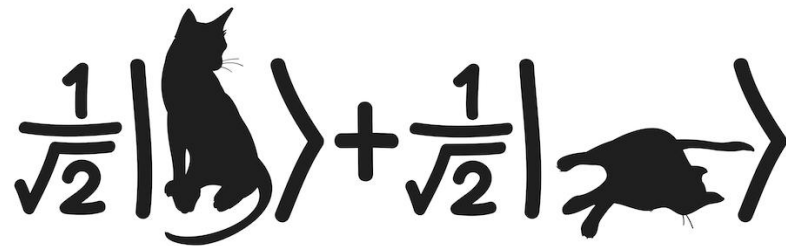


Quantum Mechanics & Quantum Computation

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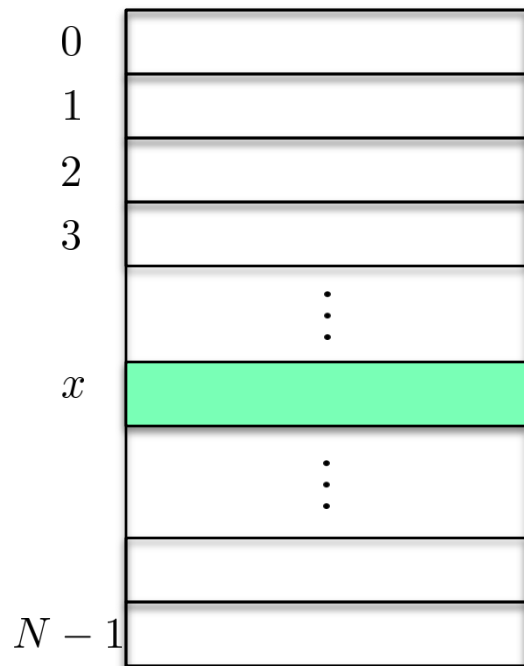
Lecture 15: Quantum Search

Grover's Algorithm

Unstructured search

$$N = 2^n$$

“Digital haystack”



Problem. Given $f: \{0, 1, \dots, N-1\} \rightarrow \{0, 1\}$,
find $x: f(x) = 1$.

Hardest case: There is exactly one $x: f(x) = 1$.

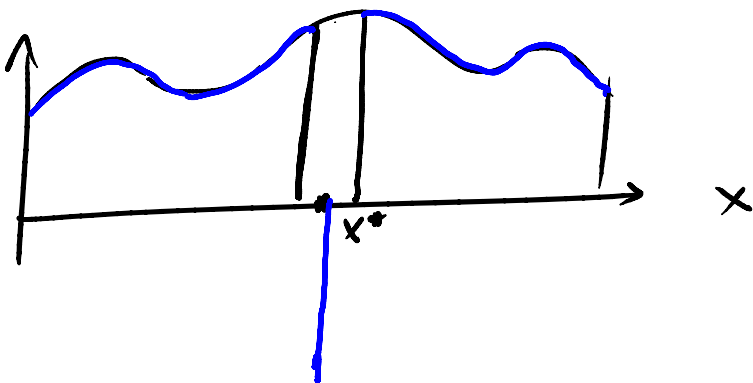
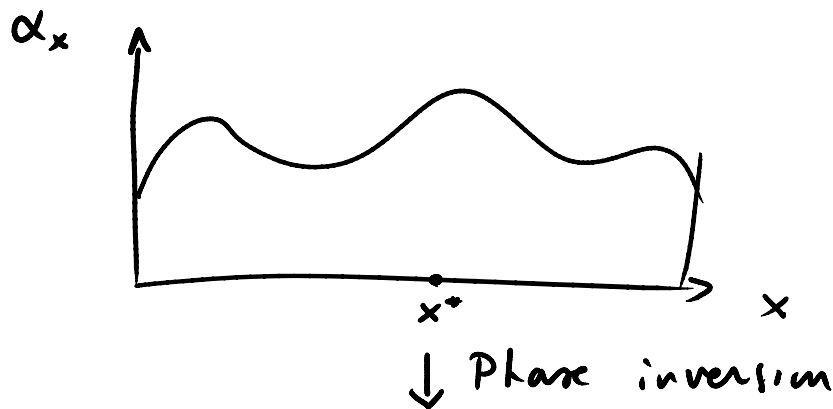
Phase Inversion

$$f(x^*) = 1$$

$$\sum_x \alpha_x |x\rangle$$

↓ Phase inversion

$$\sum_{x \neq x^*} \alpha_x |x\rangle - \alpha_{x^*} |x^*\rangle$$

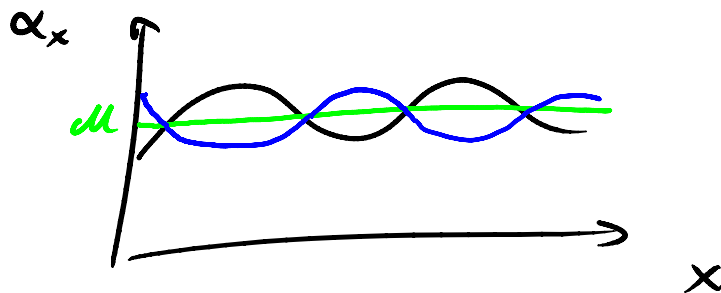


Inversion About Mean

$$\sum_x \alpha_x |x\rangle$$



$$\sum_x (2\mu - \alpha_x) |x\rangle$$

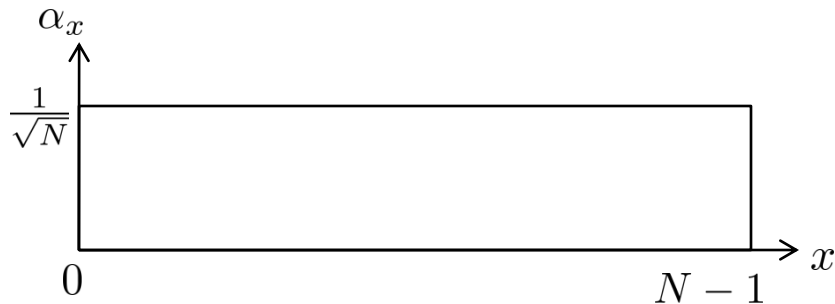


$$\alpha_x \rightarrow \mu + (\mu - \alpha_x)$$

$$\mu - \alpha_x$$

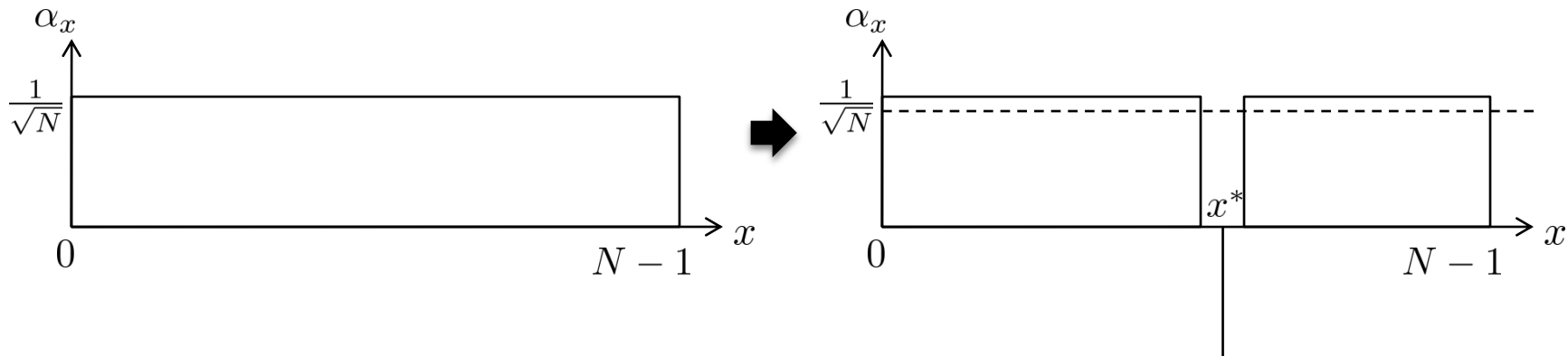
Grover's algorithm

Problem. Given $f : \{0, \dots, N - 1\} \rightarrow \{0, 1\}$ such that $f(x) = 1$ for exactly one x , find x .



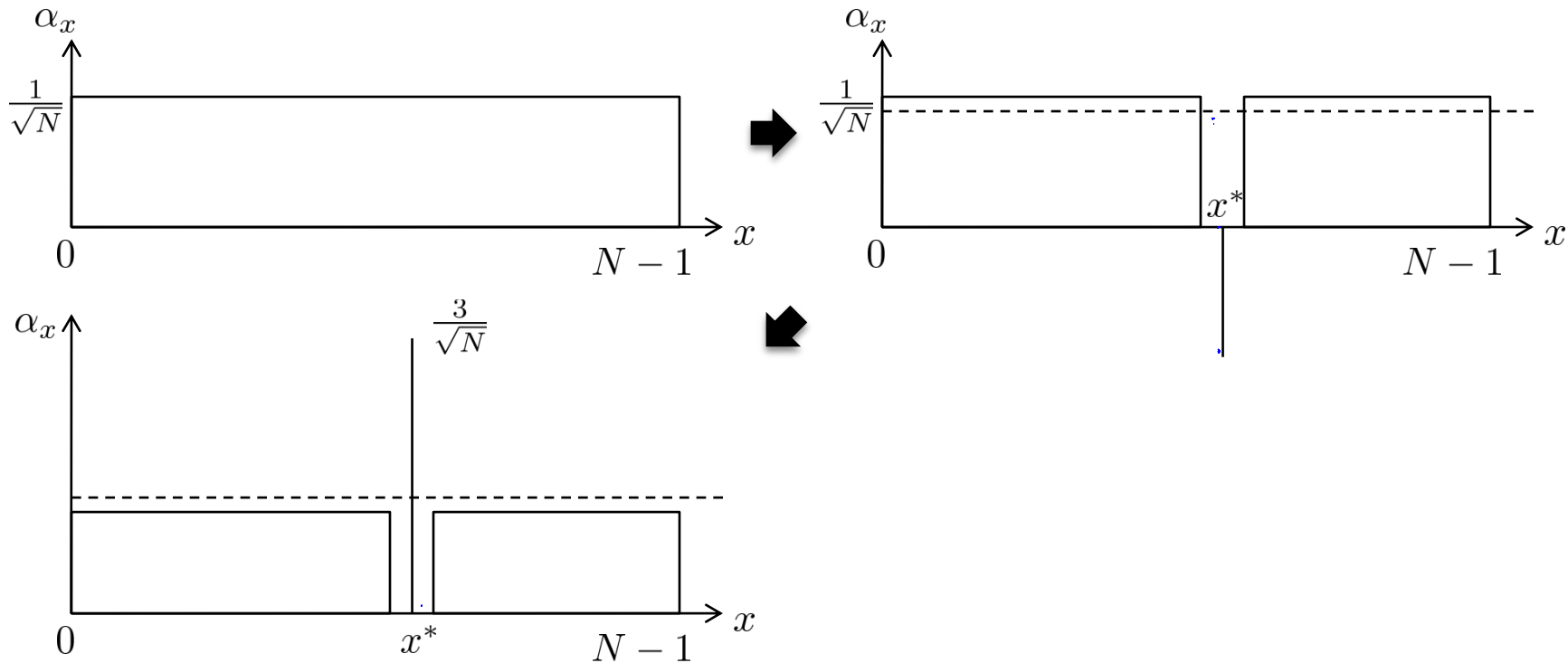
Grover's algorithm

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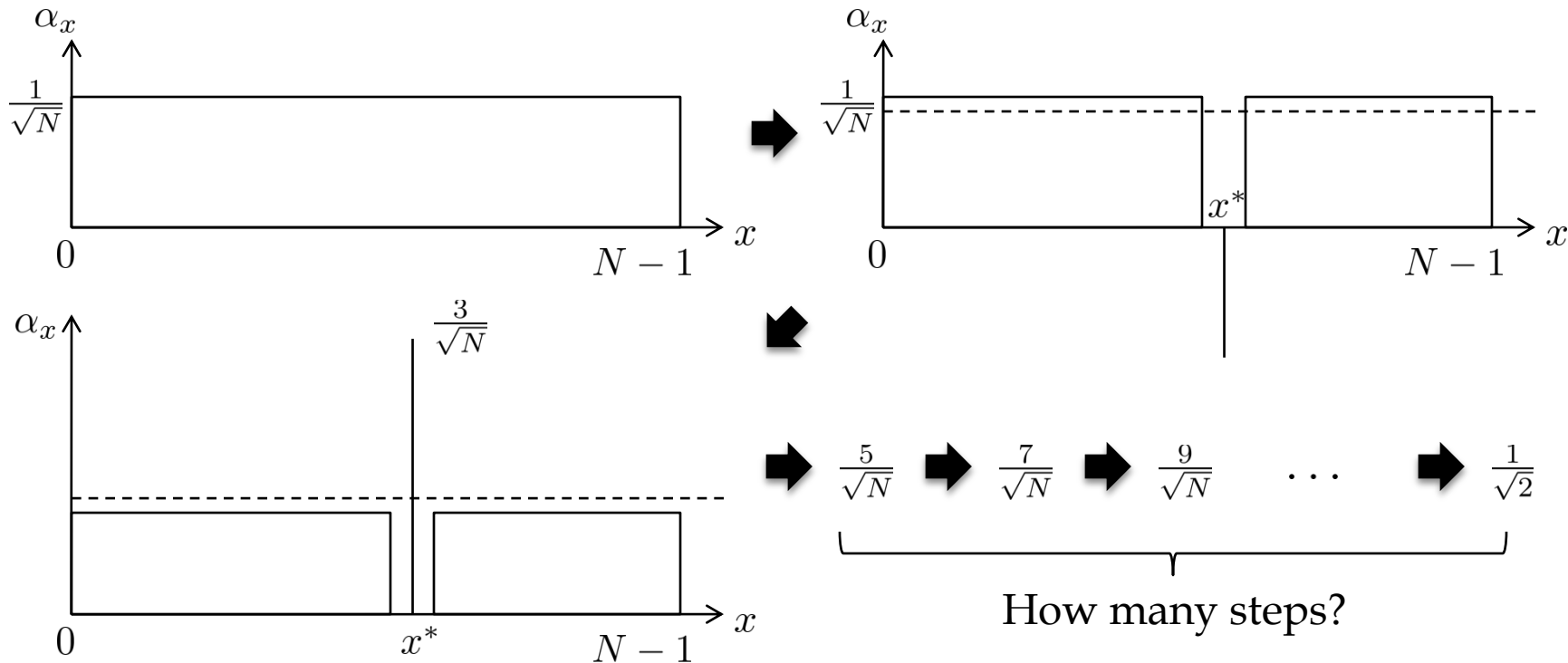
Grover's algorithm

Problem. Given $f : \{0, \dots, N-1\} \rightarrow \{0, 1\}$ such that $f(x) = 1$ for exactly one x , find x .



Grover's algorithm

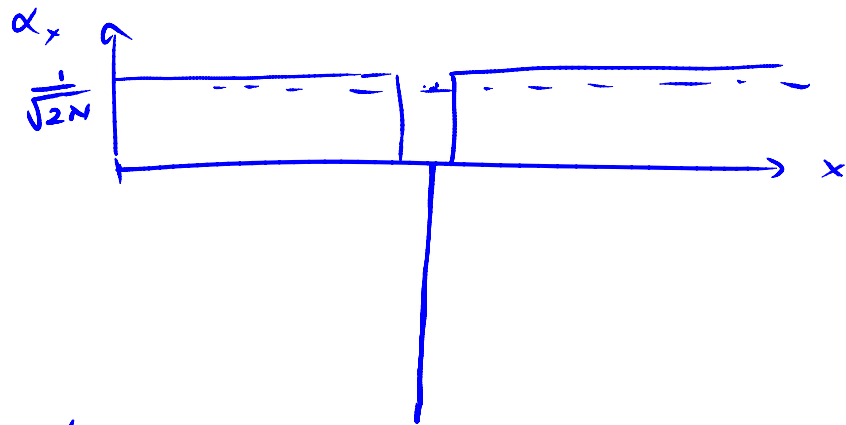
Problem. Given $f : \{0, \dots, N-1\} \rightarrow \{0, 1\}$ such that $f(x) = 1$ for exactly one x , find x .



Grover's algorithm

What is the amplitude of the rest when the needle has $\frac{1}{\sqrt{2}}$? $\frac{1}{\sqrt{2N}}$

At this point how much improvement are we making per step? $2 \times \frac{1}{\sqrt{2N}} = \sqrt{\frac{2}{N}}$



We will reach $\frac{1}{\sqrt{2}}$ in $O(\sqrt{N})$ steps.

$$\frac{1/\sqrt{2}}{1/\sqrt{2N}} = \frac{\sqrt{N}}{2} \text{ steps.}$$