Quantum Mechanics & Quantum Computation

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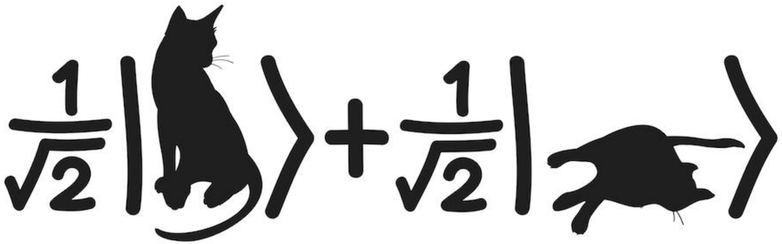


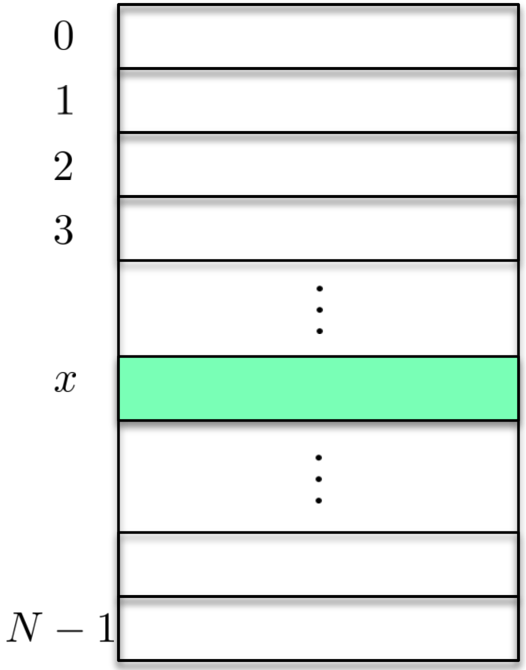
Lecture 15: Quantum Search

Needle in a haystack

Searching for a needle in a haystack

Unstructured search

"Digital haystack"



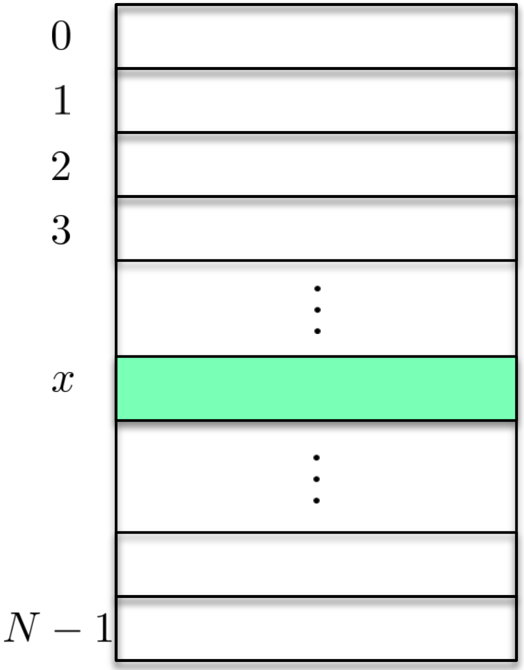
Goal: Search for the marked entry.

Classically: try random entries.
O(N/2) expected time.

Quantum??

Unstructured search

"Digital haystack"

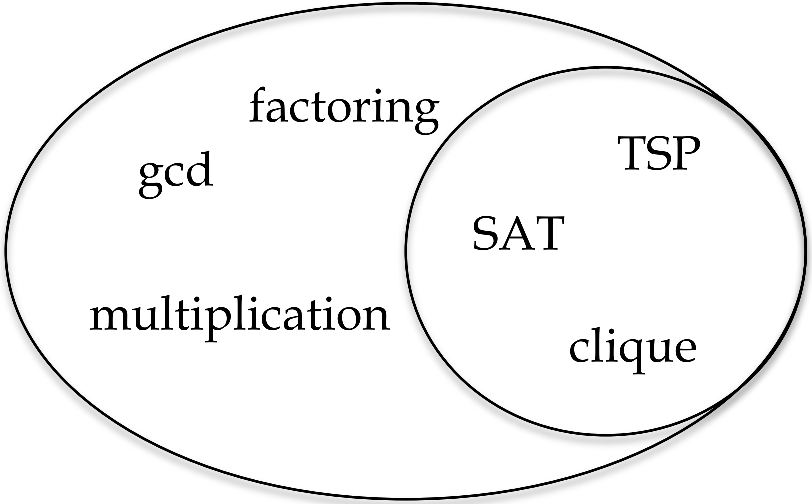


Quantum??

NP-Complete Problems:

What does it mean?

For most computational problems, finding an answer is very difficult, but checking an answer is easy.



Finding a solution to an NP-complete problem can be viewed as a search problem.

$$(x_1 \vee \neg x_2 \vee x_3) \wedge (x_2 \vee \neg x_5 \vee x_6) \wedge \cdots$$

Is there a configuration of $x_1, x_2, \cdots$ that satisfy the above formula?

There are 2^n possible configurations.

Unstructured search

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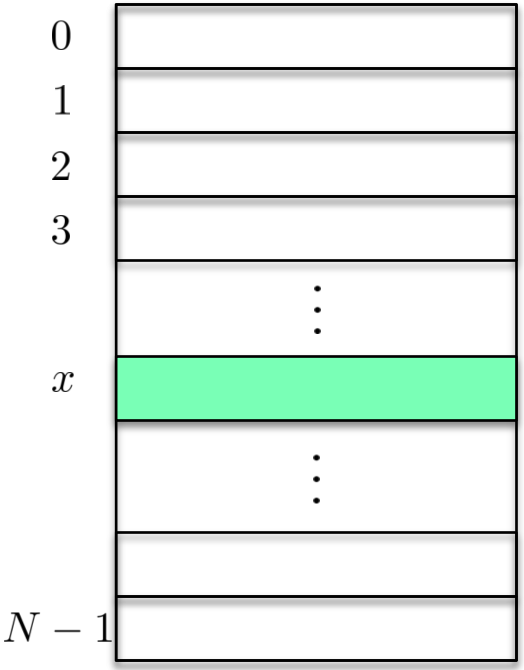

$N = 2^n$

NP-Complete Problems:

Satisfiability:

Finding a solution to an NP-complete problem can be viewed as a search problem.

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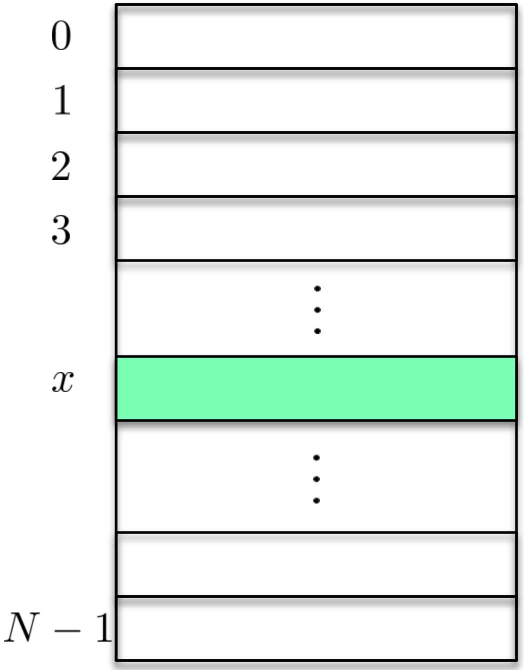


n qubits $\quad 2^n = N$

$$\frac{1}{\sqrt{N}}\sum_{y=0}^{N-1}|y\rangle \longrightarrow U_f \longrightarrow \sum_y |y\rangle|f(y)\rangle$$

Random y

No better than probing random entry!!

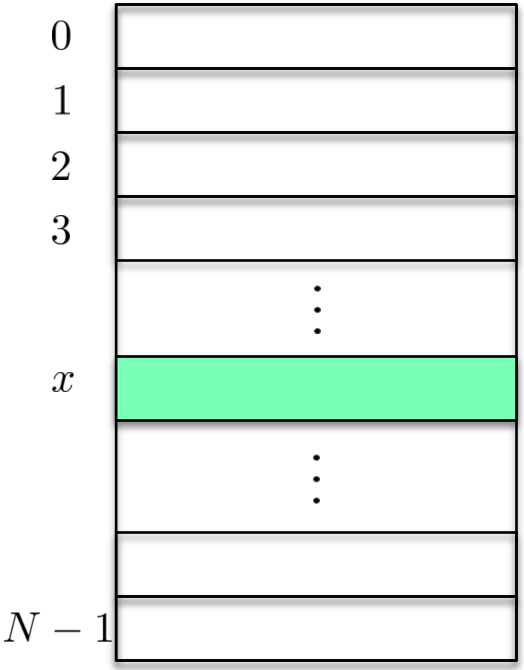
$$f(x) = (x_1 \vee \bar{x}_2 \vee x_3) \wedge (\quad) \wedge (\quad) \cdots \wedge (\quad)$$

$x = x_1 x_2 \ldots x_n$

Unstructured search

$N = 2^n$

"Digital haystack"



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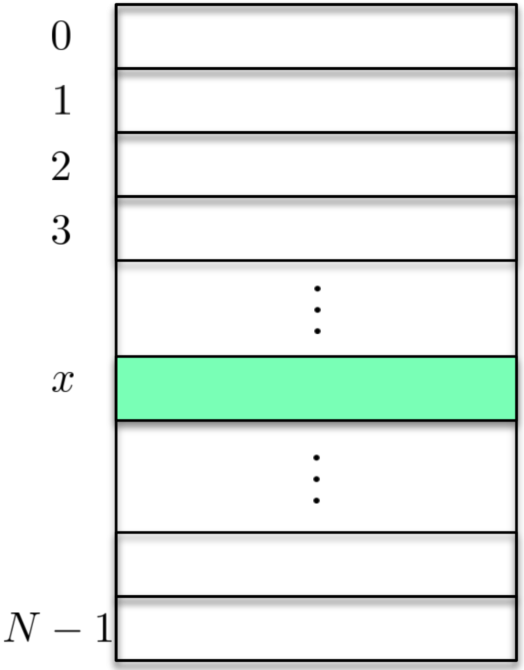
Quantum??

Theorem: Any quantum algorithm must take at least $\sqrt{N}$ time.

$= 2^{n/2}$

Unstructured search

"Digital haystack"



Quantum??

Theorem: Any quantum algorithm must take at least √N time.

Grover's Algorithm: Quantum algorithm for unstructured search that takes O(√N) time.

Unstructured search

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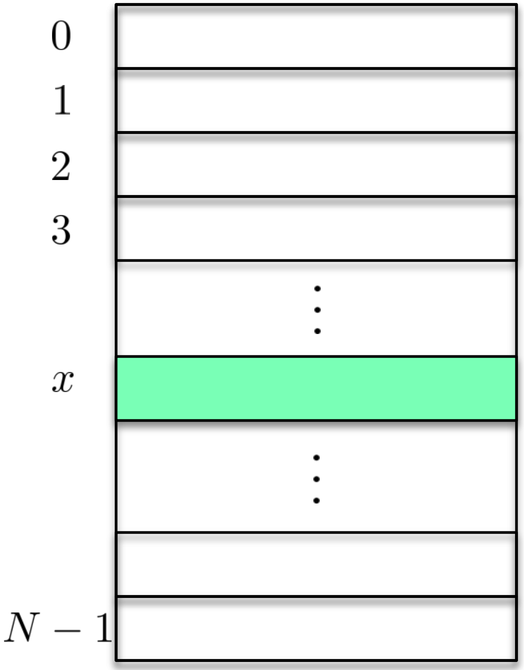

Problem. Given f:{0, 1, …, N−1} → {0,1}, find x: f(x) = 1.

Hardest case: There is exactly one x: f(x) = 1.