Quantum Mechanics & Quantum Computation

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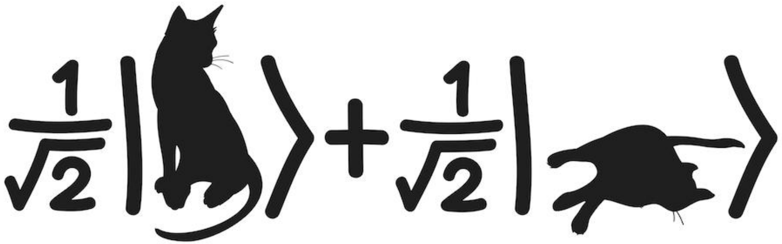


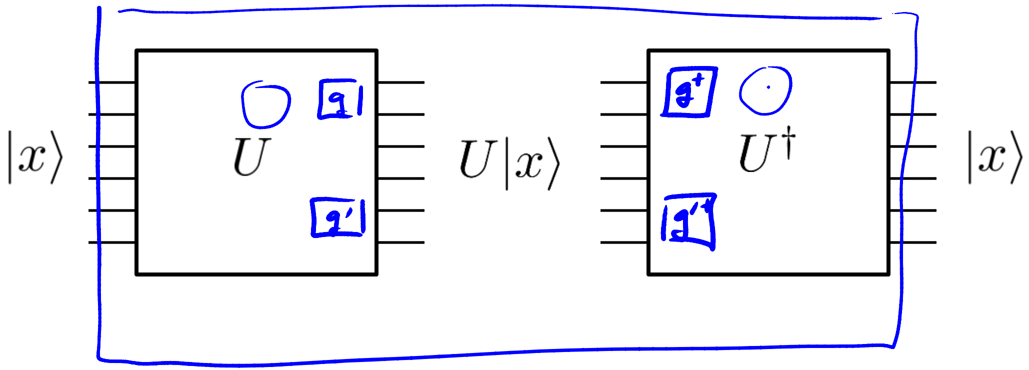
Lecture 11: Quantum Circuits

Reversible Computation

Reversible computation

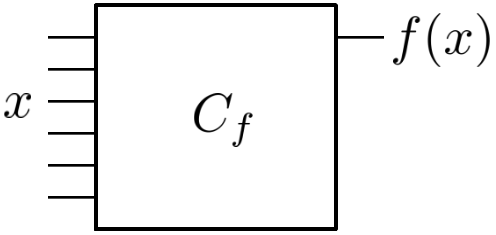
- Quantum computers are reversible.

- Why?



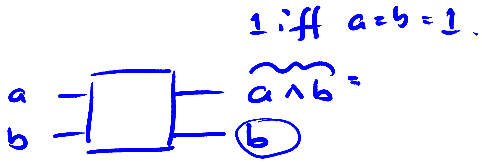
$UU^\dagger = U^\dagger U = I.$

Implementing classical circuits

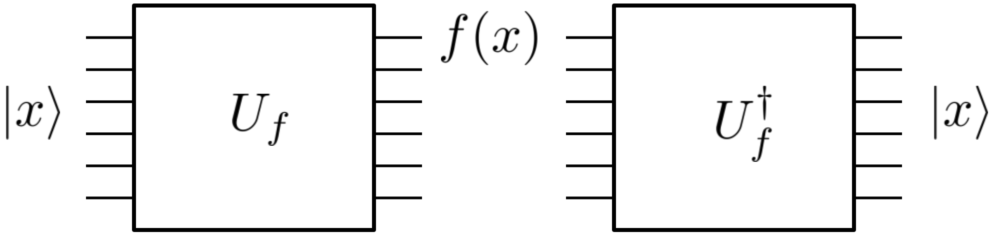


Classical circuit for computing a boolean function

$$f : \{0,1\}^n \to \{0,1\}$$



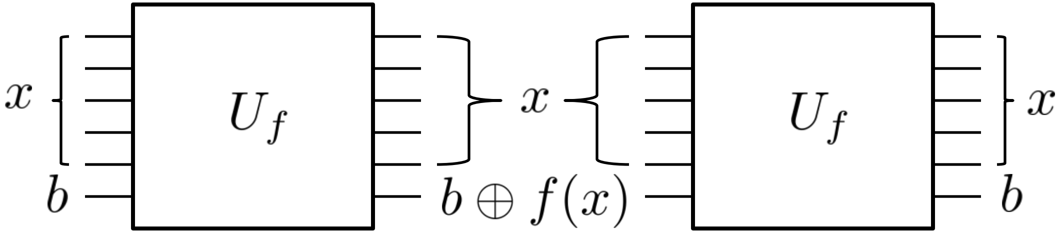
Imagine a quantum version of C_f:



Have to be reversible.

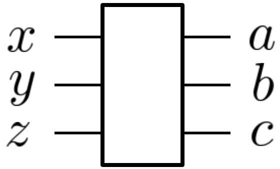
But classical gates throw away information!

Classical reversible computation



$b = 0$

Consider C-SWAP gate:



$$a = x$$

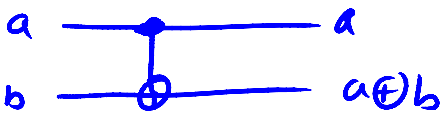
$$x = 0 \quad \Rightarrow \quad b = y, c = z$$

$$x = 1 \quad \Rightarrow \quad b = z, c = y$$

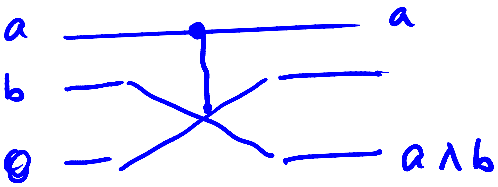
NOT

$|0\rangle - \boxed{X} \rightarrow |1\rangle$

CNOT

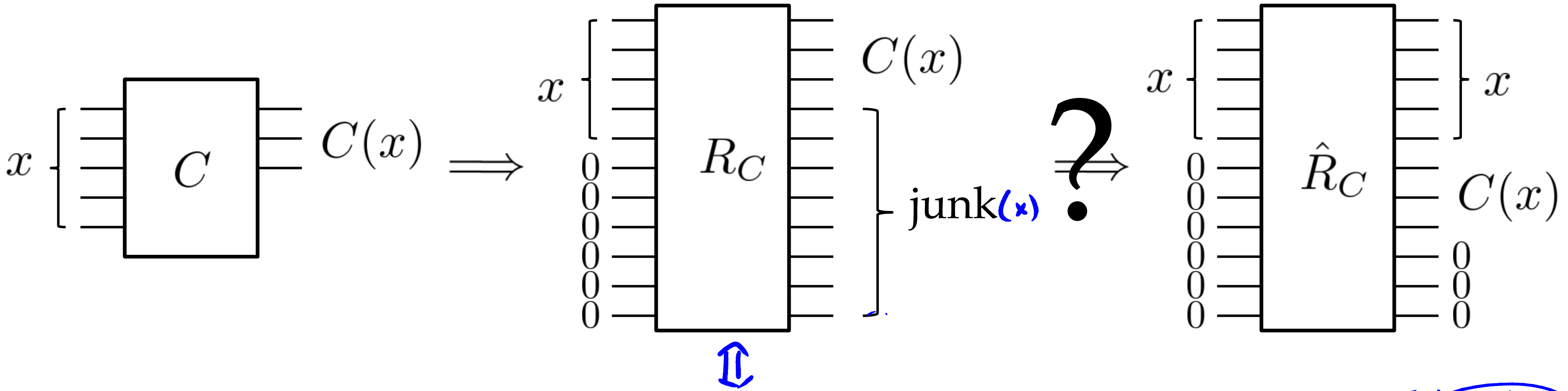


C-SWAP

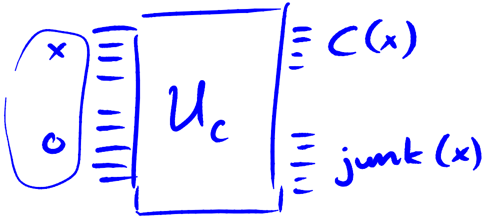


if a=0	0
if a=1	b

Classical reversible computation



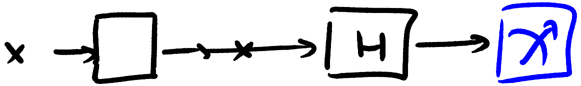
$\sum_x \alpha_x |x\rangle |0\rangle$



$\sum_x \alpha_x |C(x)\rangle \cancel{|junk(x)\rangle}$

Why remove junk?

Prevents interference.

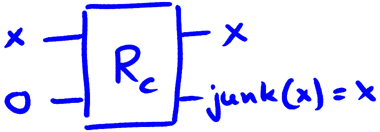


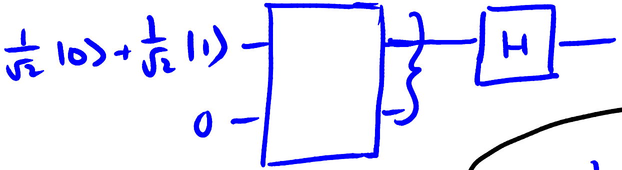
$$\sum \alpha_x |x\rangle \longrightarrow \sum \alpha_x |x\rangle$$

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \longrightarrow |0\rangle$$

$$\frac{1}{\sqrt{2}}|0\rangle \xrightarrow{H} \frac{1}{2}|0\rangle + \cancel{\frac{1}{2}|1\rangle}$$

$$\frac{1}{\sqrt{2}}|1\rangle \xrightarrow{H} \frac{1}{2}|0\rangle - \cancel{\frac{1}{2}|1\rangle}$$



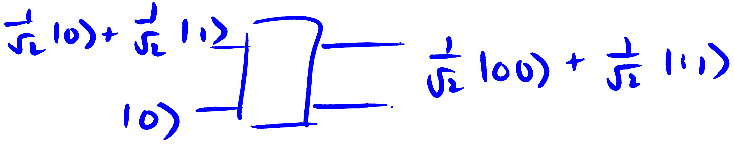


$$\underbrace{\frac{1}{2}|00\rangle + \frac{1}{2}|10\rangle} + \underbrace{\frac{1}{2}|01\rangle - \frac{1}{2}|11\rangle}$$

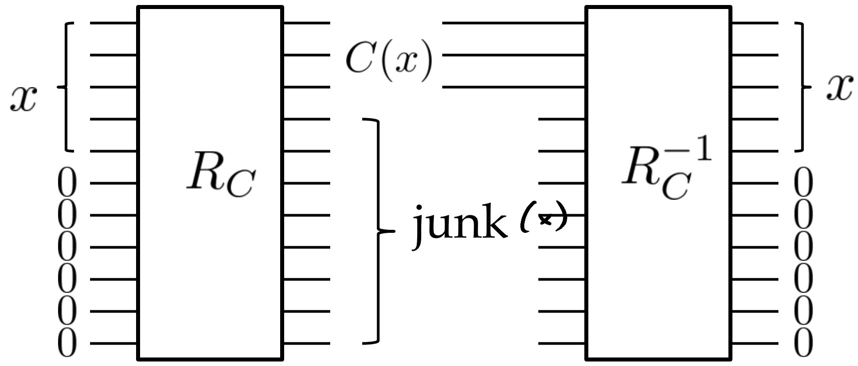
$$\underline{\underline{\frac{1}{\sqrt{2}}|00\rangle}} + \underline{\underline{\frac{1}{\sqrt{2}}|11\rangle}}$$

Why remove junk?

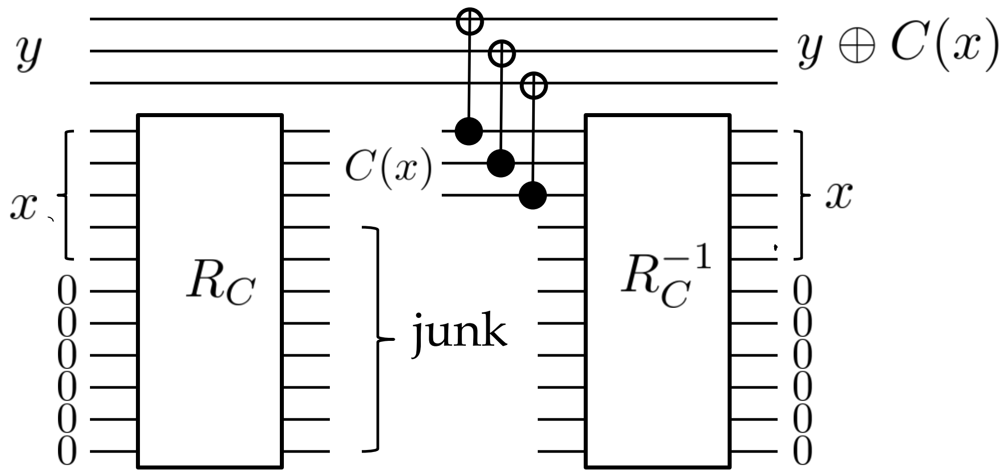
- Can’t we just throw away the junk qubits?

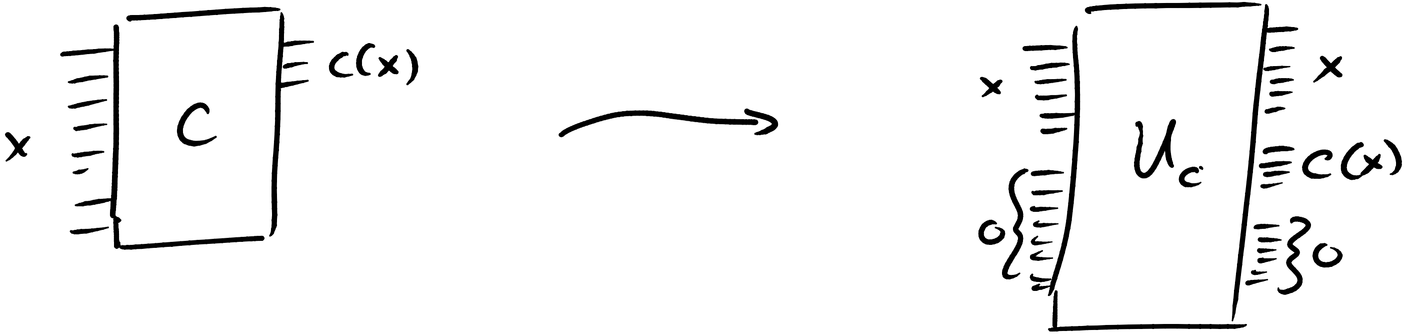


Classical reversible computation



Classical reversible computation





$$\sum_x \alpha_x |x\rangle |0\cdots0\rangle \xrightarrow{U_c} \sum_x \alpha_x |x\rangle |C(x)\rangle |0\cdots0\rangle$$

$$|xy\rangle = |x\rangle|y\rangle = |x\rangle \otimes |y\rangle$$