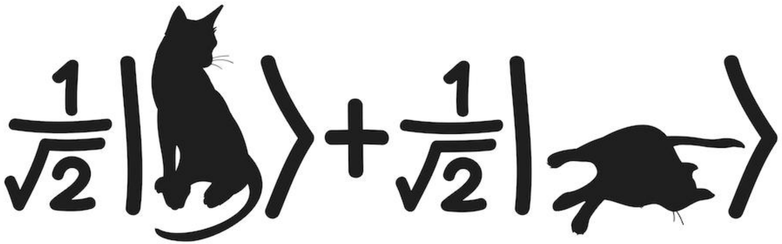
Quantum Mechanics & Quantum Computation



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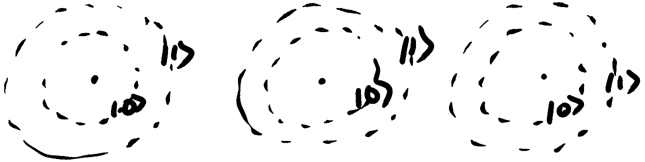
Lecture 11: Quantum Circuits

n qubit systems

Exponential Growth



- One qubit $\in \mathbb{C}^2$ $\alpha_0|0\rangle + \alpha_1|1\rangle$
- Two qubits $\in \mathbb{C}^4$ $\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$
- Three qubits $\in \mathbb{C}^8$ $\alpha_{000}|000\rangle + \alpha_{001}|001\rangle + \alpha_{010}|010\rangle + \alpha_{011}|011\rangle + \cdots + \alpha_{111}|111\rangle$

  $= \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$

- n qubits $\in \mathbb{C}^{2^n}$ $\alpha_{0\ldots00}|0\cdots00\rangle + \alpha_{0\ldots01}|0\cdots01\rangle + \cdots + \alpha_{1\ldots11}|1\cdots11\rangle$

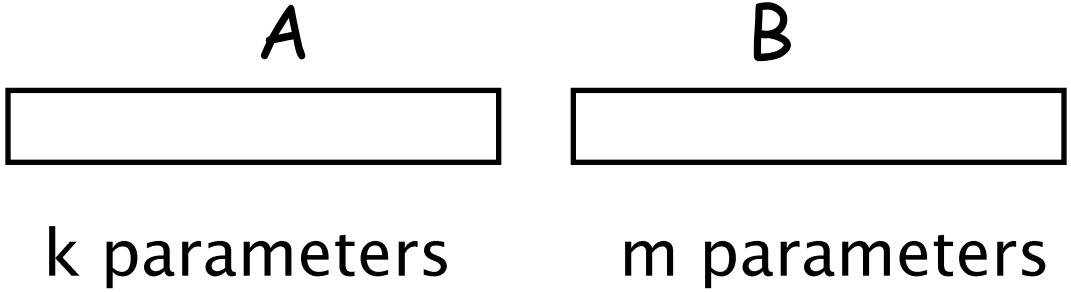
  $\mathbb{C}^{2^n} = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2$

$n \approx 500$

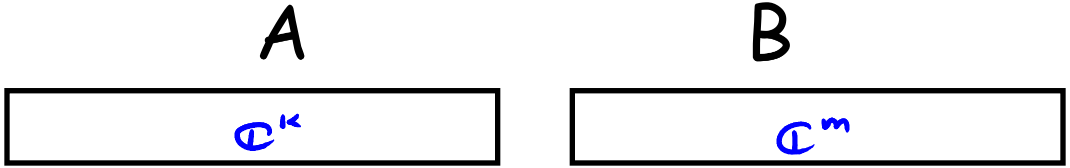
$2^n = 2^{500}$

$>$ (# particles in the universe) $\cdot$ (age of universe in femtoseconds)

Tensor Products

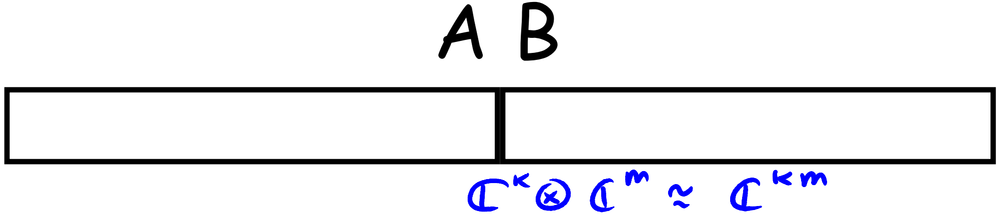


Tensor Products



k parameters

m parameters



km parameters

Quantum entanglement

Exponential Growth

Axiom 1: Superposition principle

n qubits

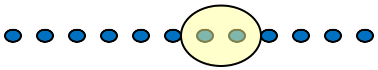
Exponential Hilbert Space

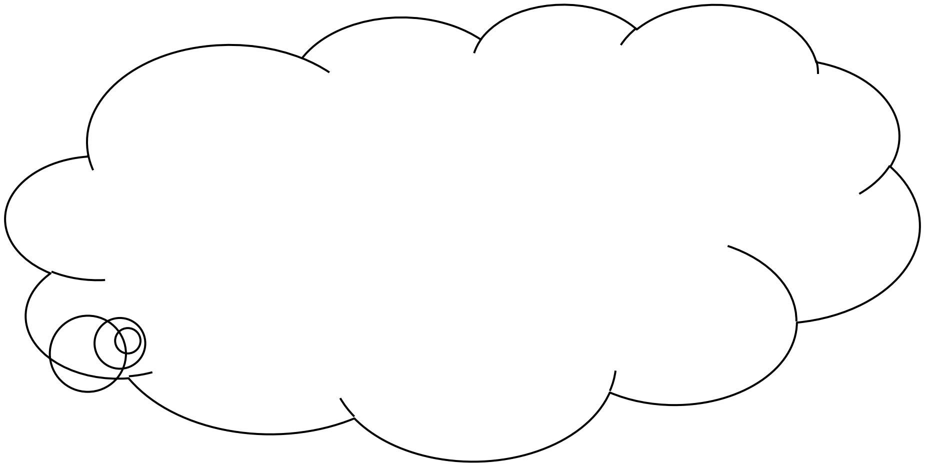
$$\Psi = \sum_x \alpha_x |x\rangle$$

$$\sum_x |\alpha_x|^2 = 1$$

all n-bit strings

Axiom 2: Unitary Evolution



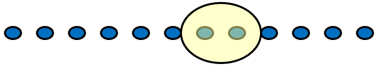


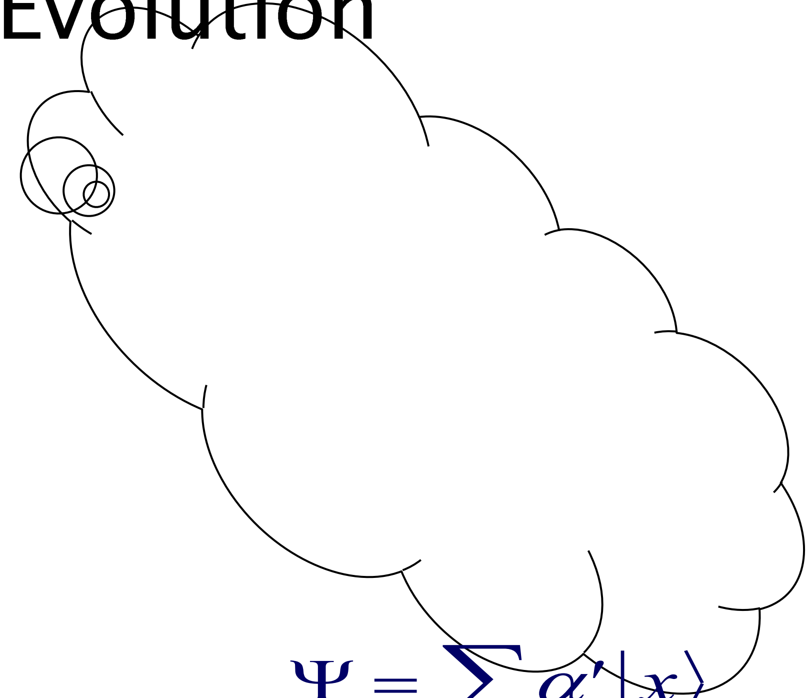
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{bmatrix} \otimes I_{n-2}$$

$$\Psi = \sum_x \alpha_x |x\rangle$$

all n-bit strings

Axiom 2: Unitary Evolution

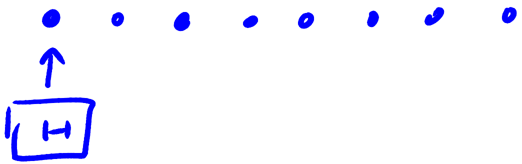




$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{bmatrix} \otimes I_{n-2}$$

$$\Psi = \sum_x \alpha'_x |x\rangle$$

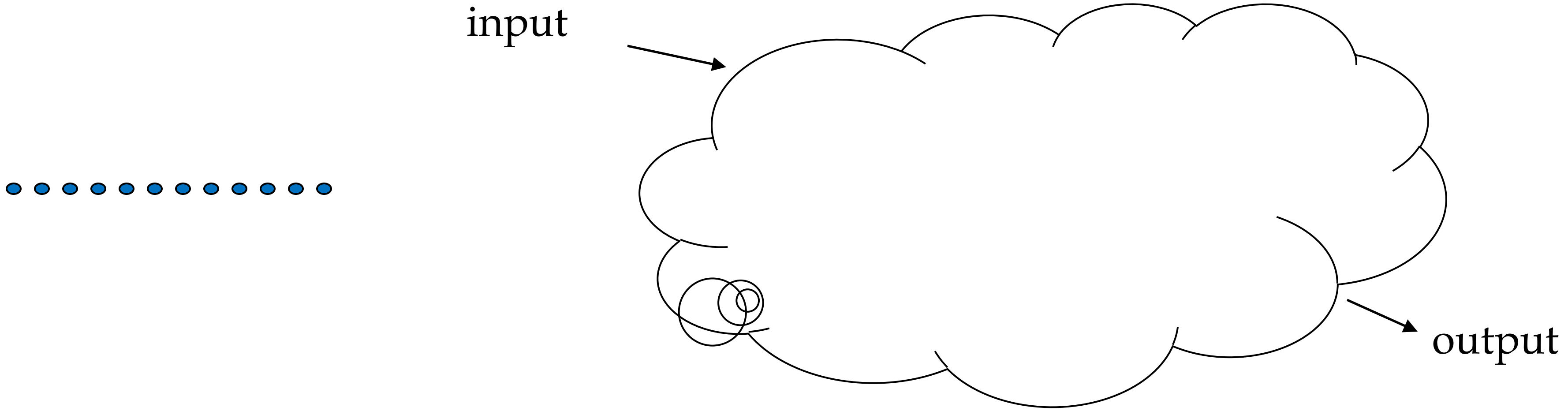
all n-bit strings



$$\left.\begin{array}{l} \alpha_{0x'} \; |0x'\rangle \\ \alpha_{1x'} \; |1x'\rangle \end{array}\right\} \qquad \begin{array}{l} \dfrac{\alpha_{0x'} + \alpha_{1x'}}{\sqrt{2}} \; |0x'\rangle \\ \dfrac{\alpha_{0x'} - \alpha_{1x'}}{\sqrt{2}} \; |1x'\rangle \end{array}$$

Updating all 2^n amplitudes α_x.

Limited Access – Measurement



$$\Psi = \sum_x \alpha_x |x\rangle$$

$$\sum_x |\alpha_x|^2 = 1$$

- Measurement: See $|x\rangle$ with probability $|\alpha_x|^2$