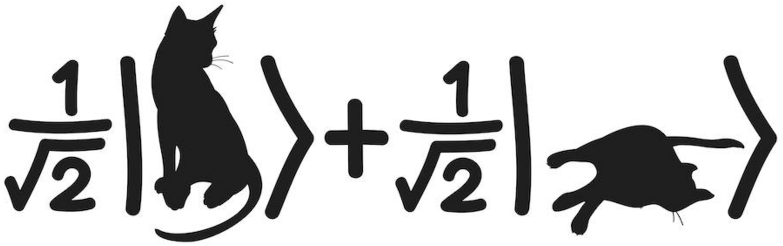
Quantum Mechanics & Quantum Computation



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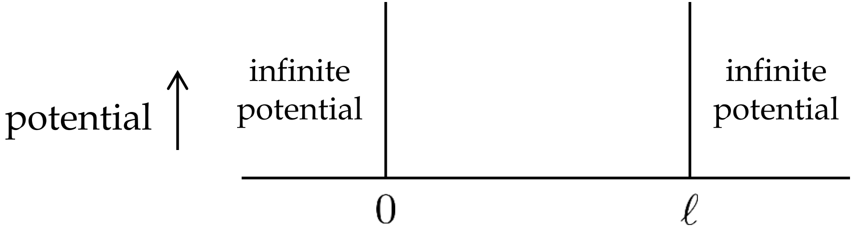
Lecture 10: Observables, Schrödinger's equation, Particle in a box

Qubits



We will solve Schrödinger's equation:

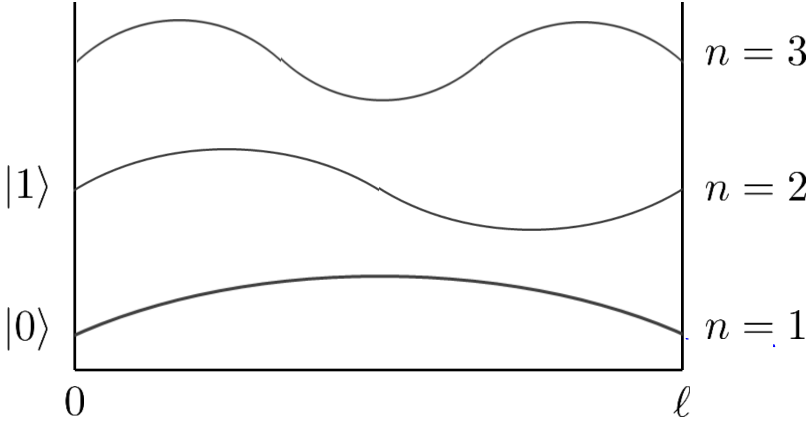
$$i\hbar\frac{\partial\psi}{\partial t} = H\psi = \frac{\hat{p}^2}{2m}|\psi\rangle + V(x)|\psi\rangle = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}|\psi\rangle$$

Boundary conditions: $\psi(0) = \psi(\ell) = 0$

Solution:

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2m\ell^2} \qquad \psi_n(x) = \sqrt{\frac{2}{\ell}}\sin\frac{n\pi x}{\ell}$$

Quantization:



Implementing qubits

- Restrict the energy to be small enough: $E < E_3$

$$|0\rangle \quad \Rightarrow \quad |\psi_1\rangle$$
$$|1\rangle \quad \Rightarrow \quad |\psi_2\rangle$$

- Suppose $|\psi(t=0)\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\sqrt{\frac{2}{\ell}}\sin\frac{\pi x}{\ell} + \beta\sqrt{\frac{2}{\ell}}\sin\frac{2\pi x}{\ell}$

- Then:

$$|\psi(t)\rangle = \alpha\sqrt{\frac{2}{\ell}}e^{-\frac{iE_1 t}{\hbar}}\sin\frac{\pi x}{\ell} + \beta\sqrt{\frac{2}{\ell}}e^{-\frac{iE_2 t}{\hbar}}\sin\frac{2\pi x}{\ell}$$

$$= \sqrt{\frac{2}{\ell}}e^{-\frac{iE_1 t}{\hbar}}\left(\alpha\sin\frac{\pi x}{\ell} + \beta e^{-\frac{i(E_2-E_1)t}{\hbar}}\sin\frac{2\pi x}{\ell}\right)$$

$\Delta E = E_2 - E_1 \approx 10\,\text{eV}$

$\nu \approx 2.5 \times 10^{15}\,\text{Hz}$