PUZZLES FROM OTHER WORLDS

MARTIN GARDNER

FANTASTICAL BRAINTEASERS FROM
ISAAC ASIMOV'S
SCIENCE FICTION MAGAZINE

PUZZLES FROM OTHER WORLDS

ALSO BY MARTIN GARDNER

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Fantastical Brainteasers from ISAAC ASIMOV'S SCIENCE FICTION MAGAZINE

Martin Gardner

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FOR DAVID B. EISENDRATH, JR.
WISE AND GOOD FRIEND FROM GRAY CITY DAYS

FOREWORD

Since *Isaac Asimov's Science Fiction Magazine* started in 1976, I have had the pleasure and honor of contributing to each issue a puzzle clothed in a science-fiction or fantasy story line. The first thirty-six of these vignettes were published in 1981 as *Science Fiction Puzzle Tales.* This book reprints the next thirty-seven. As in the previous collection, I have added to the final answers some additional comments about whatever I think will most interest readers. In many cases these remarks derive from pleasant correspondence with readers of Asimov's magazine.

Almost every puzzle leads to a second and related puzzle, which in turn usually leads to a third, and occasionally to a fourth. The answers on all four levels are gathered at the back of the book, where they are numbered to correspond with the problems. Needless to add, you will get much more fun as well as instruction from the book if you try seriously to solve each puzzle before you turn to its solution.

—MARTIN GARDNER

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PUZZLES

PUZZLES

1

CHESS BY RAY AND SMULL

Two young mathematicians in the computer shack of the spaceship *Bagel,* Ray and Smull, were enjoying a few hours of leisure by inventing unusual chess games to play with VOZ, the ship's computer.

"I've got a great idea," said Ray. "We'll ask VOZ to put the five black pieces—king, queen, bishop, knight, and rook—on five randomly selected squares of the board. We'll tell him not to display the pieces on the screen, but only to star the five cells where he puts them. You and I will sit at two consoles and wear earphones so each of us can ask VOZ questions, but neither of us can hear what he says to the other."

"And so?" said Smull.

"Each question," Ray went on, "will be about any designated square of the board. We'll ask VOZ how many pieces are attacking that cell."

"Can we ask about a starred cell?"

"Certainly. Of course no piece attacks the cell it is on. If we ask about a starred cell, the answer can be 0,1,2,3, or 4. If we ask about an empty cell, it can be 0,1,2,3,4, or 5. Instead of asking how many pieces attack a cell, we can ask if a certain piece is on a certain cell. To such questions VOZ will answer yes or no."

"I think I can anticipate," said Smull. "VOZ will keep a record of the number of questions we each ask until we have determined the positions of all five pieces. Whoever asked the fewest questions is the winner."

"You've got it!" said Ray. "It's a sort of chessboard version of the old twentieth-century game of Master Mind."

The two men soon became fascinated by the game, even though VOZ complained that it was trivial and a big waste of his valuable time. In the course of their play they discovered a number of remarkable problems. For example, consider the computer display shown in Figure 1.

FIGURE 1

8				*				
7	2						*	
6								
5		*					2	
4								
3			*		*			
2								
1				2				
	a	b	c	d	e	f	g	h

The five pieces are on the starred cells. VOZ has indicated that three other squares are each attacked by just two pieces. This information is all you need to determine which piece is on which starred cell. It's a fine exercise in chess logic. You are urged to try to solve it before checking the answer.

2

THE POLYBUGS OF TITAN

After the spaceship *Bagel* returned from its last mission, the crew enjoyed a month's vacation on Earth before the ship embarked again. Its new mission was to land on Titan, the largest of Saturn's moons, to determine what sort of life, if any, flourished beneath the satellite's thick cover of yellow clouds.

The trip to Titan was uneventful. The *Bagel* made a cautious landing on the side of the moon facing Saturn. Larc Snaag, the ship's captain, and Stanley G. Winetree, an exobiologist, were the first to venture outside the ship.

"What a cold, gloomy world!" exclaimed Snaag as they tramped slowly across a gooey surface between pools of liquid nitrogen. Looking up through the dense nitrogen atmosphere, they could barely make out the huge outline of Saturn and its rings even though the planet was almost wholly illuminated by the sun.

"Halloo! What's this?" shouted Winetree, lowering his head so that a bright beam of light from the top of his space helmet was directed downward. Myriads of tiny life-forms, all gray in color, were scurrying about over the black goo like a swarm of ants.

With the gloved hand of his spacesuit, Winetree scooped about a thousand of the little creatures into the large specimen box he was carrying. Back in the *Bagel's* biochem lab, close inspection of the life-forms disclosed some astonishing facts. Each body was a hard crystal, about half a centimeter in diameter, in the shape of a perfectly formed convex polyhedron! The appendages, which served as legs, varied in number from six to twenty. There were no indications on the body of eyes or a mouth.

A convex polyhedron is a solid with flat polygon faces. Convex means that if any two points in or on the solid are joined by a straight line, the line is wholly in or on the solid. All of Titan's "polybugs," as Winetree named them, were "simple." This means there were no holes going through them.

Every conceivable variety of polyhedron with no more than 30 edges seemed to be represented. Careful examination of the polybugs in the collected sample disclosed that the number of edges on a polybug could be any number from 6 through 30 with just one exception—the number 7.

"I can't understand it," said Winetree to Ronald Couth, the ship's top mathematician. "I know the simplest polyhedron is a tetrahedron. It has 4 corners, 4 faces, and 6 edges. Lots of our polybugs are tetrahedrons. And lots of others have 8 edges. Why not 7?"

Couth broke into a laugh. "You've forgotten your elementary solid geometry, Stan," he said. Then he went on to explain why no polybug had 7 edges. What was his explanation?

3

CRACKER'S PARALLEL WORLD

"You mean," exclaimed Ada Loveface, "there really *are* parallel worlds?"

"The evidence is overwhelming," said Professor Alexander Graham Cracker. "I know you like classic science fiction, so you must have read H. G. Wells's great utopia novel *Men Like Gods*. If so, you may recall that his protagonist, Mr. Barnstaple, along with several other persons, gets transported to a parallel earth with a history almost the same as ours, but not quite."

"I know the book well," said Ada. "Wells modeled his Rupert Catskill on Winston Churchill, and Father Amerton on Gilbert Chesterton. Most science-fiction fans don't know that."

"To tell you the truth," said Cracker, looking surprised, "*I* didn't know it. I'll have to read the book again sometime. Anyway, you'll be pleased to know I've discovered a way of entering parallel worlds provided they're no more than half a centimeter away from us along the fourth space coordinate."

At that time Cracker was a research physicist at Columbia University, in Manhattan, and Miss Loveface was his companion and top assistant. It took six months to construct the parallel-world machine, with its ingenious hyperspace Dean drive, and to surround it with supercooled superconductors capable of creating a magnetic field strong enough to allow the drive to displace the machine half a centimeter through 4-space.

Ada joined Cracker in their first test. They squeezed themselves into the machine, which was about the size of a telephone booth. Cracker pushed a button. There was a loud firecracker sound and a bump that jarred their backsides. Apparently the machine had dropped half a centimeter onto the laboratory's cement floor.

Cracker opened the hatch and they twisted themselves out. The laboratory looked exactly the same.

"You've done it again, Alec," said Ada. "The experiment is a bust. Maybe you'll win another Uri." (The annual Uri awards, started back in 1980 by magician James Randi, are bent-spoon trophies that go to the year's most crackbrained science projects.)

"Don't be so sure," said Cracker. "Remember—the differences between parallel worlds less than a centimeter apart may be so slight it won't be easy to recognize them. Let's do some exploring."

It was a sunny October afternoon. As they strolled about Morningside Heights the campus looked no different than before. Several students, recognizing the professor and Ada, nodded to them as they walked by. The pair paused beside the large bronze replica of Auguste Rodin's famous statue *The Thinker.* The statue stands, or rather sits, on a cube-shaped granite pedestal about five feet high, in front of Columbia's philosophy building, as if to say, "Here, folks, is where all the deep thinking goes on."

Rodin's muscular nude was in the old familiar posture, chin resting on the back of a curved right hand, right elbow on right thigh, and the lowered face lost in meditation. Looking northwest, between the corners of St. Paul's Chapel and Low Memorial Library, Cracker could see the gray tower of Union Theological Seminary through the drifting leaves.

"Look, Alec!" shouted Ada, pointing to the statue. "I was wrong! We *are* in another universe!"

What did Ada see that convinced her?

4

THE JINN FROM HYPERSPACE

John Collier Fletcher had always wanted to be an opera star. He was a big man, but unfortunately his singing voice was on the small side—difficult for audiences to hear without electronic amplification. At college he gave up his dream, got a doctorate in mathematics, and became a professor at New York University. His specialty was number theory. For many years he struggled without success to prove Fermat's last theorem.

(Fermat's last theorem asserts that the equation $a^n + b^n = c^n$ has no solution in positive integers if n is greater than 2. The case of $n = 1$ is trivial. When $n = 2$ there is an infinite number of solutions, called Pythagorean triples, of which the simplest is $3^2 + 4^2 = 5^2$. Pierre Fermat had made a note in the margin of a book saying he had a marvelous proof of his theorem, but that the margin was too small for it. To this day no one has found such a proof or a counterexample to the theorem.)

One wintry evening, when Fletcher was tramping through snow and slush to his bachelor's apartment in the SoHo (*So*uth of *Ho*uston) area near NYU, he passed a small store that he could not recall having seen before. A sign above the dirty window said: "RAY PALMER'S OLD BOTTLE SHOP."

A shelf behind the window held a dozen or so curious bottles. One caught Fletcher's eye. It looked as if—yes, it surely was!—a Klein bottle.

(A Klein bottle is a closed surface without edges, like the surface of a sphere. A sphere's surface has two sides, outside and inside. An ant crawling outside cannot get inside unless there is a hole. But a Klein surface is one-sided like a Moebius strip. Outside is continuous with inside. Without going through a hole an ant can walk to any spot on both "sides" of the surface.)

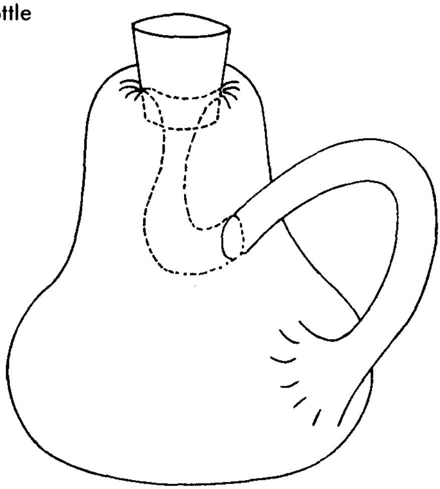
Fletcher had always wanted to own a Klein bottle to show to his students. The shop's heavy door creaked ominously as he opened it. Through some tattered curtains at the back emerged an old man about four feet high, with white hair and watery blue eyes.

"Is that a Klein bottle in your window?" asked Fletcher.

"Well, not exactly," said the gnome. "It's just a crude model [see Figure 2]. You'll observe that the stem goes

FIGURE 2

Model of Klein Bottle



through a hole where it enters the bottle. A true Klein bottle has no hole. The surface nowhere self-intersects because the stem twists around through the fourth dimension."

"I know, I know," said Fletcher. "I teach topology at New York University."

The gnome seemed unimpressed. "I do have a few genuine Klein bottles in stock. But they're more expensive. And they can be troublesome."

"Troublesome?" said Fletcher. "Why?"

"Because they twist through four-space. You never know what sort of creature from hyperspace might crawl out when you unstopper the bottle. It could be a friendly angel or jinn, but it might be something evil like a demon or a dero. Don't laugh. I'll show you one of the things."

Fletcher checked his laugh—actually it sounded more like a thin cackle—while the gnome disappeared behind the curtain. He emerged a moment later with a pear-shaped bottle almost as large as himself. It seemed to be made of rather fragile pink glass. It was a Klein bottle all right, except that where the stem usually plunged through a hole there was a spherical region of intense whiteness that shimmered and glowed like ball lightning. The gnome pointed to it with a black-edged fingernail.

"That's where the miserable thing bends through hyperspace," he said. "Naturally you can't see the twist. But if you drop anything into the bottle, it will fall into the fourth dimension and you'll never recover it."

Fletcher was so intrigued that he bought the bottle at once even though it cost much more than he had anticipated. In his apartment he put the bottle in the center of his living room, then knelt on the rug beside it and tried to figure out what caused that cloud of scintillating light.

He tried to feel the cloud, but his hand simply vanished into a region of intense cold. When he removed his hand, his fingers were so frozen that he had to warm them under a hot-water faucet.

The opening at the top of the bottle was plugged by a black rubber stopper almost six inches across. By working it from side to side, Fletcher finally succeeded in pulling it out.

A loud popping sound was accompanied by a rush of icy air, a billowing cloud of purple smoke, and a strange Istanbul smell that seemed to mix sewage odors with aromatic spices. An enormous jinn, dressed as if he had popped straight out of *The Arabian Nights,* materialized from the cloud and made a low bow.

"I am at your command," he said in a deep resonant voice that Fletcher envied. "You have the usual three wishes. What is your first desire?"

After Fletcher recovered his composure he said hesitantly: "I've always wanted to sing like Caruso."

"To hear," said the jinn, "is to obey."

Fletcher felt a sudden surge of energy pulse through his lungs and it seemed as if his chest had enlarged several inches. He sang a few notes. Magnifico! The tone was perfect, the vibrato exquisite.

"Bravissimo!" said the jinn. "And your second wish?"

Fletcher thought for only a few seconds. "I would like a proof of Fermat's last theorem."

"I beg your pardon?" said the jinn.

Fletcher quickly scribbled an equation on a sheet of paper. "It's the greatest unsolved problem in number theory. If I can prove that this has no solution in integers when *n* exceeds 2, I'll be more famous than Isaac Asimov."

The jinn studied the equation. "I have a poor head for figures. This will require consultation with a higher authority. Don't leave. I'll be back in a few Earth minutes."

Somehow the jinn managed to flow into the pink bottle. A moment later, out he popped in another purple burst of strange-smelling smoke, and handed Fletcher the paper he had taken with him. Under the equation in small but legible handwriting, was a short proof.

Fletcher read the proof with mounting embarrassment. In his excitement he had written the wrong equation! He had interchanged the *n*'s with *a, b,* and *c.* The equation had, so to speak, been inverted like this:

$$n^a + n^b = n^c.$$

The easy-to-follow proof of impossibility when *n* exceeded 2 was certainly watertight. Can you devise such a proof?

5

TITAN'S LOCH METH MONSTER

Four months ago we left Larc Snaag, captain of the spaceship *Bagel,* and his exobiologist, Stanley G. Winetree, exploring the murky surface of Titan, Saturn's largest moon. After analyzing Titan's tiny life-forms—Winetree called them "polybugs"—the two men left the ship for their second exploration of the ruddy satellite.

The temperature on Titan's surface, about minus 300 degrees Fahrenheit, was so low that the methane in Titan's nitrogen-rich atmosphere could form a liquid. The intrepid explorers were not surprised when they came upon an enormous lake of methane.

They rotated their heads from side to side so that the light beams from their helmets played over the edge of the black ominous liquid. A line from Edgar Allan Poe jingled through Snaag's mind:

Resignedly beneath the sky
The melancholy waters lie.

Suddenly a creature emerged from the heaving liquid. An enormous snakelike monster, with a single red eye that glowed at the center of its head, crawled out on the dark sand and moved slowly toward Snaag.

Winetree, who was standing off to one side, was the first to act. He yanked a laser gun from his belt and sliced the sea serpent neatly into thirds by two parallel straight cuts.

Assume for puzzle purposes that the serpent was cut by two parallel planes into three pieces of identical length. The length of each piece was ten meters plus half the length of one of the pieces.

How long was the creature?

6

THE BALLS OF ALEPH-NULL INN

Aleph-Null Inn is an infinitely huge resort hotel inside the Black Tube, a John Wheeler "worm hole" that joins the black hole at the center of the Milky Way galaxy to the black holes of other universes. The inn has what mathematicians call a "countable infinity" of rooms, a number of rooms that can be put in one-to-one correspondence with the counting numbers.

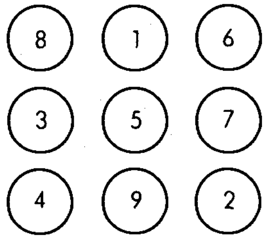
Behind the inn lies an enormous playground. It is cluttered with sand piles, swings, slides, seesaws, and jungle bars for children who come from Earth, and there are countless exotic play items for children from other planets.

Colored rubber balls, each bearing a number, are available free at the Natural Number Wall. Along this remarkable wall is a row of holes, each half a meter above the ground. As you may have guessed, the wall and its row of holes extend forever. The holes are labeled with the positive integers 1, 2, 3, . . . , and so on to infinity. If a child presses a button by a number, a rubber ball with that number rolls out of the hole. Naturally there is an infinite supply of balls for every integer.

One day Yin, a black girl from Africa, was playing with a Chinese boy named Yang. Yin had obtained nine balls numbered 1 through 9 and had amused herself by arranging them on the gray grass to make the familiar magic square shown in Figure 3.

Each row of three—horizontal, vertical, and diagonal—adds to the magic constant 15. Not counting rotations and

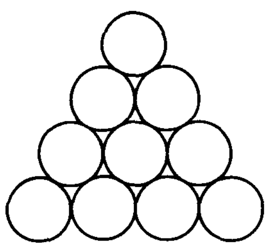
FIGURE 3



reflections as different, this is the only magic square that can be made with the first counting numbers.

"I wonder," said Yang, "if there are any magic triangles." He pushed the button by the tenth hole to obtain a 10-ball. For almost an hour Yin and Yang tried to arrange the ten balls in the triangular pattern shown in Figure 4 so that each row (horizontal or slanting) that consisted of two, three, or four balls had the same magic sum.

FIGURE 4



Try as they would, they couldn't find a solution. Suddenly Yin slapped her forehead. "How stupid can we be? Of course there's no solution!"

What insight did Yin have?

7

SCRAMBLED HEADS ON LANGWIDERE

Somewhere in the Milky Way a small planet called Langwidere spins around a sun known to Earth astronomers as Oz-1856. The planet is inhabited by a humanoid race of intelligent beings who have the unusual ability to exchange heads whenever they please.

When Langwiderians in good health die, their unconscious, blood-drained heads are preserved by cryogenic techniques and sold for prices that vary with the age, quality, and beauty of the head. Poor persons cannot afford extra heads, but the moderately wealthy may own two or three, and a wealthy man or woman may acquire a headrobe of several hundred. There is an old proverb on Langwidere:Three heads are better than two. Most hotels on Langwidere maintain supercooled head lockers where guests can store their spare heads.

A trio of attractive young Langwidere ladies, whose first names were Dot, Trot, and Zot, together checked into a resort hotel one summer for a vacation. Each lady brought along just one extra head. When they placed their spare heads in the hotel's head locker, a dim-witted clerk (he had once lost his head in an accident and now wore one with a low-order brain) mixed up the three claim checks and gave them to the ladies in a random order.

"Are you sure our heads will be safe?" asked Dot.

"You can count on it," said the clerk. "The hotel has never had a head stolen or spoiled yet. Enjoy your stay!" What is the probability that at least one lady got her correct head check?

8

ANTIMAGIC AT THE NUMBER WALL

Two chapters back, in "The Balls of Aleph-Null Inn," we described a playground behind the inn where there is a Natural Number Wall. By pressing buttons above an infinitely long row of numbered holes, a child can obtain a rubber ball with any desired positive integer printed on it, like the balls used in pool-table games on Earth.

In our previous puzzle tale, a black girl named Yin and a Chinese boy named Yang amused themselves by using balls bearing consecutive numbers starting with 1 to experiment with small-size magic squares, triangles, and hexagons. "Magic" means that all straight rows of two or more balls have the same sum. Yin and Yang made a magic square and proved that magic triangles cannot exist.

"Do you suppose it's possible," Yin asked one day, "to make a square that's antimagic?"

"Meaning?" said Yang.

"Meaning that the sum of every row, column, and main diagonal is different."

The two children found it easy to prove that there is no 2 × 2 antimagic square. (Curiously, every possible arrangement of 1, 2, 3, and 4 produces a square with sums of 3, 4, 5, 5, 6, and 7.) They also found it easy to make 3 × 3 antimagic squares. It was so easy, in fact, that they tried imposing other constraints to make the task more interesting.

Yin and Yang were unable to make an order-3 antimagic square with all eight sums in consecutive order, nor were they able to make an order-3 antimagic square in which the sums of broken diagonals (a diagonal of two numbers plus the corner number opposite) were included. They did find two antimagic 3 × 3 squares with the following property. If you put a chess rook on 1, you can make a rook-move of one cell to 2, then a rook-move of one cell to 3, and so on to 4, 5, 6, 7, 8, and 9. One solution is:

3	4	5
2	1	6
9	8	7

Can you find the other one? Of course rotations and reflections of a pattern are not considered different.

9

PARALLEL PASTS

You may recall from a few chapters back how Alexander Graham Cracker and his curvy assistant, Ada Loveface, tested Cracker's theory that an infinity of parallel universes flourish side by side like the pages of a book. Cracker's parallel-world machine was capable of moving half a centimeter along the fourth spatial coordinate to the "next" universe—a universe almost (but not quite) exactly like our own.

The reason Cracker and Ada failed to encounter their twins when they entered the next world was that their doubles had just left it in their own machine. Of course no sooner had Cracker and Ada gone, so to speak, from page n to page $n + 1$ than their doubles from $n - 1$ appeared in their n laboratory. When Cracker and Ada returned to n, all the pairs simultaneously shifted back to their original worlds.

After another year of intensive research, Cracker improved his machine so that he could visit any desired spot on an adjacent earth at any time in its parallel past. For his first test he decided to go back two decades to 1981.

"In eighty-one," said Cracker, pointing a finger, "there was a television set over in that corner. It was an old-fashioned color set with a flat picture. We won't even have to leave the lab to check on 1981 in $n + 1$. We'll just watch some $n + 1$ television programs and see if we can observe any significant differences between our past and that of the world we'll be in."

Cracker and Ada twisted their bodies into the small machine. Cracker adjusted the dials and a moment later they untwisted themselves into the newer and cleaner lab of 1981. Fortunately no one was in the room at the time. The old television set was there, just as Cracker had said. Ada switched it on.

Johnny Carson was chatting with Mickey Rooney. "You've been in the show business ever since you were a small lad," said Carson. "And you've worked with dozens of big stars. In fact, you were once married to one—Ava Gardner. Who's the lady star you've most enjoyed working with?"

"Oh, that's hard to say," replied Mickey. "They were all so wonderful. Of course I loved working with Judy. Judy was great. But they were all great. I loved them all."

"Is it possible," said Cracker, "that our machine is still in our own spacetime? Are we back in our own past?"

Ada's red hair swirled as she shook her head. "Impossible!" she exclaimed. "You can be sure that those two on the screen are not *our* Johnny and Mickey. Didn't you listen to their conversation?"

What had Ada heard that convinced her they were in the past of a parallel world?

10

LUKE WARM AT FORTY BELOW

And now there came both mist and snow,
And it grew wondrous cold.
—Coleridge, THE ANCIENT MARINER

A planet known as STC, in a solar system neighboring our own, is slightly smaller than the Earth, with a diameter of about 10,000 kilometers. The planet's first exploration, led by Earth astronauts Luke Warm and Hotta Stuph, is underway.

When home base was set up on the planet's equator, the weather was so warm that members of the landing party had to keep their space suits cooled. But while Luke was several kilometers from the base, on an expedition to gather rock samples, there came a sudden, inexplicable drop in temperature. A thick violet mist rolled over the land, and something resembling Earth snow, except it was dark purple, began to blanket the ground. Luke adjusted the dial on his belt to raise the temperature of his suit's heating system.

"What the devil is going on?" he shouted over the intercom to Hotta. She was monitoring his calls back at the base.

"How the Hell should I know," said Hotta. "You're the weather expert on this caper. What's the temperature out there?"

Luke checked the thermometer on his wrist. "Holy Asimov, I can't believe it! It's forty below!"

This conversation was being broadcast live to most television stations around the Earth. In Tulsa, Oklahoma, an eighth-grade science class was watching the screen with rapt attention.

A boy in the class raised his hand. "Does Luke mean forty below on the Celsius or the Fahrenheit scale?"

The teacher scowled. "To tell you the truth, Bascom, I don't know. You'd think by now that one scale would have become standard. But as we learned last month, Americans recently became so hotly divided over the question that our Republican Congress passed a law making it legal to use either scale. I'm really not sure *how* Luke's thermometer is calibrated."

"I know exactly what temperature he's reporting," said Babs, a girl in the back row. "It's minus forty Celsius."

"Are you certain his thermometer is Celsius?" the surprised teacher asked.

"No," said the girl. "But I'm positive I'm right."

How could she be so certain?

11

THE GONGS OF GANYMEDE

Old cults seldom die. They have a way of rising from their ashes like the fabled Phoenix. Thus it was that in the mid-twenty-first century—by a pleasant coincidence it was in Phoenix, Arizona—the ancient Greek Pythagorean Brotherhood was revived by Dr. Matrix III, great-grandson of the famous numerologist.

In the year 2053, Dr. Matrix III and several hundred members of his Church of Pythagorology emigrated to Ganymede, Jupiter's largest moon. Indeed, it is the largest in the solar system, exceeding even Saturn's Titan, though it is considerably less dense. At the center of the large crater (which Dr. Matrix III named Iva in honor of his great-grandmother) the Church established its base under an enormous geodesic dome.

Ganymede was selected not only because of its size but because of its extraordinary "resonance lock" with two sister satellites, the giant moons Io and Europa. Ganymede orbits Jupiter in about one Earth-week. Europa makes the circuit in exactly half of Ganymede's time, and Io in exactly half of Europa's time. This lock of ratio 1:2:4 is the only triple lock known for three astronomical bodies. Because patterns of natural numbers are at the heart of Pythagorology, Ganymede seemed an ideal spot for the colony.

At the age of 21 Earth-years (21 being the sum of the squares of 1, 2, and 4) every man and woman in the colony is required to make a ceremonial journey around the inside of the crater's rim. The length of the circular path is exactly 21 kilometers. Spaced at integral distances (in kilometers) along this path are five large statues of eagles. They commemorate the great eagle who carried Ganymede, a beautiful Trojan youth, to Mount Olympus, where he became cup-bearer to the gods. One eagle is gold, the other four are bronze. Beside each eagle is a small dome, where the walker can obtain food and a night's lodging. Inside each dome is an enormous brass gong.

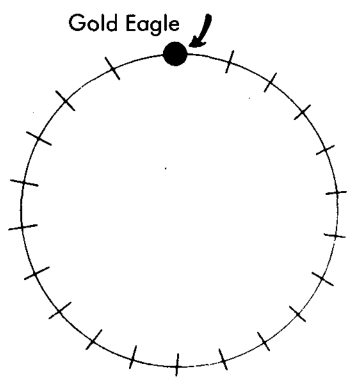
The ceremonial walk begins at the gold eagle. If the walker is male, he travels clockwise around the path. At the first bronze eagle he stops, enters the dome, and strikes the gong as many times as the number of kilometers he has just walked. While the sounds reverberate, he meditates on the secret Pythagorean significance of that number. He then proceeds to the next eagle, where he again hits the gong as many times as the kilometers he has traveled from the last eagle, while he meditates on this second number. The ritual is repeated at each eagle until he returns to the gold statue, where he started. He will have struck five gongs and meditated on five different integers.

The man now begins the second phase of his walk by continuing clockwise, but this time he stops at every *second* eagle. As before, he strikes the gong as many times as the number of kilometers just walked. Two circuits around the path return him to the gold eagle. He will have hit each gong once and meditated on five more numbers, none duplicating a previous one. On the third phase of his walk, he stops at every *third* eagle, following the same procedure. This brings him back to the gold statue after three circuits. Next he stops at every *fourth* eagle, returning to the gold eagle after four circuits. A fifth and final walk takes him just once around the path to the fifth statue, which of course is the gold one. There he hits the gong 21 times while meditating on the beauty and mystery of 21.

The five eagles are cleverly placed. When the man has completed eleven circuits he will have honored every integer from 1 through 21, though not in consecutive order. Women who make the walk must travel counterclockwise. Males take the retrograde direction of Jupiter's outermost moons. Females follow the direction of Ganymede, Europa, and Io, the innermost of the Big Four—the giant moons discovered by Galileo.

There is one and only one way (not counting a mirror reversal as different) to place the five eagles so that the ceremonial walk is possible. (Of course if it works in one direction it will work in the other.) We can describe the problem as follows: Place four more points on the 21-kilometer path shown in Figure 5 so that every integer from 1 through 20 corresponds to a distance on the circle between two points.

FIGURE 5



12

TANYA HITS AND MISSES

Colonel Ronald Couth, chief computer scientist on the spaceship *Bagel,* had just received information about the ship's new orders from Earth.

"What's the mission, Father?" asked Tanya, the colonel's twelve-year-old daughter.

"A new solar system has just been discovered near the Galaxy's rim. We're on our way to investigate."

"Are there life-forms on any of the planets?"

"Yes. I've been told that three of the first five planets from the central star have some form of life."

"Which one do we explore first?"

"The fifth."

Tanya clapped her hands. "How exciting! I wonder if its life is carbon based."

Colonel Couth's eyebrows went up. "How do you know the fifth planet isn't one of the two barren ones?"

The colonel laughed when Tanya explained. What did she tell him?

13

MYSTERY TILES AT MURRAY HILL

An earlier tale, reprinted as Puzzle 15 in my *Science Fiction Puzzle Tales* (1981) reported on the work of French archeologists of the 25th century in unearthing the ruins of what had once been New Jersey. The state had been totally destroyed by the great nuclear war of 2037. The same group, after completing its Secaucus dig, turned its attention to Murray Hill in the hope of finding some remnants of Bell Laboratories.

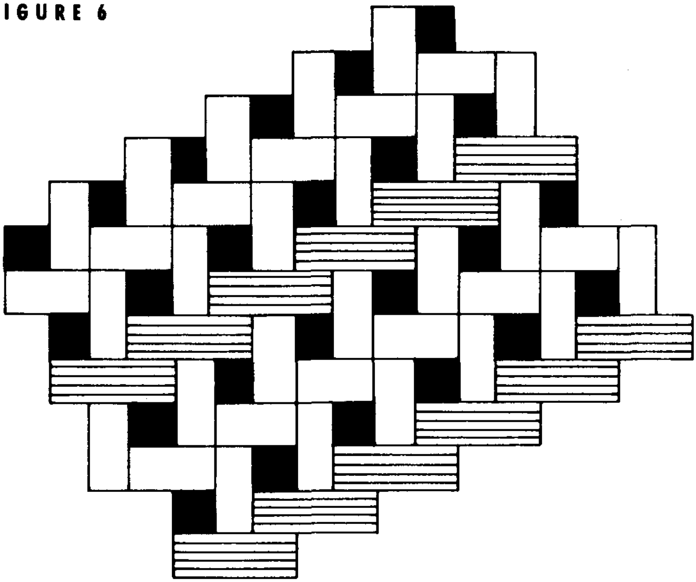
Their efforts were soon rewarded. Large portions of the lab's main building were found intact. One small office, on a floor where Bell's mathematicians worked, had a wall covered with a curious periodic pattern of brightly colored ceramic tiles. Figure 6 shows what a fragment of the wall looked like.

Back in Paris, a group of French mathematicians studied the pattern. "It looks like a conventional tiling with three kinds of straight polyominoes," said Clyde Barge, a noted graph theorist.

"I can't believe it's not more than that," said Doris Snapshutter, Clyde's associate. "After all, we know the office belonged to Ronald L. Graham, the famous combinatorialist who discovered the Graham tile."

The Graham tile, found in 1986, is the only known shape that tiles the plane only in a nonperiodic way. It is a nonconvex polygon with 71 sides. Graham was perhaps better known to the general public in his day for his work with Marvin Minsky in constructing the first robot capable of juggling 100 balls.

FIGURE 6

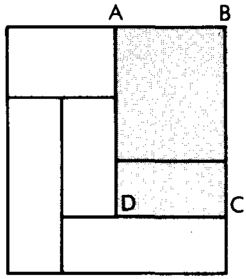


"I see what you mean" said Clyde. "It's unlikely a man of Graham's interests would have had his wall tiled in anything but some sort of remarkable pattern. See if you can find something about it in the literature of the period."

Doris spent several days at her computer console, searching old twentieth-century math journals for papers on polygonal tiling. She finally found what she was looking for. A paper in a 1982 issue of *Mathematics Magazine,* written by Graham and four friends (Fan Chung, E. N. Gilbert, J. B. Shearer, and J. H. van Lint), described the pattern.

The five mathematicians had set themselves the following unusual task. A rectangle is divided into smaller rectangles, all sides of which are integral, and in such a way that no sub-rectangle is formed by two or more rectangles. Such a tiling is said to be "irreducible." For example, the

FIGURE 7

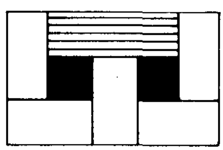


pattern in Figure 7 is not irreducible because of the subrectangle *ABCD*.

The problem is this. What is the smallest average area of tiles that can be obtained by tiling a rectangle according to the rules? Of course without the proviso about irreducibility, the minimum average is 1. You simply divide the rectangle into unit squares.

Figure 8 shows how a rectangle can be divided to obtain 1.875 as the average area of a tile. (The average is the total area of 15, divided by the number of tiles, 8.) Can this average be lowered?

FIGURE 8



It can. In 1980 Graham and his colleagues discovered the infinite pattern that Graham had placed on his wall. By a clever use of what are called "weighting functions," they were able to show that this is the only irreducible pattern of integral rectangles that tiles the entire plane in such a way that the average area of a tile is minimized.

"But can a rectangle be tiled so as to achieve this minimum?" Clyde asked.

"No," replied Doris. "But it can come arbitrarily close. You simply take a very large portion of the infinite pattern, as nearly rectangular as possible, then fill it in around the ragged edges. By going to larger and larger hunks of the pattern, you can reduce the average area of a tile to a value as close to the minimum as you please."

What is this minimum average? It is easily calculated from the pattern.

14

CROSSING NUMBERS ON PHOEBE

In the middle of the twenty-first century, when all government controls over biological research were lifted in the United States, genetic engineering got a bit out of hand. Hundreds of strange humanoid types were created, some of which bred rapidly. One of the most intelligent species was called *toroidus bimandibulus,* or tor for short, because its members were shaped like doughnuts and they had two mouths.

Arkay Guy, an ordinary human who was studying graph theory at the University of Michigan, was sitting in a campus hangout having beer with an attractive tor named Phoebe Snow.

"Professor Frank Hoorayri introduced us to crossing numbers this afternoon," said Arkay. "Do you know what a crossing number is?"

Phoebe shook her snow-white, hairless head.

"It's really very simple. A graph, you know, consists of points joined by lines. If a line connects every pair of points, the graph is called complete. Take, for instance, the complete graph for four points."

Arkay took out his pen and made four points on a paper napkin in the pattern shown in Figure 9.

FIGURE 9

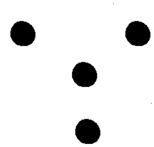
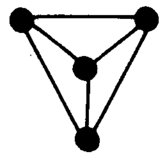


FIGURE 10

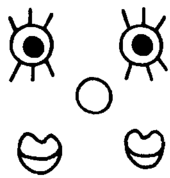


Phoebe put a hand over her left mouth and giggled. "It looks just like you."

"So it does," said Arkay, joining all pairs of points as in Figure 10.

"As you see," he went on, "I've made a complete graph without any lines intersecting one another. If you can draw a graph like that, it's said to have a crossing number of zero. Now let's add a fifth point and make it look like *your* face."

FIGURE 11



"Suppose," said Arkay, "you drew lipstick lines on your skin and tried to connect every pair of your five facial features—your beautiful green eyes, your snub nose, and your two purple mouths. The lines can twist around any way you like provided they don't go through any of the points. You'll find you can't do it without at least two lines crossing somewhere. So the crossing number for *your* face is one. Of course you could draw the complete graph with more than one crossing. But the crossing number is defined as the *smallest* number of crossings required. It's one of the graph's fundamental topological properties."

"Sorry," said Phoebe with her right mouth, while her left mouth guzzled beer, "but you couldn't be more wrong. My face has a crossing number of zero, just like yours."

"Impossible!" snorted Arkay, slapping the table.

Phoebe took a lipstick cylinder from her shoulder bag and quickly drew on her skin a complete graph for her eyes, nose, and two mouths. There was not a single crossing! How did she do it?

15

SF'S AND F'S ON FIFTY-FIFTH STREET

Readers of my puzzle tales may remember that on two occasions I have written about a curious shop run by a gnomelike old man named Raymond Dero Palmer. His shop has a way of moving mysteriously around the streets of Manhattan.

I never expected to find the shop in Chicago; but a few months ago, when I was attending a conference on number theory at the University of Chicago, I decided to stroll along Fifth-fifth Street to see how much it had changed since I had lived there many decades ago. A few blocks from the Illinois Central Railroad tracks, I passed the familiar dirty window of a tiny shop that sold old SF books and magazines. A crudely lettered card on the doorknob said:

OUT TO LUNCH FROM 80 MINUTES PAST NOON TO 1:20 P.M.

I glanced at my watch before I realized that the sign was telling me the proprietor was *never* out to lunch.

Sure enough, when I walked in, there sat Mr. Palmer, his large head barely projecting above a battered desk. Two pale blue eyes peered at me over a cheese sandwich he was nibbling.

"You again," he said.

"Me again," I replied, looking around. On a dusty table in back was a large pile of *Isaac Asimov's Science Fiction Magazines*. Beside them was the sign reproduced on the following page:

SPECIAL SALE!
BACK ISSUES OF ISAAC ASIMOV'S
SCIENCE FICTION MAGAZINE
All different! Fifty cents each!

The science fiction stories and science fact articles in *Isaac Asimov's Science Fiction Magazine* are the product of years of scientific study, followed by years of writing experience.

—R. D. Palmer

The magazines were neatly stacked so that their spines alternated with their page-edge sides. I counted 20 spines on one side of the stack. Then I walked around to the opposite side and counted 20 edges.

How many magazines were in the stack?

16

HUMPTY FALLS AGAIN

I had been reading in bed, chuckling over the manuscript of Professor Raymond Smullyan's remarkable book, *Alice in Puzzleland.* Just as I was drifting off to sleep, I heard the sound of someone moving about in the kitchen.

I padded to the kitchen on bare feet. Who should be standing there, by the oven, but Humpty Dumpty himself.

"Good morning," said the large egg. (It was 3 A.M.)

"May I ask what you're doing here?" I said.

"You may," replied Humpty.

There was a long pause.

"Well." I said.

"Well, what? Aren't you going to ask your question?"

"What," I repeated, "are you doing here?"

"Cooking," said the egg. "Remember the blue caterpillar?"

"Of course."

"I've just broiled some of his mushroom for you."

Humpty took a dish out of the oven, picked up a piece of mushroom, and handed it to me.

"Which side did it come from?" I asked. "Will it make me grow or shrink?"

"It came from the right side," said Humpty.

By now I realized I was in a Carrollian dream. It was too interesting to be disturbed by waking myself up, so I popped the mushroom into my mouth and instantly became three inches high.

Humpty extracted a pocket mirror from his red vest and placed it on the kitchen counter, mirror side up. Then he picked me up by the collar of my pajamas and set me down on the mirror. Well, not exactly. I dropped through the mirror as if I had dropped through a trap door.

Thump! I landed in a sprawl on some soft grass. When I picked myself up, there was Humpty, grinning down at me from a tall, narrow stone wall on which he was precariously perched.

"That wasn't a bad fall, was it?" he said.

"I could have broken a leg."

"But you didn't. You people with bones don't know how lucky you are. Now let me show you why I brought you here."

He leaned forward, almost toppling off the wall, to point to some strange Arabic-looking letters painted in black on stone blocks just below his feet. The letters are reproduced in Figure 12.

FIGURE 12

"Tell me what those marks mean," said Humpty, hanging his thumbs in his vest pockets, "and I'll introduce you to the Tweedle brothers. They have a message they want you to give Smullyan."

Can you answer Humpty's question?

17

PALINDROMES AND PRIMES

VOZ, the computer on the spaceship *Bagel,* was getting bored. The ship was speeding toward the edge of the Milky Way to check out a newly discovered solar system, but it would be another three months until it got there.

Gigo, the youngest of the hackers in the computer shack, had just been checkmated by VOZ for the fifth time, even though VOZ had been playing without his queen.

"Must we play again?" VOZ asked while Gigo was setting up the pieces. "Another ten minutes and it will be your chow time."

Gigo glanced at the clock above VOZ's console. "You're right," he said. "And I know how tedious these games must be for you."

"I process in parallel, remember?" said VOZ. "Isn't there some unsolved problem I could be working on while we play these stupid games?"

"I'll ask the colonel," said Gigo.

The colonel was Colonel Ronald Couth, head of the ship's computer division. Couth rubbed his chin while Gigo spoke to him at lunch. "There are thousands of unsolved questions in number theory," he said, "but most of them are intractable in the sense that it would take VOZ a few million years to answer one of them. Wait a minute! There *is* a curious unsolved problem about which very little is known. We'll give VOZ the palindrome conjecture."

"What's that?"

"It goes back to the 1930s. You start with any positive integer. Reverse the digits to make a different number, then add the two numbers. Do the same thing with the sum. The conjecture is that if you keep doing this, after a finite number of steps you'll produce a palindrome—a number that's the same backward as forward. Of course if you start with a palindrome, you reach a palindrome in no steps. Let's try it with the current year, 2058."

Couth jotted 2058 on a napkin. After four steps of reversing and adding he reached the palindrome 56265.

"Does it always work?"

"That's just it," said Couth. "Nobody knows. If you begin with an asymmetric two-digit number, and their digits add to less than 10, you obviously get a palindrome in one step. You also get a palindrome in one step if the digit-sum is 11. If the sum is 10, 12, or 13, you reach a palindrome in two steps. If the sum is 14, it takes three steps. If they add to 15, it takes four steps. And if the sum is 16, it takes six steps.

"What about 17?"

"Seventeen is the maverick. Only two two-digit numbers have a digit sum of 17. They are 89 and 98. Start with either number and it takes twenty-four steps to make a palindrome."

Gigo entered 89 on his pocket calculator. After the 24th step, the readout showed the palindrome: 8813200023188.

VOZ was delighted with the problem. At once he began to check the integers in counting order. Every number reached a palindrome in fewer than 24 steps, except for 89 and 98, until he reached 196. After a few minutes VOZ had performed 1,000 steps on 196 without encountering a palindrome. It took several hours to extend the calculation to a million steps. Still no palindrome.

"I could go on to a billion," VOZ said to Gigo, "but I think it would be a waste of my time. I ran several sophisticated probability checks while I was calculating, and the chances of ever hitting a palindrome are close to zero. It's strange. I may try writing a program to see if I can prove the conjecture is false."

The conjecture is believed to be false by Charles W. Trigg, the first to discover that 196 is the smallest number that could be a counter-example. The conjecture has been proved false only for binary notation, and any notation based on a power of 2. The smallest binary counter-example is 10110, or 22 in the decimal notation. It produces an asymmetric pattern that expands in a systematic way that precludes the formation of a palindrome.

Another famous conjecture about palindromic numbers is that there is an infinity of palindromic primes. The first such prime (not counting single-digit numbers) is of course 11. The sequence continues: 101, 131, 151, 181, 191, 313, 353, 373, 383, 727, 757, 787, 797, 919, 929, 10301,. . .

Note how the sequence jumps from three-digit primes to five-digit primes. Can you prove that, except for 11, no palindrome with an even number of digits can be a prime?

18

THIRTY DAYS HATH SEPTEMBER

Thirty days hath September
All the rest I can't remember.
—Anonymous

It was a sultry Sunday afternoon in the month of Fort, 2032. Myrtle was working on a high school term paper about Ronald Reagan.

"Let's see," Myrtle said aloud. "Reagan was born on February 6, 1911. That would have been a Friday."

"Not necessarily," said Myrtle's father, putting down his copy of *Isaac Asimov's Science Fiction Magazine.* "The old Gregorian calendar was still in use in 1911. That means you can't tell from a month-date what day of the week it is unless you look it up or do a lot of calculating with formulas I don't remember."

In 2032 Reagan was best known for having initiated the great movement in which show business took over the nation's political system. Television had become such a potent force in election campaigns that only seasoned show people were able to project the kind of TV image that would win mass votes.

The 1990s saw a steady decline in country music as science fiction, accompanied by computer-composed electronic music, rose in popularity on the TV networks. Times Square was officially renamed Isaac Asimov Square by New York City's first black mayor, Gary Coleman.

Gary had become famous in the 1990s as the captain of the spaceship *Bagel* in a television series based on puzzle stories in Asimov's magazine. Coleman was then a SF fan and an ardent disciple of Charles Fort. When he was elected Democratic president of the United States, in 2000, one of his first reforms was to replace the clumsy Gregorian calendar with the calendar that had been adopted by the Fortean Society back in the early twentieth century.

The Fortean calendar has thirteen months of 28 days each. Each month starts on Sunday and ends on Saturday. Thus, to the further annoyance of all triskaidekaphobes (those who fear the number 13), every month has a Friday the thirteenth. The thirteenth month, Fort, separates June from July. Because $13 \times 28 = 364$, an extra holiday, called "World Day," occurs without a date between December and January. Another dateless holiday, "Asimov Day," occurs every leap year between June and Fort.

Now back to Myrtle and her father.

"How did people ever remember the number of days in each month?" said Myrtle.

"We had a little jingle about it," said her father. "I remember as a boy how Gary Coleman liked to recite it when he was agitating for calendar reform. He used to say it this way:

"Thirty days hath September,
April, June, and November,
All the rest have thirty-one,
Except for February . . .
Is that fair?"

Later in the day, when Myrtle was practicing on the piano, she suddenly exclaimed: "Dad! I've just figured out a way to use the piano keyboard as a mnemonic for the number of days in each month of the old calendar. It's neater than that silly rhyme you recited."

What did Myrtle discover?

19

HOME SWEET HOME

Silently, swiftly, invisibly, an alien intelligence from hyperspace floated through the streets of Los Angeles, gathering information on the region's life-forms. Here is an excerpt, translated into English, from a monumental report that the alien later submitted to its superior:

We observed a young lady, riding a bus, who held on her lap a small object shaped like a rectangular parallelepiped. The lengths of the object's edges were in the rough proportions of 1 to 14 to 20. The object consisted entirely of thin laminations made from dried plants, each lamination covered with complex chemical patterns. One face of the object had stamped upon it a replica of a life-form we have not yet observed anywhere on the planet. At the lower left corner of the same face, a rectangle contained 37 horizontal black lines of varying thicknesses. At the top of the face were six large symbols that apparently form a Russian word we have not yet deciphered.

What object is the alien describing?

20

FINGERS AND COLORS ON CHROMO

The planet Chromo is inhabited by a not-so-intelligent race of three-eyed humanoids. There are three sub-races on Chromo, one with pink skin, one with blue skin, and one with green skin.

Tourmaline, ruler of the pinks, was planning a state banquet that required the seating of 60 Chromos in one large hall, three to a table

"Have you decided who sits where?" the ruler asked Coralie, one of her aides. "Remember, all three colors will be represented, and we want to mix the colors as much as possible at each table."

"I understand, Your Majesty," said Coralie. "I'm still working on the seating plan. No matter how I divide the sixty guests into triplets, at least one person in every triplet must be a pink."

Exactly how many pinks, how many blues, and how many greens are among the guests?

21

VALLEY OF THE APES

"My dear Lulu, how would you like to live for a few years in equatorial West Africa?" asked Dr. Fanzine Patterfanny, Stanford University's expert on ape languages.

Lulu, one of Miss Patterfanny's most intelligent gorillas, pondered the question for several minutes while she paced back and forth. Finally she signed: "It would be a welcome change from this lousy cage, beloved Fan. When do we start?"

Dr. Patterfanny's purpose in sending Lulu to Africa was to see if Lulu could teach sign language to a tribe of gorillas that lived in a large valley of the Congo. The experiment was a great success. In three years the apes not only had learned how to sign, but they had modified ASL (American sign language) so drastically, and added so many new signs, that when Miss Patterfanny returned to the Congo, she had enormous difficulty understanding the new language.

To convey some notion of Luluish (as the new language came to be called), here is how three simple phrases are signed:

Eat red ant: Turn a back flip, scratch left eyebrow, thumb nose.

Big red berry: Poke pinkie in right ear, thumb nose, stick out tongue.

Eat berry quick: Raise left foot, turn back flip, poke pinkie in right ear.

Given this information, how would you sign the phrase "Big ant" in Luluish?

22

DR. MOREAU'S MOMEATERS

"Who would have guessed," said Dr. Moreau III, a noted geneticist at Kings College, London, "that such a small alteration of the genetic code of this fish would produce such a big change? What shall we call the new species?"

"How about momeater?" suggested Montgomery, Dr. Moreau's assistant. Montgomery was a chimpanzee whose intelligence had been raised by genetic engineering to a level almost equal to that of Dr. Moreau himself.

Momeater was an appropriate name for the tiny fish because of its peculiar breeding habits. Each female produces exactly ten eggs, which hatch inside a pouch on the mother's underbelly. When the ten baby fish leave the pouch, a strange thing happens. They kill and devour the mother! Since gestation takes only a few days, and the fish live for years, a population of momeaters grows at an explosive rate.

Dr. Moreau put ten newly hatched momeaters in a large tank. "I want you to keep a careful count of them every day," Moreau said to Montgomery. "Let me know when the tank holds 5,000 fish."

Montgomery scratched his chest and thought a moment. "If you mean 5,000 precisely," he said, "it's not possible. Assuming no fish die, except of course the females eaten by the young, the closest we can get to 5,000 is 4,996."

Is Montgomery right? Or did he make a mistake in his calculations?

23

AND HE BUILT ANOTHER CROOKED HOUSE

If you ever read Robert Heinlein's classic story about a tesseract, or four-dimensional cube, you may recall the plot. It began when Quintus Teal, architect extraordinary, built a crooked house somewhere near Los Angeles for his rich oilman friend, Homer Bailey.

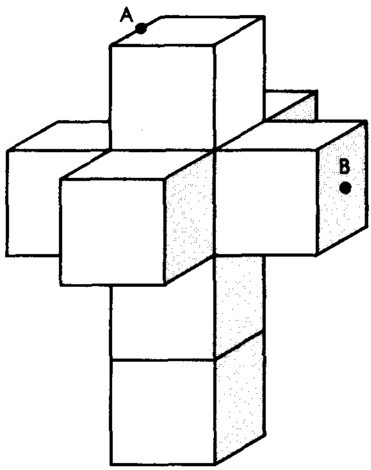
Imagine six cardboard squares joined along edges to make a cube. You can cut along seven edges and unfold the cube to make a flat Latin cross. In an analogous way, a four-dimensional creature could join eight cubes at their faces to make a tesseract. If certain of these faces are cut, the tesseract will unfold to make a solid three-dimensional cross of eight attached cubes. This was the structure that Teal built. Unfortunately, a small California earthquake caused the house to collapse along its natural joints into a stable tesseract. After some bizarre incidents took place inside the four-dimensional house, a more severe earthquake caused the structure to drop into another space and totally disappear.

A few months ago, when I was visiting some magician friends at the Magic Castle in Hollywood, I thought of Quintus Teal. On an impulse, I checked the Los Angeles Telephone directory. To my surprise, there was his name! He was still living on Lookout Mountain, not far from Laurel Canyon.

"Of course I know who you are," he said on the phone. "I've been enjoying your puzzle tales in Asimov's magazine ever since the thing started."

Naturally, I drove out to see him. To my amazement, I

FIGURE 13



found him living alone in another crooked house, exactly like the first one except that he had turned the three-dimensional cross right-side up instead of upside-down as it had been before. Figure 13 shows how it looked. Salvador Dali had used this same structure for the cross in his well-known painting of the crucifixion, *Corpus Hypercubus.*

"I built it much sturdier this time," said Teal, a bushy-bearded, bald-headed man in his late sixties. "If the San Andreas Fault acts up, I've got nothing to worry about."

Teal had a strong interest in recreational mathematics. In fact, we spent most of the afternoon and evening discussing puzzles that were related in some way to the structure of his house.

One of Teal's best ideas was to search for geodesics, or shortest possible paths, between two points on the outside of the house. For example, suppose a spider started from point *A,* at the middle of an edge, and crawled along the outside surface of the house to point *B,* at the center of a square face. (See Figure 13.) If the spider took the shortest possible route, how far did it travel?

24

PIGGY'S GLASSES AND THE MOON

As the letters departments of SF magazines testify, readers who know their science take great delight in calling attention to scientific blunders in stories by authors who should know better. I had the pleasure in 1982 of attending Skycon 2, in Asheville, North Carolina, and hearing Hal Clement speak about such things. Mr. Clement mentioned two glaring goofs that he had come upon in William Golding's classic of anthropological SF, *Lord of the Flies*. I am ashamed to admit that when I first read this somber novel, back in the fifties, I didn't notice either of them.

If you enjoyed *Lord of the Flies,* or saw the British movie version, you will recall Piggy, the rotund intellectual among the stranded schoolboys. He was so myopic that without his thick glasses he could hardly see his own hands, and the world beyond his hands seemed covered by what Golding calls a "luminous veil." Chapter 2 describes how the boys used Piggy's glasses to start a much-needed fire:

> There was pushing and pulling and officious cries. Ralph moved the lenses back and forth, this way and that, till a glossy white image of the declining sun lay on a piece of rotten wood. Almost at once a thin trickle of smoke rose up and made him cough. Jack knelt too and blew gently, so that the smoke drifted away, thickening, and a tiny flame appeared. The flame, nearly invisible at first in that bright sunlight, enveloped a small twig, grew, was enriched with color and reached up to a branch which exploded with a sharp crack. The flame flapped higher and the boys broke into a cheer.

What whopping blunder in high school physics did Golding make when he wrote that paragraph?

25

MONORAILS ON MARS

Around 1900 the American astronomer Percival Lowell wrote three popular books about Mars and its canals. They contained detailed maps showing networks of lines on Mars which Lowell thought he had observed through his telescope on Mars Hill, in Flagstaff, Arizona.

In 2049 the famous California parapsychologist Dr. Harold Backoff published a controversial paper in the *International Journal of Precognition.* Perhaps, Dr. Backoff suggested, Lowell was not as self-deluded as most astronomers assumed. Could it be that Lowell was a psychic with strong precognitive powers? Was he observing Mars as it would appear 150 years in the future?

Dr. Backoff based his theory on the fact that by 2049 Mars was indeed crisscrossed by hundreds of lines, most of them straight, that resembled the lines on Lowell's maps. They were not water canals, as Lowell mistakenly assumed, but transportation monorails built to connect the bases that had been established on Mars by U.S. and U.S.S.R. explorers.

Mars has a diameter of about 4,200 miles. The first three bases constructed by American engineers were named Asimov, Bradbury, and Clarke. Call them *ABC.*

B and *C* are each 6,000 miles from *A.* Which base, *A* or *B,* is the nearest to *C?*

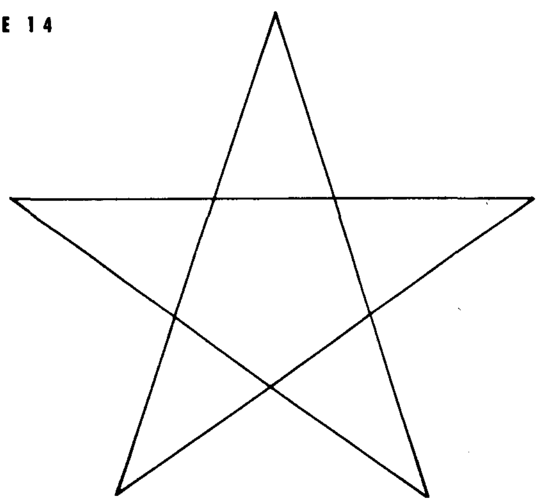
26

THE DEMON AND THE PENTAGRAM

Ever since I read Goethe's *Faust* I have been fascinated by the strange ritual that enabled Faust to transform his poodle into Mephistopheles, and by the role played by a pentagram, or "devil's foot," that Faust drew on the floor at the entrance to his study. The pentagram is the five-pointed star (shown in Figure 14) that every child learns to draw in one continuous line of five straight segments. Because Faust failed to close completely one of the star's points, Satan's attendant was enabled to enter the study, but the closed sides of the interior pentagon prevented him from leaving.

Use of the pentagram for magic purposes goes back to the ancient Greek Pythagoreans. Throughout the Middle

FIGURE 14



Ages it represented Christ if drawn with one point straight up, Satan if drawn with a point straight down. It was widely used for charms and magic spells, and became a fundamental symbol for such occult groups as the Rosicrucians and such secret orders as the Freemasons.

You can imagine my excitement when there fell into my hands an old parchment from Germany that gave in detail the exact ritual by which Faust had summoned Mephistopheles, the second greatest of the fallen angels. One stormy winter night, when my wife was sleeping soundly, I climbed the stairs to my attic study. Uri, our black cocker spaniel, followed at my heels.

I locked the door. On the inside of the door, with red chalk and a shaking hand, I drew a large pentagram, pointing it downward and carefully leaving a tiny gap at the bottom corner. For weeks I had been secretly gathering all the paraphernalia required by the ritual: thirteen black candles, a broken crucifix, a goblet of human blood, the eye of a newt, the heart of a frog, and assorted oils and rare unguents.

Uri began to whine and cower as I neared the end of the ceremony. A reddish haze enveloped the dog, whose form began to blur and writhe until it resembled a baby dragon. A soft explosion produced a cloud of dark sulphurous smoke. When the smoke cleared, Uri was gone. In his place stood a tall, evil-looking man dressed neatly in a business suit.

"Well done, my friend," said Mephistopheles. "Your pentagram has trapped me as you planned. If you'll kindly erase it, I'll be on my way."

"I'll do nothing of the sort," I replied, "until you make me an acceptable offer."

"I expected that," said the demon. "How would you like a spotless first edition of Goethe's *Faust,* signed by the author? It's worth a fortune."

"And what must *I* do?"

"Just erase that damnable diagram. I'll return as soon as you replace it with another pentagram, provided you draw it a certain way. You're a mathematician—at least you pretend to be—so you probably know that each corner of a regular pentagram is an angle of 36 degrees. This makes a total of 180 degrees for the five points."

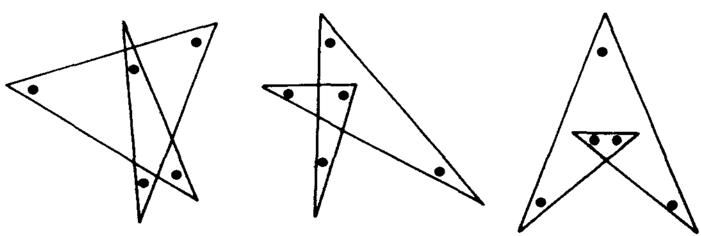
"Yes, I'm aware of that."

"Good," said the demon. "Now, when you draw your new pentagram, skew its lines a bit so that the sum of the five angles is either more or less than 180 degrees by at least one degree. Such a figure allows me to come and go as I please. I will not return unless you draw it properly. As soon as you do, I'll bring you Goethe's priceless volume."

"Let me make sure I understand," I said. "I must draw the star with five continuous straight lines as before. But I must make the star sufficiently irregular so that the sum of its points will differ from a straight angle by one or more degrees."

"Precisely. It doesn't even have to look like a star." The demon took a pencil and sheet of paper from my desk and drew the three diagrams shown in Figure 15, adding black spots to mark the interior angles of each. "As you see, the diagram on the left is recognizable as a distorted star. But

FIGURE 15



you may also draw, if you prefer, degenerate stars like the other two diagrams, stars with one or two points on the inside. Make the sum of the five angles anything you like except 180."

The proposition seemed reasonable enough. We shook hands. When I erased my regular pentagram Mephistopheles vanished instantly. I never saw him or Uri again.

No, the demon had not lied. I simply failed to fulfill my part of the bargain. Can you explain why?

27

FLARP FLIPS A FIVER

Lieutenant Flarp, a navigation officer on the *Bagel,* finished his second Mars martini. He and Ensign Pulfer, a new officer who had just joined the crew, were sitting in the ship's cocktail lounge a few hours after blasting off from the Mars Space Center. As regular readers of this feature may recall, the *Bagel* is a giant spaceship shaped like a bagel. It rotates continually in space to produce an artificial gravity field inside the toroidal hull.

"Let me have the tab," said Pulfer, reaching for it.

Flarp grabbed a bit faster. "No, no. This is your first drink aboard. I'm paying."

The two men argued for a few minutes, then Pulfer took an aluminum fiver from his pocket. A fiver, or five-dollar coin, with Asimov's profile on one side, purchased what a nickel would have bought back in the late twentieth century.

Pulfer handed the coin to Flarp. "You spin it on the table and I'll call heads or tails."

"If you insist," shrugged Flarp. "But spinning is out of the question. I'll flip it instead."

Why did Lieutenant Flarp refuse to spin the coin?

28

BOUNCING SUPERBALLS

The February 1958 issue of *Galaxy* contained an amusing tale by Walter Tevis called "The Big Bounce." It was about a ball so elastic that once it started bouncing, each bounce would be higher than the last one. Eventually all of the ball's heat was used up as energy for bouncing, so the thing froze and shattered before it could do any damage.

As Isaac Asimov pointed out in an article in *The Smithsonian* (May 1970), such a bouncing ball would violate the second law of thermodynamics, but let's ignore this and assume that such a superball could actually be made. We also assume that the ball's first bounce is exactly one foot. The next bounce is half a foot higher, the third bounce adds one-third of a foot to the height, the next adds one-fourth, and so on. In brief, each nth bounce increases the height of the bounce by $1/n$ feet. Of course we must idealize all the parameters: a perfectly flat surface, a uniform gravity field, no loss of energy from air resistance, and so on. When we speak of the ball's height, we mean the height of the point at the ball's center.

Now for a crazy question. If this superball is allowed to bounce for a long enough time, will it ever bounce as high as a mile? The answer leads into some fascinating theorems about a famous infinite series of fractions.

29

RUN, ROBOT, RUN!

A few centuries from now a young man was walking his robot dog, Farfel, through Central Park, in Manhattan. There was a November chill in the air, and a thin blanket of snow lay on the ground.

When the man reached the park's reservoir he stopped to set Farfel's trotting speed at 10 mph. Then he took from his pocket a small rubber ball.

"While I circle the reservoir," he said to the dog, "I'll keep tossing the ball. I want you to fetch it each time and bring it back. Understand?"

"I understand," said Farfel, wagging his tail.

As he walked around the reservoir, the man thought: "Farfel's battery is getting low and I'd like to make it last as long as possible. The greater the distance he covers, the more current he uses. If I want to minimize the distance he runs, should I toss the ball ahead of me, or throw it backward along the path, or off to the side away from the water?" It was a perplexing question.

Assume that the man walked at a constant speed of 5 mph, the dog trotted at a constant speed of 10 mph, and neither dog nor man paused during their trip around the reservoir. Assume also that the man and dog started together, and were together when the man completed his circuit.

How should the man toss the ball so as to minimize the distance covered by the dog? Ahead, behind, or to one side? And does it matter whether he throws the ball a short distance or far away?

30

THANG, THUNG, AND METAGAME

The gods, who live in a realm utterly beyond our space-time, play a variety of intellectual games that are impossible to describe. One of the most popular is played on a grid in a space of 37 dimensions. There are more than a million pieces, and an average game lasts for about a thousand years of earth time.

The gods have found it convenient to divide their games into two broad classes: finite and infinite. A finite game is one that must always end after a finite number of moves, such as our checkers, chess, go, ticktacktoe, bridge, and so on. An infinite game is one that is permitted by its rules to continue forever. Such games would be wearisome on earth, but what is time to the gods?

One day a clever little god named Thang looked up from the board of an infinite game he was playing and said to his opponent, Thung:

"I've just thought, Thung, of something curious. It's a new game. I call it metagame. The rules are simple. The first player makes his first move by choosing a finite game. The other player then makes the first move in the finite game, and the game proceeds until it ends."

On earth, for example, the first player of metagame might choose chess. The players would then sit down at a chess board and the second player would open by moving, say, a pawn to queen four.

"What a stupid notion," snorted Thung. "We all play metagame every time two of us meet at the club and decide what to play."

"I know," said Thang. "But hear me out. Metagame leads into a marvelous paradox when you consider this question: Is metagame finite or infinite?"

Thung scratched his heads.

"It must be one or the other," said Thang. "But in either case we run smack into a logical contradiction. Assume metagame is finite. The first player is allowed to choose any finite game, so suppose he picks metagame. Now the second player, making the first move in metagame, must pick a finite game. Suppose he again chooses metagame. Then the first player does the same. Obviously this can go on forever. Therefore the assumption that metagame is finite is false."

"Ye gods!" exclaimed Thung. "I see what you're getting at. If metagame is not finite, it must be infinite. Okay. Assume it's infinite. Now the first player *can't* choose metagame because it's not finite, so he has to pick a finite game. But all finite games end. Again our assumption is contradicted. Metagame can't be infinite!"

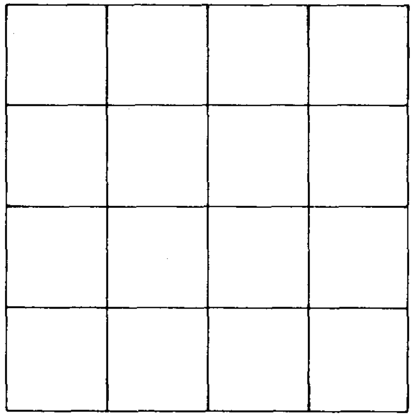
Thang and Thung are still debating this curious paradox while they play their eternal game. Who knows when or how they will resolve it?

In the meantime, let us consider a solitaire game played on the 4-by-4 grid shown in Figure 16. It can end in a finite number of moves if you start with certain initial patterns, but will never end if you start with other patterns.

To play the game, put 16 pennies on the 16 cells. Each penny may be heads or tails in any pattern you like. The object of the game is to turn over the pennies, according to the rules, until all of them are heads. The rules are as follows:

You may turn any horizontal row or any vertical column provided you turn all four coins. You may turn any diagonal row. "Diagonal" here includes the two main diagonals of four coins, the four diagonals of three coins, the four diagonals of two coins, and each of the four corner coins.

FIGURE 16



Thus each corner penny is considered a diagonal of one. As before, when you turn a diagonal you must turn *all* its coins.

Try a few games with random patterns of heads and tails, and you will find that sometimes it is easy to get all the coins heads. (If you can get all heads, of course you can also get all tails by merely turning four rows.) However, with other starting patterns you will find yourself turning coins endlessly without ever achieving the goal.

There is a very simple way to glance at a pattern and know at once whether or not it is solvable. This makes for an amusing betting game. You can explain the game to a friend by setting up a few solvable patterns, and demonstrating how easy it is to change the pattern to all heads. Let him experiment with some random initial patterns of his own. As soon as he sets up an unsolvable pattern you can bet a thousand dollars against his dime that he can't solve it in an hour.

What's the secret? How can you tell so quickly whether a pattern is solvable or not?

31

THE NUMBER OF THE BEAST

Here is wisdom. Let him that hath understanding count the number of the beast: for it is the number of a man; and his number is Six hundred threescore and six.

—REVELATION 13:18

When the year 1000 approached there was a vast outpouring throughout the Christian world of sermons and literature dealing with the approaching appearance of the Antichrist, the Battle of Armageddon, and the Second Coming of Jesus. Now as the year 2000 looms ahead, it is happening again. Adventist sects are growing rapidly, and the electronic evangelists, including Billy Graham, are talking more and more about Satan's coming incarnation as the Beast, and how the number 666 will apply to the archfiend.

Being curious about how this trend will fare in the eighties and nineties, I put myself into a precognitive trance, using a secret technique developed last year by parascientists at Stanford Research International. The information obtained was as astonishing as it was bizarre.

Hysteria about the Second Coming reached a crescendo in 1999, and not just because this was the last year before 2000. If you turn 1999 upside down, its numerals begin with 666. But there were other reasons for the excitement. It was in June 1999, the sixth month of the year, that the Reverend Sun Moon announced that he was the new messiah. By that time the Moon cult had become the largest adventist sect in America, far surpassing in numbers and wealth the Mormons, the Seventh Day Adventists, and Jehovah's Witnesses.

In past centuries thousands of political and religious leaders have been identified with the Beast. The number 666 was usually derived from their name by numbering the letters of an alphabet in a plausible way, then adding the values of the letters in the person's name to obtain a sum of 666. Early Christians applied 666 in this way to Nero, other Roman tyrants, and Muhammad. Protestants applied it constantly to the names of popes, and Catholics retaliated by finding 666 in the names of Luther, Calvin, and other reformers. It was applied to Napoleon, and later to Hitler, Mussolini, Stalin, and other dictators.

One elegant way to get 666 from HITLER is to use the simple code: A = 100, B = 101, C = 102, and so on. Adding the letters of HITLER produces 666.

I was startled to learn, in my trance, that in July 1999 I published a discovery I had made earlier that year. It used the simplest of all codes: A = 1, B = 2, C = 3, and so on. The value of each letter in a name is then multiplied by 6 and the products added. When this system is applied to SUN MOON the result is 666. Publishing this aroused such hostility on the part of the Moonies that for months I was bitterly attacked in their press, and even narrowly missed being killed by a crazed Moonie who, fortunately, was a poor shot with her revolver.

After emerging from my trance, I went carefully through my extensive files on 666. It occurred to me that readers might be interested not only in what I had precognized, but also in a few of the remarkable properties of 666.

Six raised to the sixth power, and that number again raised to the sixth power, gives the number of alternate universes that can be entered by the device in Robert Heinlein's novel *The Number of the Beast*.

The sum of the first 36 counting numbers is 666, and note that $36 = 6 \times 6$.

The sum of the squares of the first 7 prime numbers (2,3,5,7,11,13,17) is 666. Note how this joins 666 to the mystical number 7.

The first 144 decimal digits of pi, Michael Steuben (a mathematics teacher in Annandale, Virginia) recently found, add to 666. Note that 144 is $(6 + 6) \times (6 + 6)$.

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 5^3 + 4^3 + 3^3 + 2^3 + 1^3 = 666$$

$$1^6 - 2^6 + 3^6 = 666$$

$$6 + 6 + 6 + 6^3 + 6^3 + 6^3 = 666$$

The next verse of Revelation after 13:18 is 14:1. It contains the number 144,000. Michael Keith, of Hightstown, New Jersey, discovered that if you divide 144,000 by 666, you get the repeating decimal number 216.216216216 . . ., and $216 = 6 \times 6 \times 6$.

I could go on for 66 more pages, but there is space only for five pleasant puzzles:

1. Using the code that was applied to Hitler, what is the most evil day of the week?

2. Prove that the Beast is a FOX.

3. Three plus signs can be inserted within the sequence 123456789 to make a sum of 666:

$$123 + 456 + 78 + 9 = 666.$$

If minus signs are also allowed (except that a minus sign is not permitted in front of the sequence), what is the only other way to obtain 666 with just three signs? The sum cannot be achieved with fewer signs.

4. Find the only way to insert four signs (each may be plus or minus) inside 987654321 to make a sum of 666. Again, there is no solution with fewer signs.

5. A common interpretation among today's fundamentalists is that 666 represents a falling short of 777, which is

taken to be a symbol of perfection. I believe there is no way to insert any number of plus and minus signs in 123456789 to get a sum of 777, and just one way to do it with 987654321. How?

32

THE JOCK WHO WANTED TO BE FIFTY

By the year 2001 the United States had made remarkable strides in all the sciences. At the same time, the rapid decline in the teaching of mathematics, which began with the twentieth-century computer revolution, had accelerated.

Dr. Ophelia Bumpers, professor of mathematics at the University of Miami, was 47 years old, but she looked and behaved like a gorgeous brunette of 27. One of her students in her Introduction to Elementary Algebra class was Lucky O'Toole, age 19, and captain of the university's football team. O'Toole was not too bright, but he was good-looking, virile, and hopelessly in love with his teacher. Ophelia didn't mind. In fact, she had instigated the romance.

One sultry afternoon, when the pair was relaxing on a sandy beach in their bathing trunks, Lucky lay on his back with his head in Ophelia's lap. "You must have heard," he said, "about the big genetic breakthrough last week. Some biologists at Harvard have found a way to age a person any number of years in just a few weeks of treatment."

"I read about it in the *National Enquirer,*" said Ophelia. "I understand the process is irreversible."

"It is," said Lucky. "But I don't mind. I've made an appointment with the Harvard group to serve as a volunteer subject. They say they can boost my age to 50. If it works, you and me can get married."

Ophelia took her hand out of Lucky's curly hair to put it over her mouth. "I can't believe it! You'll lose thirty of the best years of your life!"

Ophelia was not only profoundly shocked. She was enormously annoyed. "Look, Lucky, you're exactly the age now you really want to be. I can prove it with algebra." She pointed to the right. "Hand me that notepad and I'll show you." Ophelia always carried a pad and pencil with her. She was a creative mathematician, and whenever an inspiration struck, she liked to jot down the essentials before she forgot them.

"We'll let n stand for your age now," she said, "and a for the age you want to be, and d for the difference between the two ages."

Lucky nodded. So far he understood.

"Obviously," Ophelia continued, "we can write this equality:

$$a = n + d.$$

"Let's multiply each side by $(a - n)$:

$$a(a - n) = (n + d)(a - n).$$

"After performing the two multiplications, the result is:

$$a^2 - an = an + ad - n^2 - nd.$$

"Subtract ad from both sides. This gives us:

$$a^2 - an - ad = an - n^2 - nd.$$

"Factor both sides:

$$a(a - n - d) = n(a - n - d).$$

"Now we cancel $(a - n - d)$ from both sides. And see what results!

$$a = n.$$

"Remember," Ophelia went on, "a is the age you want to be and n is the age you are now. I've just proved—and you can't argue with algebra—that the two ages are identical. You're just the age you prefer!"

Lucky scratched his head. He had learned enough algebra to follow each step of the proof. Every transformation of the original equation seemed legit.

Of course Ophelia was not serious. To Lucky's embarrassment she cackled with merriment when she saw the perplexed look on his face. Something obviously is wrong with her proof, but what?

33

FIBONACCI BAMBOO

Everybody knows how widely bamboo is used in the Orient, for a thousand different things, but few are aware that it is the fastest growing of all woody-stemmed plants. In some areas, such as Sri Lanka, it can grow at a rate of 16 inches in a single day, often reaching a height of more than 100 feet. If the tip of a bamboo stalk is viewed through a powerful microscope, you can actually *see* it grow!

It was near the middle of the twenty-first century that Professor Mitsu Matsu, a Tokyo geneticist, succeeded in engineering a new bamboo species that he called Fibonacci bamboo because of its remarkable way of growing. Not only did it grow at a steadily accelerating rate, but the rate conformed precisely to what mathematicians call a generalized Fibonacci sequence.

"Yes," said Dr. Matsu one sunny afternoon to Dr. Beatrice Mince, a visiting geneticist from Philadelphia, "each day's growth is exactly equal to the growth of the two preceding days. If a stalk grows a feet the first day, and b feet the second day, the third day's growth of c feet will equal a plus b."

"And the fourth day the growth will be b plus c?" asked Dr. Mince, gazing in wonder at the tall hollow "trees" that were sprouting in the dense bamboo forest through which they were strolling.

"Exactly. Every twenty-four hours the growth for that period is precisely the sum of the growths on the preceding two days. Fortunately, when a stalk reaches a height of about five hundred feet it stops growing."

"I know that Fibonacci sequences are involved in the growth patterns of many plants," said Dr. Mince, "but this is really fantastic." She reached out to touch a bamboo joint that was making a faint humming sound. She could feel it vibrating. "How much will this stalk grow today?"

"One hundred feet," Dr. Matsu replied.

"And when did it start to grow?"

"Six A.M. last Sunday. Today is Saturday. By tomorrow morning at six it will have grown for exactly one week."

"How much did it grow on the first and second days?"

"I don't recall," said Dr. Matsu. "Different strains of Fibonacci bamboo grow at different rates during the first two days. I'll have to check my records."

This presents us with a pretty problem in number theory. Assume that the seven integers that represent the stalk's growth on each of the seven days form the longest possible Fibonacci chain that ends in 100. Assume also that the second day's growth exceeds that of the first day, and that each day's growth is an integral number of feet. You now have all the information you need to determine how many feet the stalk grew on the first day and on the second day.

34

TETHERED PURPLE-PEBBLE EATERS

Back in the mid-twentieth century there was a popular children's riddle that went: What's purple and eats pebbles? Answer: A purple pebble-eater. These problems are not about purple pebble-eaters, but about purple-pebble eaters.

The surface of Gillikan, a curious planet discovered by the spaceship *Bagel* on one of its exploratory missions to other solar systems, is completely covered with small purple pebbles. Life on the planet is silicon based. The most intelligent of Gillikan's bizarre animal forms is a small turtlelike creature that lives in a hard gray shell shaped like a perfect cube about three inches on the side. Its intelligence is near that of a squirrel.

When members of the exploring party, led by the *Bagel*'s exobiologist, Stanley G. Winetree, first saw the gray cubes, they had no idea they were alive. But when Lieutenant Flarp reached down to pick one up, a tiny head, with a perfectly flat top and two black beady eyes, suddenly pushed out from a side of the cube and emitted a high-pitched growl. Flarp quickly put down the cube. Four legs instantly emerged from the cube's base, and the creature scurried off.

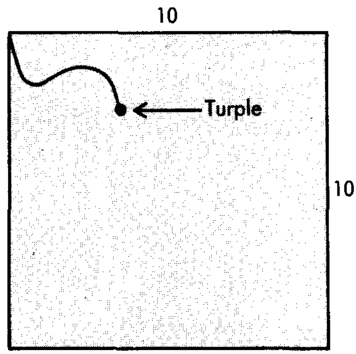
At the spot where the animal had been resting were a number of small purple cubes, hard as cement. The explorers later learned that these were the animal's excreta.

"Fantastic!" exclaimed Winetree, slapping his gloved palms against the sides of his helmet. "I can't wait until I get a chance to dissect one of these things."

"You may have to cut it open with a buzz saw," said Flarp.

One of the beasts was eventually captured and taken back to the *Bagel,* where it quickly became known as a "turple." The ship's carpenters built a square enclosure on the pebbly ground outside the ship, ten feet on the side, and tethered the turple to one of the corners, as shown in Figure 17. The turple at once freed itself by gnawing through the rope. Someone found a steel chain that was too hard for the turple's teeth. An hour later it was feeding quietly on the purple pebbles, all rather soft and crumbly.

FIGURE 17



Now for our first puzzle. Assume that the chain allowed the turple to graze over exactly one-fourth of the square field. It can, of course, feed as far as the chain's length. How long is the chain? Only some elementary geometry is needed for the solution.

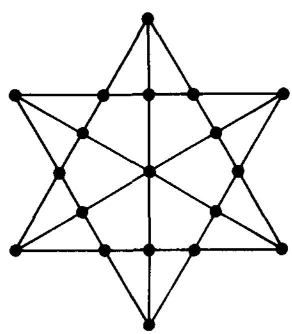
35

THE DYBBUK AND THE HEXAGRAM

I am obliged to plant a grove,
To please the pretty girl I love.
This curious grove I must compose
Of nineteen trees in nine straight rows;
And in each row, five trees must place,
Or I may never see her face.
Now, readers brave, I'm in no jest.
Pray lend your aid and do your best.

The above doggerel, from an old puzzle book, is elegantly answered by the pattern of Figure 18:

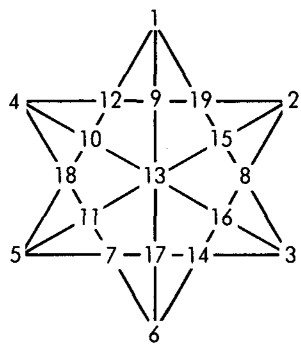
FIGURE 18



The diagram is the familiar Jewish Star of David (known to the Cabalists of the Middle Ages as the Seal of Solomon), with three lines added to join opposite corners of the star. Without the added lines, the two interlocked triangles are sometimes called a hexagram. Like the pentagram, or five-pointed star, it was widely used in past ages as a mystic diagram for warding off evil.

Late one night, when I was poring over a copy of a Cabalistic manuscript—it had been sent to me by an Israeli archeologist who had found the original parchment in a cave—I came upon a hexagram drawn just as you see it above. But there was an additional feature. Each point was labeled with a number from 1 through 19 in the remarkable manner shown in Figure 19. I realized at once that the numbers were magic. Each row of five added to 46.

FIGURE 19



The more I studied the magic numbers, the more I was impressed by their symmetries. Opposite corners of the star add to 7, like opposite faces of a die. Opposite numbers on the interior hexagon add to 26. In the center is the evil number 13. A translation from the Hebrew (supplied by my Israeli friend) explained an elaborate secret ritual by which the star could be used to summon a dybbuk from the spirit world.

Dybbuks, in ancient Jewish folklore, are the souls of dead persons who are permitted at times to return to this world, and who are capable of taking possession of the body of someone living. Readers who recall my account of how I used a pentagram to summon Mephistopheles will understand my eagerness to try a similar experiment with the hexagram.

There are two reasons why I cannot describe the Cabalistic ritual in detail. Portions of it are unsuitable for a general audience, and I don't want to be responsible for the fate of any reader tempted to indulge in ancient black arts. I will say only this: The ritual requires that the hexagram, and its powerful magic numbers, be carved in wood by a knife with a black handle of sheep's horn, and a blade that has been washed in a mixture of hemlock juice and the blood of a black cat.

Now it happens that I own a large black Persian cat named Furicle. After obtaining the proper knife and washing its blade in the prescribed manner, I retired to my study at midnight, armed with all the other necessary paraphernalia. Furicle must have sensed that something extraordinary was about to happen because he whined loudly and leaped to the top of a tall file cabinet when I lit the incense and started carving the diagram on the room's wooden paneling.

An uncouth gesture completed the twenty-minute ceremony. There was a sizzling burst of bright light from the hexagram, and a moment of total silence; then suddenly my cat began to chuckle.

"Congratulations, Gardner," said Furicle, his big blue eyes staring down at me from the file cabinet. "You performed that ceremony admirably."

"Am I talking to a dybbuk?" I asked.

"You are indeed," said the cat. "I could have entered *your* body just as easily, but this way we can converse. Don't you recognize my voice?"

I shook my head.

"I'm your old friend Jekuthiel Ginsburg. We used to meet regularly in Manhattan, thirty years ago, when I edited *Scripta Mathematica.*"

It was Ginsburg's voice all right. He had been an expert on number theory who used to hold monthly meetings of recreational math enthusiasts in the New York City area. Yesterday, he told me, when he learned of my plans to perform the ritual, he had requested permission to respond. It was the first time, he said, that the hexagram had been used this way since the sixteenth century. We chatted for several hours about number theory, while I took extensive notes on problems I may be giving in future publications.

"Will I have to exorcise you," I asked, "or will you leave voluntarily?"

"Not to worry," he said. "I'm a friendly dybbuk. But I won't go until you answer a little question about the star. How many distinct triangles can you trace in its lines?"

It took me about ten minutes to make the count. While I was working on it, Furicle prowled about my bookshelves, occasionally taking down a puzzle book and pawing through its pages. When I announced the correct number of triangles, he meowed twice, then came over to rub his back against my leg.

What number did I give? See if you can count all the triangles (some are easily missed) before you check the answer. Assume, of course, that the lines of the diagram are unbroken by spots or numerals.

36

1984

Nineteen eighty-four, the year George Orwell chose for the setting and the title of his famous science-fiction novel (in the tradition of such negative utopias as Aldous Huxley's *Brave New World,* and H. G. Wells's *When the Sleeper Wakes*) is upon us. To honor the year of this book's setting I have constructed the Orwellian magic square shown in Figure 20.

FIGURE 20

4	8	6.4	16	10
2.5	5	4	10	6.25
2	4	3.2	8	5
1	2	I.6	4	2.5
3.1	6.2	4.96	12.4	7.75

It is magic in a much more remarkable way than the conventional magic square, which has a constant sum for each of its rows, columns, and main diagonals. This square is magic in the sense that if you freely choose five of its numbers, according to a simple procedure, then multiply them together on a calculator, the product is certain to be 1984!

If you don't believe it, try the following. Select any of the 25 cells you like, and draw a circle around its number. Now cross out all the other numbers in the same horizontal row as the chosen number, and similarly cross out all the other numbers in the same vertical column. To avoid damaging the page you might want to photocopy the square or draw it on a sheet of paper. You'll need many copies if you want to repeat the trick or show it to friends.

Now select a second number. Circle it, and again cross out the numbers in the same row and same column. Repeat this five times. Each time, of course, you must choose a number not circled or crossed out. When you finish, there will be five circled numbers. Although they were selected at random, when you multiply them on your calculator, you will see 1984 on display as the final product!

It is not generally known that before Orwell picked 1984 as the time for his novel, G. K. Chesterton had used the same year for his novel *Napoleon of Notting Hill.* Orwell predicted that by 1984 the United States would be part of a tyranny under the control of Big Brother, a dictator who resembled Stalin. Of course it hasn't happened. Chesterton's novel went the other way. It predicted a revival in England of local patriotism so intense that Notting Hill, then a suburb of London (today it is one of London's slum districts), revolts against the government of England, waging a war to become a separate nation. This didn't happen either. Both novels, indeed, illustrate a fact about predicting history that Chesterton himself described in the first paragraph of his novel:

> The human race, to which so many of my readers belong, has been playing at children's games from the beginning, and will probably do it till the end, which is a nuisance for the few people

who grow up. And one of the games to which it is most attached is called, "Keep to-morrow dark," and which is also named (by the rustics in Shropshire, I have no doubt) "Cheat the Prophet." The players listen very carefully and respectfully to all that the clever men have to say about what is to happen in the next generation. The players then wait until all the clever men are dead, and bury them nicely. They then go and do something else. That is all. For a race of simple tastes, however, it is great fun.

All numbers are grist for puzzle-making, and 1984 is no exception. For instance, is it possible to use just the ten digits (0 through 9) to form a set of numbers that will add to 1984? It is easy to get 1980 in this way: 6 + 28 + 407 + 1,539 = 1980. Curiously, it is *not* possible to get 1984. However, there is one digit, and one only, such that if you omit it, the remaining nine digits *will* form sets of numbers that add to 1984.

What digit must be omitted? In other words, what set of nine different digits will form a group of numbers (using each digit only once) that have a sum of 1984? To answer this question it is not necessary to experiment with numbers. All you need is a bit of elementary number theory of the sort that accountants used to know before computers took over all their calculating tasks.

37

THE CASTRATI OF WOMENSA

Travel around the galaxy, by way of shortcuts through Wheeler wormholes, became commonplace in the twenty-third century. As a result, thousands of small planets with earthlike atmospheres became colonized by adventurous earthlings, and over the centuries each planet developed its own unique culture.

Womensa was such a planet. Its original settlers were German women of extremely high intelligence who belonged to an organization for which their planet was named. For breeding purposes they took with them a supply of males who had been genetically engineered to combine handsome faces with muscular bodies and low-grade intelligence. Low, that is, by Womensa standards. By twentieth-century standards they had the IQ of an Asimov.

The colony was a monogamous and matriarchal dictatorship ruled by a supreme dictator named Fidelia. No rules governed the love lives of unmarried men or of women, married or single, but all husbands were expected to remain permanently faithful. If it could be proved that a man committed an infidelity, the punishment was swift and severe. He was castrated and placed in the Fidelia Castrato Choir, a group of men that sang on festive occasions and at official state functions.

One day Fidelia was approached by her Minister of Morals, who, like Fidelia, had no husband. "O Great One," said the minister, "it has come to my attention that there has been an alarming rash of infidelities in our capital city. Ten husbands are suspected of having been unfaithful."

Fidelia was shocked. "Issue a decree at once," she said, between angry chomps on her cigar, "stating that the government is aware of at least one infidelity, perhaps more, among the husbands of our city. Announce also that when a wife is certain her husband has been unfaithful, she must report it to your office at once. The cad will be castrated later that day and sent to the choir. His wife will be free to remarry."

"It will create a furor, *mein Führerin,*" said the minister, bowing low and giving the Womensa salute with her middle finger, "but it shall be done."

To understand the bizarre consequences of this decree, you must accept the following posits about Womensa folkways:

1. As soon as a husband is unfaithful, gossip spreads so rapidly that within an hour every married woman in the city hears of it except the wronged wife.
2. Knowledge of an emasculation spreads with the same speed.
3. Any statement made by *die Führerin* is accepted as true.
4. All females of Womensa are perfect logicians. They immediately grasp the implications of any argument and can reason with unerring accuracy.

After the decree was issued, nine days went by with no reports of infidelities. Then on the tenth day, ten wives reported the names of their guilty husbands.

Can you explain the ten-day delay?

FIRST ANSWERS

1

CHESS BY RAY AND SMULL

The piece on d8 must be either a queen or rook, otherwise c3 and e3 would both have to hold knights that attack d1. Since there is only one knight on the board, d1 must be attacked by a queen or rook on d8.

The knight must be on c3. If it were on e3 there would be no way that two pieces could attack a7. The piece on e3, in order to attack a7, must be a queen or bishop. If a queen, then the rook must be on d8, and no piece is available to go on g7 that can attack a7. Therefore e3 must hold the bishop. Note that the bishop is now attacking g5.

The piece on g7 must be the rook or queen because it must attack both a7 and g5. It can't be the rook, because that would put the queen on d8 and we would have three pieces, not two, attacking g5. Therefore the queen is on g7, the rook on d8, leaving the king to go on b5.

Figure 21 (see next page) shows the unique solution.

After playing the game for several weeks, Ray and Smull decided to make the game harder by telling VOZ not to indicate the positions of the five pieces. Otherwise, the game was played as before.

Figure 22 (see next page) shows the number of pieces attacking each of twelve cells. Where are the five pieces?

FIGURE 21

8				R				
7	2						Q	
6								
5		K					2	
4								
3			N		B			
2								
1				2				
	a	b	c	d	e	f	g	h

FIGURE 22

8								0
7		3	2	1				
6		2						
5		1						
4					1	2		
3					2		2	
2						2	3	
1								
	a	b	c	d	e	f	g	h

2
THE POLYBUGS OF TITAN

It is impossible for a simple polyhedron, convex or not, to have just 7 edges. If you don't believe it, try to cut a potato into a 7-edged polyhedron! But first, let's prove that polyhedrons can have any number of corners, or any number of faces, greater than 3.

A tetrahedron has 4 faces. Slice off a corner and you produce a fifth face. Slice off another corner and you add a sixth face. By cutting off more corners you can form a convex polyhedron with any number of faces greater than 3.

A tetrahedron has 4 corners. Think of it as a pyramid with a triangular base. Change the base to a square and you have a pyramid resembling the Great Pyramid of Egypt. It has 5 corners. Change the base to a pentagon and you get 6 corners. By successive additions of sides to the base you can make a pyramid with any number of corners greater than 3.

The edges are more complicated to analyze. There is insufficient space for a formal proof that 7 edges are impossible, but here is one way to go about it. A well-known formula, by the famous Swiss mathematician Leonard Euler, says that for any simple polyhedron, with simple polygon faces, $V - E + F = 2$. V stands for vertexes (corners), E for edges, and F for faces.

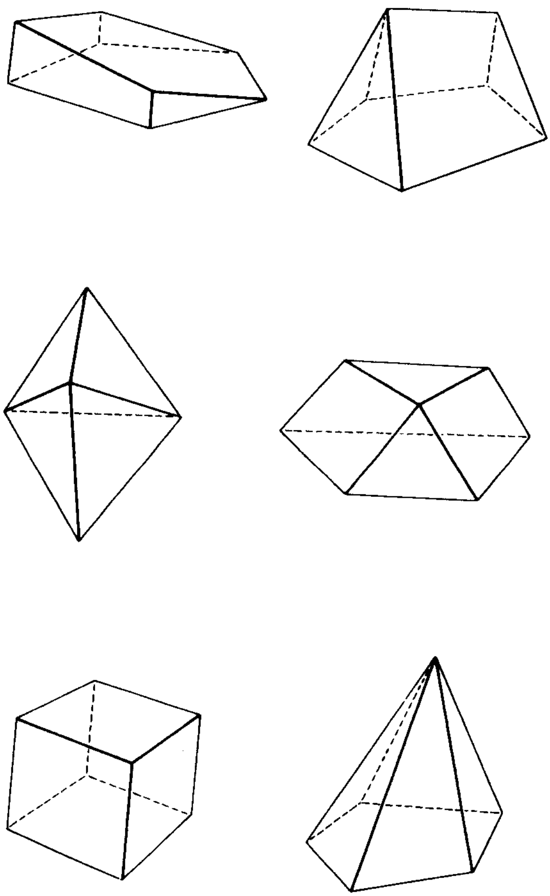
Because at least 3 edges must meet at every corner, and at least 3 edges must surround every face, it is not hard to show that two inequalities must hold:

$3V$ is equal to or less than $2E$.

$3F$ is equal to or less than $2E$.

Let $E = 6$. There is only one way to plug two other positive integers into Euler's formula and simultaneously satisfy

FIGURE 23



the two inequalities: $V = 4$, $E = 6$, and $F = 4$. We have described the structure of the tetrahedron, the only polyhedron with four faces.

Let $E = 8$. Again there is only one solution in positive integers that satisfies the three formulas: $V = 5$, $E = 8$, and $F = 5$. We have described the Great Pyramid. Let $E = 7$. A little experimentation will convince you there are no integral values for V and F that will satisfy Euler's equation and the two inequalities. In other words, no simple polyhedron has just seven edges.

Another way to look at the problem is to imagine a tetrahedron altered so it acquires a fifth corner. There is no way to do this without causing the number of edges to jump from 6 to 8. Above 8, a convex polyhedron can have any number of edges.

Further inspection of Titan's polybugs showed that there were just seven varieties of hexahedra—convex polyhedra with 6 faces. Six are shown in Figure 23. They are topologically (or combinatorially) distinct in the following sense. It is impossible to deform the skeleton (structure of edges) of one to the skeleton of another by varying the lengths of edges and the angles between them. A cube, for example, is combinatorially identical with a 4-sided pyramid that has had its top corner sliced off.

There is a seventh convex hexahedron not shown. Can you sketch its picture?

3

CRACKER'S PARALLEL WORLD

In our universe Rodin's Thinker has his right elbow on his *left* thigh.

I am writing this in 1980, a year that has exactly 36 divisors. (We count 1 and the year itself as divisors.) This is surprising, considering that the previous year, 1979, is a prime with no divisors except 1 and itself, and that you must go back as far as 1800 to find an earlier year with as many as 36 divisors.

Can you determine the year of Cracker's parallel-world experiment if I tell you it is the first year later than 1980 with as many as 36 divisors, and that, like 1800 and 1980, it has just 36 divisors?

To find the divisors of a number, first determine its prime factors, then multiply all possible combinations of prime factors to obtain the nonprime divisors. For example, the prime divisors of 1980 are $2 \times 2 \times 3 \times 3 \times 5 \times 11 = 1980$. Thus its divisors form a sequence of 36 numbers beginning with 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 15, . . ., and ending with 1980.

4

THE JINN FROM HYPERSPACE

I am indebted to Douglas Hofstadter for his proof of the upside-down version of Fermat's last theorem. Hofstadter introduces the theorem in his Pulitzer Prize-winning book *Gödel, Escher, Bach: An Eternal Golden Braid.* No proof is there given because, as the text says, the marvelous proof "is so small that it would be well-nigh invisible if written in the margin."

The equation $n^a + n^b = n^c$ obviously has no solution in positive integers if $n = 1$ because it then reduces to the false equality $1 + 1 = 1$. It has an infinity of solutions if $n = 2$. We have only to let $a = b$, and $c = a + 1$. For example: $2^2 + 2^2 = 2^3$.

Suppose n is greater than 2. If n is the base of a number notation, then all powers of n have an n-ary representation that is 1 followed by a string of 0s. Thus in our base-10 notation, all powers of 10 have the form: 10, 100, 1000, 10000, and so on.

In the upside-down equation either $a = b$ or a does not equal b. If $a = b$, the sum of the two equal powers, written in base-n notation, will be the sum of two numbers, each written as 1 followed by a zeros. The sum will have the form of 2 followed by a zeros, which obviously cannot be a power of n.

Suppose a is not equal to b. Each power will be written in base-n notation as a 1 followed by a string of 0s, but now the strings will be of different lengths. Therefore the sum will have the form of 1 followed by a string of 0s that will contain another 1 somewhere in the string. Once more, a number of this form cannot be a power of n. Since a must either equal or not equal b, we have proved the theorem by *reductio ad absurdum.* (You may wonder why this proof doesn't apply to binary notation when $a = b$, but a little experimentation will make it clear.)

The jinn scowled while Fletcher explained his mistake. "We did supply what you requested," he said. "Therefore it must count as a fulfilment of your second wish. What is your third?"

"I desire a proof that *this* equation," Fletcher boomed in his new stentorian voice, "has no solution when n is greater than 2!" This time he wrote the equation correctly.

"To hear," said the jinn, bowing, "is to obey. But I must again check with my superiors."

The jinn flowed back into the bottle. Several minutes passed. Fletcher could not stifle another impulse to test his voice. He sang a familiar aria from *Rigoletto,* ending on a high note. He belted out the note with all the lung power he could muster.

The pink bottle shattered into a thousand pieces.

Of course Fletcher never saw the jinn again. Nor could he locate the old bottle shop. It seemed to have vanished as completely as the jinn.

Fletcher changed his name and occupation. Perhaps you have heard of John Luciano Pavoletti, said to be the greatest tenor since Caruso.

Admirers of the fantasy of John Collier may recall his classic short story "Bottle Party," about a jinn and an unfortunate fellow named Frank Fletcher. If you are interested in a simple way to construct a Klein bottle out of paper, see Chapter 2 of my *Sixth Book of Mathematical Games from Scientific American.*

5

TITAN'S LOCH METH MONSTER

Call the serpent's length x. The length of each piece is $x/3$. We can now write the equation:

$$x/3 = 10 + x/(2 \times 3).$$

Solving the equation gives x a value of 60 meters.

Unfortunately, this answer presupposes a fact that is not given in the statement of the problem. What is this unjustified assumption, and what is the problem's correct answer when the assumption is *not* made?

6

THE BALLS OF ALEPH-NULL INN

Here is one way to prove that balls 1 through 10 cannot form a magic triangle. Suppose there is a solution; then consider the following diagrams:

FIGURE 24

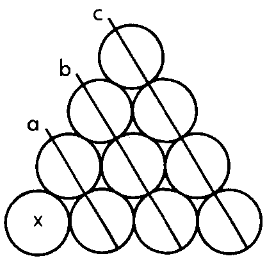
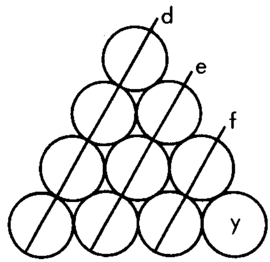


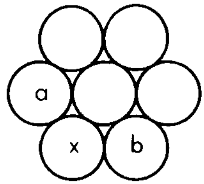
FIGURE 25



On the left, slanting rows *a, b, c* must each have a sum that is one third of the total of numbers on the nine balls that form these three rows. This sum is clearly equal to 55 (the total of all ten numbers) minus the number on ball *x*, and the result divided by 3. Expressed algebraically, the constant is $(55 - x)/3$ on the right. For the same reasons, the magic constant for rows *d, e, f* must be $(55 - y)/3$. Because the magic constants for both diagrams must be the same, we are forced to conclude that $x = y$. But x cannot equal y because each ball has a different number. Therefore our original assumption is false. We have proved by *reductio ad absurdum* that no magic triangle of numbers 1 through 10 is possible. The proof generalizes in an obvious way to triangles of any size.

After Yin explained her proof, the two children decided to investigate hexagonal patterns. First they tried a pattern

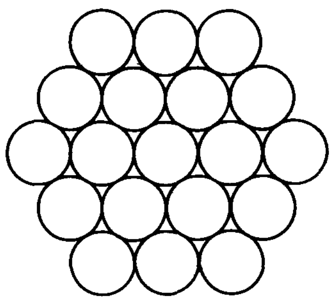
FIGURE 26



of balls 1 through 6, but they quickly saw that this had no solution.

A corner ball in the above pattern, such as the one marked *x,* belongs to two rows of two balls each. This would require that balls *a* and *b* have identical numbers to give the two rows the same sum. Yin and Yang next turned their attention to the 19-ball pattern shown below.

FIGURE 27



Several hours later they found a solution. That night one of their parents, a mathematics teacher, told them that not only was this the only possible solution (not counting rotations and reflections as different), but that no hexagon of any larger size could be magic. This is not true of magic squares. The larger the square the greater the number of solutions. A magic square of four by four (numbers 1 through 16) has 880 basic solutions. In 1973 a computer program found that there are 275,305,224 basic magic squares of five by five. The number of six-by-six magic squares is so enormous it has not yet been determined. Yet for magic hexagons, regardless of size, there is just one specimen!

Your problem is to put numbers 1 through 19 in the cells so that every row of three, four, or five cells, in any direction, has the same magic sum. The sum is 38. This is easily obtained by taking the sum of numbers 1 through 19, which is 190, and dividing by 5, the number of parallel rows in any direction.

It is a formidable task. You are urged to work on it for a while with numbered pieces of cardboard before you give up and turn to the solution.

7

SCRAMBLED HEADS ON LANGWIDERE

Call the ladies D, T, and Z. Their three head checks can be permuted in just six equally possible ways:

DTZ TDZ ZDT DZT TZD ZTD

Assuming the correct order is DTZ, it is easy to see that in four of the six cases at least one lady gets her correct check. The probability of this occurring, therefore, is 4/6 = 2/3 = .6666. . . .

For one person and one check, the probability of getting the right check obviously is 1. For two people with randomized checks the probability at least one check is right goes down to 1/2. As we have seen, for three people it jumps back up to 2/3. For four persons there are 24 equally possible permutations, of which 15 have at least one check correct, so the probability lowers to 15/24 = 5/8 = .625. As n (the number of persons) increases, the probability alternately bobs up and down, but with rapidly diminishing increments. In the long run the sequence "strangles" (converges on) a limit. What is this limit?

8

ANTIMAGIC AT THE NUMBER WALL

The only other rook-wise connected order-3 antimagic square is:

7	6	5
8	9	4
1	2	3

Each pattern is the "complement" of the other because it is obtained by changing each digit to its difference from 10. In both solutions the rook's path is a spiral from 1 to 9.

Yin and Yang next turned their attention to antimagic triangles, and quickly discovered that the triangle formed with 1, 2, and 3 is antimagic:

1 2
3

"How pretty!" exclaimed Yin. "If we add the three corner numbers, we get 6. That makes four sums in consecutive order—3, 4, 5, and 6."

When the children began exploring antimagic triangles of the next larger size they found them so plentiful that they started searching for a pattern in which the sums formed by each row of two or more digits, the three corner digits, and the three interior digits were not only distinct, but were in consecutive order. Yang was able to prove that the eight consecutive sums had to be 6, 7, 8, 9, 10, 11, 12, and 13. To their delight, Yin and Yang found a solution and proved it was the only one.

Can you put the digits 1 through 6 in a triangular array so that the eight sums (formed by all rows of two, all rows of three, the three corner digits, and the three interior digits) are the numbers from 6 through 13?

9

PARALLEL PASTS

It was Johnny Carson's first question. Nobody, but nobody, in show business in this United States, circa 1981, speaks of "the" show business.

Cracker and Ada decided not to linger in 1981. They were eager to travel to the parallel pasts of cities other than New York. Plans had already been made to visit Stratford, England, on April 23, 1616, then make a quick trip to Madrid on the same day to check on the deaths of Shakespeare and Cervantes. As is well known, both writers died on April 23, 1616. Here is how Robert Service described the coincidence in a poem:

Is it not strange that on this common date,
Two titans of their age, aye of all Time,
Together should renounce this mortal state,
And rise like gods, unsullied and sublime?
Should mutually render up the ghost,
And hand in hand join Jove's celestial host?

At Stratford-upon-Avon, Cracker and Ada witnessed Shakespeare's funeral and burial. But when they moved to Madrid, on the same day, they discovered that Cervantes had died ten days earlier!

"I'm staggered by this disparity," said Cracker, when they were back to 2001 in the apartment they shared on 106th Street. "In view of our past experiences with parallel pasts, I would never have anticipated such a big difference."

However, as Ada later learned, the difference was greatly exaggerated. What careless mistake had they made?

10

LUKE WARM AT FORTY BELOW

Minus forty is the only value at which the Fahrenheit scale intersects the Celsius (or centigrade) scale. In other words, minus forty on either scale exactly equals minus forty on the other!

Because 100 divisions on the Celsius scale equal 180 divisions on the Fahrenheit, each Fahrenheit degree is 5/9 as large as a Celsius degree. And zero Celsius, the temperature at which water freezes, is 32 degrees Fahrenheit. Knowing these facts, it is not hard to arrive at the following equation: $C = 5/9(F - 32)$.

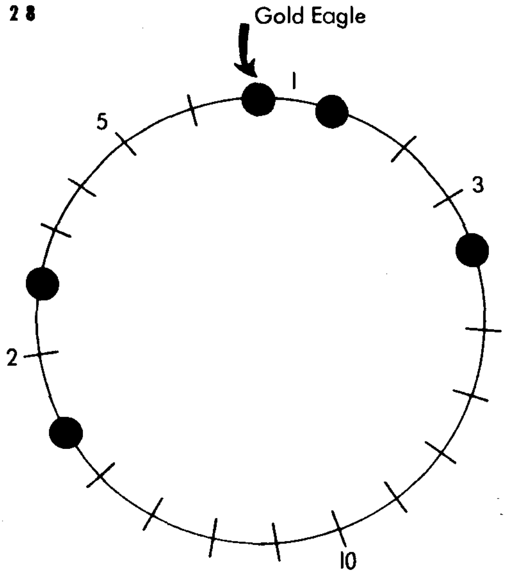
If $C = -40$, the equation gives F a value of -40. Incidentally, Luke's thermometer could not have operated with mercury because mercury freezes at −39C.

You may have noticed that on Fahrenheit thermometers for taking your temperature there is a slight constriction in the tube that keeps the mercury from dropping down again until the thermometer is shaken. How come? If the mercury, when it expands, can go up through the constriction, why can't it go down when it contracts? And why will the reading drop for a moment if you plunge the thermometer in hot water?

11

THE GONGS OF GANYMEDE

FIGURE 28



The spots in Figure 28 show where the five eagles must go. Check and you will see that every number from 1 through 20 is represented once and once only by a distance on the circle between a pair of spots.

Let us generalize to n points (eagles). If $n = 1$ the pattern is trivial: a single point on a circle of length 1. If $n = 2$ the pattern is almost as trivial: a circle of length 3. When $n = 3$ the only pattern is a circle of length 7 with three points that have distances between them of 1, 2 and 4—the triple resonance lock! Every integer from 1 through 6 is given once only by a distance between two points.

For $n = 4$ there are two essentially different solutions. The circle's length is 13, and the spacings are either 1, 2, 6, 4 or 1, 3, 2, 7. We have already seen the only solution for

$n = 5$. For $n = 6$ the circle is length 31 and there are five solutions:

1, 2, 5, 4, 6, 13
1, 2, 7, 4, 12, 5
1, 3, 2, 7, 8, 10
1, 3, 6, 2, 5, 14
1, 7, 3, 2, 4, 14

We can define a "Ganymede circle" as one on which n points can be placed so that the unit distances between pairs of points are the counting numbers from 1 through $n(n - 1)$. The circle's circumference will be $n(n - 1) + 1$. In 1938 James Singer, an American mathematician, proved that Ganymede circles exist for every n equal to a prime plus 1 or a power of a prime plus 1—more formally, if $n = p^k + 1$, where p is a prime and k is any positive integer. It is not known if there are Ganymede circles that escape this proviso.

Singer conjectured that the number of different patterns (not counting reflections) for a Ganymede circle with more than two points is equal to Euler's phi function for the circumference, divided by $6k$. Euler's phi function for integer n is the number of integers not greater than n and relatively prime—that is, with no common divisors other than 1 and n—to n. For example, when $n = 6$ the circle's length is 31. Euler's phi function for 31 is 30. (The number 1 is always included among the relatively prime numbers. For any prime p, the phi function is always $p - 1$.) Writing 6 in the form $p^k + 1$ gives $5^1 + 1$, so $k = 1$. Our formula takes the values $30/6 = 5$, which tells us that for $n = 6$ there are five distinct solutions, not counting their reversals. David A. James, writing on "Magic Circles" in *Mathematics Magazine* (Volume 54, May 1981, pages 122–125; see also his letter on page 148 of the same issue), reports that he has proved the impossibility of a circle for $n = 7$, and that Singer's conjecture holds for all n less than 18.

On any Ganymede circle, *every* integral distance, however large, can be measured (and in only one way) if we permit as many circuits as we like around the circle in either direction. One day a devout Pythagorologist, after finishing his ceremonial walk, gazed up at the brilliant night sky. Jupiter was below the horizon, but Io was almost full, and he marveled at its random, disheveled surface of wavy dark lines—such a striking contrast to the rigid order of the counting numbers!

"I think I'll continue my walk," he said to himself, "by taking every sixth dome, then every seventh, and so on, until I strike a gong exactly 100 times."

Can you determine how many circuits he will have made (including the eleven circuits of his ceremonial walk) before he strikes 100?

12

TANYA HITS AND MISSES

"Because, dummy," said Tanya, "you wouldn't have said three out of the first five planets had life-forms unless the fifth were one of them. If the fifth were barren, you'd have said three out of the first *four*. Do you know how many planets there are altogether?"

"I do." The colonel nodded. "And I also know how much you like puzzles, so let me put it this way. There are more than seven planets."

"Go on," said Tanya, who had found a pencil and a piece of paper on the computer's console.

"Counting from the star, the first, second, and sixth planets are barren."

Tanya wrote it down, then looked up.

"The eighth planet, counting the other way, from the outermost planet toward the star, is also barren."

"And . . ." said Tanya, holding up the pencil.

"And between the sixth planet from the star, and the eighth planet from the other end, there are just three planets. All three have life-forms."

Tanya made the sketch shown in Figure 29, shading each planet she assumed was barren. "It's trivial," she said. "Obviously there are seventeen planets. I know four are barren, and I know six have life, but you haven't told me anything yet about the outside seven."

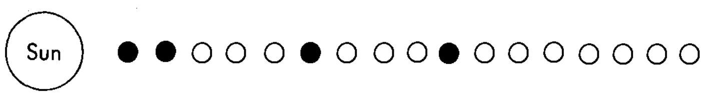
Colonel Couth chuckled couthly. "Your answer is wrong, my dear," he said. "But I haven't told you one more thing. There are fewer than fifteen planets in the system."

"That's impossible!" the girl exclaimed.

"No," said her father. "But you'll have to give it some more thought.

Tanya puzzled over her diagram for ten minutes or so before the *aha!* insight struck her. How many planets are in the system?

FIGURE 29



13

MYSTERY TILES AT MURRAY HILL

FIGURE 30

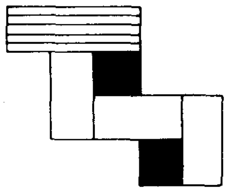


Figure 30 shows a "fundamental region" of the infinite tiling pattern. It is easy to see that by translating (sliding) this shape along the plane, without rotating it, you can tile the entire plane. The area of the shape is 11 unit squares, and since it contains six tiles, the average tile area is 11/6 or 1.83333. . . .

"There's one thing I haven't told you yet," said Doris to Clyde. "Although no rectangle can achieve the 11/6 minimum, there is one rectangle, and one only, that actually goes *below* the lower bound."

"I'm not surprised," said Clyde. "Strange exceptions like that have a way of turning up in tiling theory. They're like spots of yin within the yang. What does the maverick rectangle look like?"

When Doris sketched it, Clyde was astounded by its simplicity. It contained only five tiles, and the average area of a tile was 9/5 = 1.8. Can you discover this exception?

14

CROSSING NUMBERS ON PHOEBE

FIGURE 31

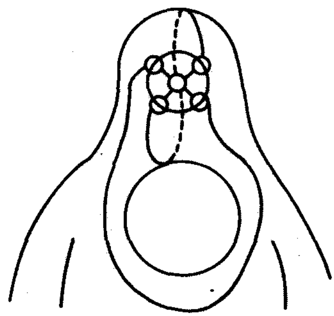


Figure 31 shows how Phoebe connected her five facial features to make a complete graph with no crossings.

"Touché!" exclaimed Arkay, while Phoebe hummed a duet with herself. "I forgot you were a tor. The crossing number for a complete graph of five points is one when the points are on a plane or on a sphere, but of course it's zero on a torus. In fact, Hoorayri discussed crossing numbers for toroidal graphs. It's hard to believe, but if your face had seven features, you could still draw a complete graph on your skin without any crossings!"

At the time this conversation took place, formulas had been found for determining the crossing numbers for complete graphs of any number of points, both on the plane and on the surface of a torus. At present, there are only conjectured formulas, and very little is known about crossing numbers on other surfaces such as the Klein bottle and the projective plane. On the plane, the crossing numbers for 6 through 10 points are known to be 3, 9, 18, 36 and 60. On the torus, for 6 through 10 points, the crossing numbers are 0, 0, 4, 9, and 23.

Now see if you can prove that on the plane the crossing number for the complete graph of five points is 1, and then use that theorem to show that no map of five regions can be drawn on the plane so that each region "touches" the other four in the sense of sharing a common part of a border.

15

SF'S AND F'S ON FIFTY-FIFTH STREET

The number of magazines in the stack could be 39, 40, or 41. When I counted 20 edges I counted the *same* set of magazines I counted before when I counted 20 spines. The magazines in the other set could be 19, 20, or 21.

"Twenty bucks seems high," I said, assuming there were 40 magazines in the stack. "I'll give you ten for the lot."

Palmer finished his sandwich, washed down the last bite with a swig of beer, and wiped his mouth on the back of a coat sleeve.

"It's a deal," he said, "if you can pass my **F** test."

"Your what test?"

"My **F** test." He aimed a pencil at the sign on the table. "You read that a moment ago. I saw you reading it. Now read it again slowly, from start to finish, and count all the **F**s on the sign. If you can tell me the correct number, you can have all the magazines for ten. If you miss, you have to pay twenty."

The offer seemed fair enough. I read the sign carefully, counting the **F**s. When I finished I called out a number.

"Wrong!" shouted the gnome. He leaped from his chair and walked toward me, rubbing his palms together and chortling.

And you know, he was right! Maybe you can do better. Go back and read the sign again *just once.* No fair checking more than once. How many **F**s do you count?

16

HUMPTY FALLS AGAIN

To read the curious lettering on the looking-glass wall, turn the page upside down and hold it up to a mirror.

I had no trouble answering Humpty's question. After all, the egg was only a figment of my dream, and I myself had invented the puzzle a few weeks before. But the egg seemed a bit miffed.

"Well, you can't win 'em all," he said sadly. "But before I let you go, see what you can make of this riddle. I thought of it while I was cooking your mushroom. What's the longest word in . . ." He paused, raised both hands, and wiggled two fingers on each side of his huge face to indicate quotation marks. ". . . the English language?"

"Hmmm," I mused. "I'm really not sure. Of course I remember some old joke answers. *Rubber,* because if it isn't long enough you can stretch it. And *smiles,* because there's a mile between the first and last letter. *Beleaguer* is even longer because there's a league between the first two letters and the last. And *endless,* because there's no end to it. And . . ."

"All old chestnuts," Humpty snorted. "My words always mean just what I want them to mean. The answer is in every pocket dictionary."

17

PALINDROMES AND PRIMES

Here's the quickest way to show that no palindrome except 11 can be a prime if it has an even number of digits.

A familiar method of testing any number for divisibility by 11 is to add all the digits in even positions, then add all the digits in odd positions. If and only if the difference between these two sums is 0 or a multiple of 11, the number will be divisible by 11. When a palindrome has an even number of digits, those in the even positions will duplicate those in odd positions. The two sums will be the same, and their difference will be zero; hence the number will be a multiple of 11 and therefore composite (not prime).

While VOZ was checking 196 to one million steps, he searched his memory bank for some palindromic number curiosities that he thought would amuse the computer hacks. Here are some he found:

The only known asymmetric number that produces a palindrome when cubed is 2201. Its cube is 10662526601. According to VOZ, this was first noted by Trigg in 1961.

The smallest palindrome prime containing all ten digits is 1023456987896543201, which was proved by Harry L. Nelson in 1980.

The largest known palindromic prime, discovered by Hugh C. Williams in 1977, consists of the digit 1 repeated 317 times. It is called a repunit prime. The only other known repunit primes are 11, and the primes formed by 19 and 23 units. The number formed with 1,031 units is probably the next largest repunit prime, but this has not yet been proved.

It is trivially true that no prime can be made by repeating any digit other than 1. But can you prove that a repunit prime must have a prime number of digits?

18

THIRTY DAYS HATH SEPTEMBER

Select any piano key F (the initial of Fort) and call it January. Now go up the keyboard, labeling the black and white keys in sequence: February, March, April, and so on to December. Every white key will correspond to a "long month" of 31 days. Every black key will correspond to a "short month" (including February).

Can you tell me how old Myrtle's father was, at the time he discussed the old Gregorian calendar with his daughter, if I tell you he was exactly x years old in the year x^2?

19

HOME SWEET HOME

The alien is describing an issue of *Isaac Asimov's Science Fiction Magazine*.

Here is another excerpt from the alien's report:

We observed the following strange event. A tall man tried desperately to reach a seven-faced polyhedron, partly buried in the ground. Two of the solid's faces were irregular pentagons. The other five faces were rectangular.

A shorter man, wearing a mask that covered his entire face, crouched near the polyhedron. He tore off his mask, dropped it to the ground, and apparently tried to prevent the tall man from reaching the polyhedron. Suddenly the tall man threw himself flat on the ground and stretched out one hand. A microsecond after he touched the polyhedron, the shorter man jabbed him in his buttocks with a small sphere, the surface of which had a pattern that resembled the yin-yang symbol of the Orient (see our former report on Tokyo).

A third man, dressed entirely in black, extended his arms and yelled a word we did not understand. Immediately a crowd of shouting men emerged from a half-underground structure, picked up the tall man, and carried him away.

What familiar event is the alien describing?

20

FINGERS AND COLORS ON CHROMO

There are 58 pinks, 1 blue, and 1 green. This is easy to prove by trying to find a triplet that will *not* contain a pink. If there are two or more blues, we could put two blues and one green at a table, and thereby contradict Coralie's statement that every possible triplet contains a pink. Similarly, if there are two or more greens. Therefore there can be only one blue and one green.

After dividing the 60 guests into triplets, Coralie found that the hall where they planned to hold the banquet was too small. Two rooms were required. Call them *A* and *B*. At first Coralie planned for 30 guests in each room; then she discovered that room *B* was larger than room *A*.

After looking over the two rooms, Coralie decided to shift just enough guests from *A* to *B* to make 15 more persons in *B* than in *A*. How many guests had to be moved in her seating arrangement?

21

VALLEY OF THE APES

The only word the first and second phrases have in common is red, and the only sign in common is thumbing the nose. So thumbing the nose means red.

The only word the first and third phrases have in common is eat, and the only sign in common is turning a back flip, so this sign means eat.

The only word the second and third phrases have in common is berry, and the only sign in common is poking a little finger in the right ear, so this sign means berry.

In phrase one, we know the signs for eat and red, so scratching the left eyebrow must mean ant.

In phrase two, we know the signs for berry and red, so sticking out the tongue must mean big.

In phrase three, we know the signs for eat and berry, so raising the left foot must mean quick.

The word order is variable. To say "Big ant" one must stick out the tongue, then scratch the left eyebrow, or make the same signs in reverse order.

A tribe of hunters called the Hiyikus live not far from the valley of the apes. They hunt occasionally for gorillas in the belief that eating gorilla meat makes them stronger. One day a hunter returned from the valley and reported to his chief that he had entered a cave containing some live gorillas.

"Did you kill some gorillas in the cave?" the chief asked.

"No," replied the hunter.

"Did you leave some gorillas alive in the cave?"

"I did not, great chief," said the hunter.

The chief look puzzled. "Did some gorillas enter or leave the cave while you were there?"

"No ape came or went," said the hunter.

"I understand," said the chief, nodding. "There were ___ gorillas in the cave when you entered it."

What number did the chief name? Yes, there is a perfectly logical answer. The Hiyikus have high IQs and very precise ways of speaking.

22

DR. MOREAU'S MOMEATERS

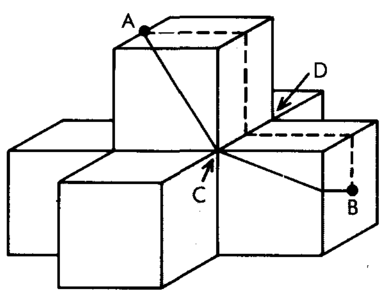
Montgomery was right. Each birth adds ten more fish to the population, but because the mother is at once eliminated, the addition is only nine. If we start with 10 fish in the tank, the population first jumps to 19, then to 28, and continues in the sequence 37, 46, 55, 64,. . . . Each term is a multiple of 9 with one more fish added. Because 5,000 is not a term in this sequence, the tank cannot contain exactly that number. The closest the population can get to 5,000 is 4,996.

Dr. Moreau was amazed by how rapidly Montgomery arrived at this result. But Montgomery had used a shortcut that is familiar to all accountants. Do you know the trick?

23

AND HE BUILT ANOTHER CROOKED HOUSE

FIGURE 32

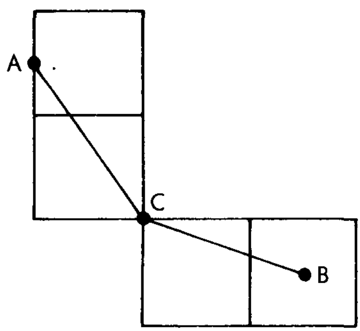


Did you decide that the shortest path is the one shown dotted in Figure 32? Assuming that each cube has an edge of 1, the length of this path obviously is 3.5. But this is not minimal. If the spider takes the path shown by the solid line, going first to corner *C*, we can calculate the geodesic from *A* to *B* by unfolding the faces as shown in Figure 33, then drawing straight lines from *A* to *C*, and from *C* to *B*. Applying the Pythagorean theorem, we find that *AB* is the square root of 3.25, or 1.8027 +, and *BC* is the square root of 2.5, or 1.5811 +. The two lengths add to 3.38+, which is slightly shorter than 3.5, the length of the dotted path.

Actually, the spider can take any of four different routes, each with the same minimal length. Instead of going from *C* to *B* as shown, it can crawl to *B* along the top of the "arm" cube. And the two paths have mirror images at the back of the house where the spider can crawl first to corner *D*, then take either of the two alternate routes from *D* to *B*.

Teal and I thought of many other pleasant puzzles based on his house, but I have space for only a few more.

FIGURE 33



1. Suppose you have a small model of the house made out of solid wood. You want to saw this polycube apart to make a set of smaller polycubes—a polycube is a solid formed by attaching unit cubes at their faces—that can be fitted together to make a 2 × 2 × 2 cube. What is the smallest number of cuts required?

2. Suppose you have a buzz saw and want to cut your wooden model into eight separate unit cubes. You may arrange the pieces any way you like before you make an-

other push past the rotating blade. How many pushes are needed to produce the eight separate cubes?

3. If you color each square face on the surface of your model a solid color, and in such a way that no pair of like-colored faces touch along an edge of one of the eight cubes, how many colors are necessary and sufficient for this task?

4. Imagine a model of the "skeleton" of the house, constructed with unit-length toothpicks, held together at the corners of the cubical cells by tiny balls of clay. You may remember that Teal used toothpicks in this way to build models of the house for his friend Bailey. What is the smallest number of toothpicks you must remove from the model so that no complete skeleton of a cube remains?

24

PIGGY'S GLASSES AND THE MOON

Myopic (nearsighted) persons wear corrective lenses that are concave. You can always tell when persons are nearsighted by the fact that their glasses make their eyes look smaller. Hyperoptic (farsighted) persons wear convex lenses, which magnify their eyes. Only convex lenses will focus light on a small spot. There is no way that the concave lenses of Piggy's spectacles could have been used to start a fire.

Golding's second mistake occurs at the beginning of Chapter 5. The day has just ended, and the stars are starting to come out. An air battle is being fought. Here is how Golding describes what the boys saw in the sky:

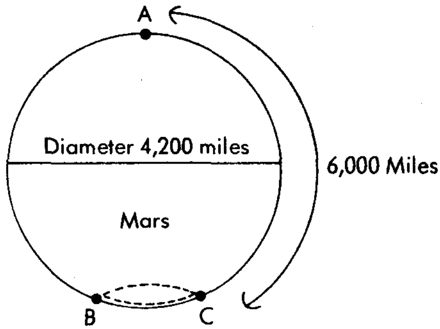
> A sliver of moon rose over the horizon, hardly large enough to make a path of light even when it sat right down on the water; but there were other lights in the sky, that moved fast, winked, or went

out, though not even a faint popping came down from the battle fought at ten miles' height. But a sign came down from the world of grownups, though at the time there was no child awake to read it. There was a sudden bright explosion and corkscrew trail across the sky; then darkness again and stars. There was a speck above the island, a figure dropping swiftly beneath a parachute, a figure that hung with dangling limbs. The changing winds of various altitudes took the figure where they would. Then, three miles up, the wind steadied and bore it in a descending curve round the sky and swept it in a great slant across the reef and the lagoon toward the mountain. The figure fell and crumpled among the blue flowers of the mountain-side, but now there was a gentle breeze at this height too and the parachute flopped and banged and pulled. So the figure, with feet that dragged behind it, slid up the mountain. Yard by yard, puff by puff, the breeze hauled the figure through the blue flowers, over the boulders and red stones, till it lay huddled among the shattered rocks of the mountain-top.

The writing is superb, but once again Golding has made a mistake that suggests how little he cared about hard science. It is surprising to learn that Golding first majored in science, at Oxford University, but after two years switched to English literature. And a good thing, too, because apparently his interest in science was minimal. What's absurd about the paragraph quoted above?

25
MONORAILS ON MARS

FIGURE 34



To see that *B* is closer to *C* than *A*, take a look at the sketch shown in Figure 34. *B* and *C* must lie on the dotted circle. Assuming *B* and *C* are as far apart as possible, the distance between them obviously is smaller than the distance from *A* to *C*. (Thanks to Mike Steuben for this one.)

About ten years after America established the first three bases on Mars, the Russians built four bases. They located them at the corners of a square. Call them *ABCD*.

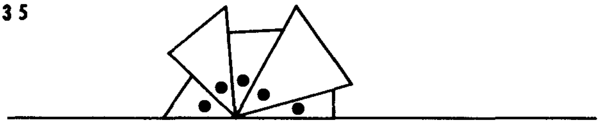
The four bases were joined by a monorail network that minimized the total length of the track. Assuming that the side of the square is 100 miles, can you calculate the network's total length?

26
THE DEMON AND THE PENTAGRAM

Surprising as it may seem, it is impossible to draw a pentagram, no matter how distorted, without having the five angles at its points add up to exactly 180 degrees.

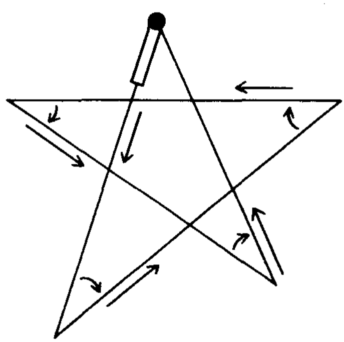
There is a crude way to demonstrate this. On a sheet of paper draw a pentagram as irregular as you please, then cut out each of its five points. Place them together and they will define a straight line as shown in Figure 35. Of course, this is not a proof of the theorem. You can prove it by plane geometry, but there's a much simpler way.

FIGURE 35



Place a match at one corner, on a side of the top angle as shown in Figure 36. Slide it down the line to the lower corner, then rotate it clockwise as indicated by the arrow (keeping one end on the vertex) until it coincides with the angle's other side. Now slide it up the line to the corner on the right, rotate the match as before, slide it to the next corner, and continue this way until the match is back at its original position. You'll find that the match has turned upside down, having rotated exactly 180 degrees. It is easy to see that this rotation has measured the sum of the five angles.

FIGURE 36



The sliding match models what mathematicians call a rotating vector. The trick can be used for proving many other theorems about the angles, both interior and exterior, of polygons. For example, it will show that the inside angles of any triangle add up to 180 degrees like the pentagram. It will show that the inside angles of any quadrilateral add up to 360 degrees, those of any pentagon to 540 degrees, and so on. The sums for all polygons are multiples of 180 degrees because the match, when it returns to its starting position, can only be in one of two states that differ from each other by a 180-degree turn.

FIGURE 37

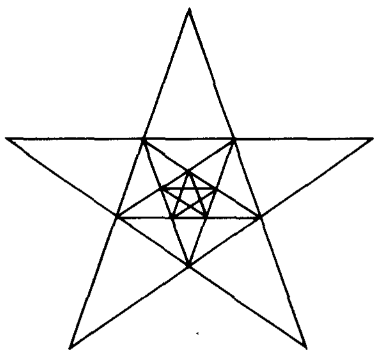
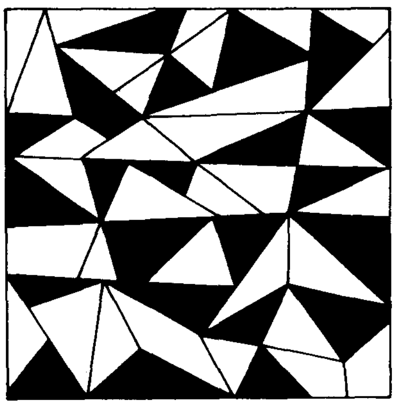


Figure 37 shows how smaller and smaller pentagrams will nest inside one another, a process that continues to infinity. Of course, you can circumscribe a regular pentagon around the original one, extend its sides to make a larger star, and proceed toward infinity in the opposite direction.

The most amazing property of this infinite set of nested pentagrams, a property well known to the ancient Greeks, is that the length of every line segment is in golden ratio to the length of the next smallest segment. The golden ratio is a famous irrational constant expressed by the infinite decimal fraction 1.61803398 . . . It is the positive root of the equation $x^2 - x - 1 = 0$, and is equal to half the sum of 1 and the square root of 5. A rectangle with sides in the golden ratio is believed to be the rectangle of most pleasing shape. If you care to learn more about this remarkable number, and why it is so important in nature, art, and recreational mathematics, see the chapter on it in my *Second Scientific American Book of Mathematical Puzzles & Diversions.*

The star border of a regular pentagram is concealed in the pattern shown in Figure 38. How quickly can you find it?

FIGURE 38



27

FLARP FLIPS A FIVER

Coins won't spin on a rotating spaceship. The inertial forces of a spinning coin make it behave like a small gyroscope. It preserves the orientation of its axis relative to the stars, just as the earth (a huge gyroscope) keeps one end of its axis pointing toward the north star. Because the torus-shaped ship rotates around an axis perpendicular to that of the spinning coin, the coin at once falls over.

Even a flipped coin would behave peculiarly unless its spin axis paralleled that of the ship. A ballet dancer on such a ship would be unable to execute pirouettes. Jugglers could not twirl plates on sticks or balls on their fingers. Yo-yos and tossed Frisbees would act strangely. Tops would not stay upright. Devices that rotated around vertical axes (overhead fans, turntables, flywheels) would have to be designed to overcome strong forces of torsion and friction. Machine parts that spun on horizontal axes would have to be carefully aligned to keep them stable. (The outside rim of the ship is of course the floor, so "vertical" here means pointing toward the center of the toroidal ship's "hole.")

This leads to an interesting paradox. According to relativity theory, there is no such thing as absolute motion. There is only motion relative to a fixed inertial frame of reference. In relativity theory, either the ship or the cosmos can be taken as the fixed frame without violating any natural laws. But if the ship is assumed to be unmoving, and the cosmos is regarded as rotating around it, how can you explain the eccentric behavior of the spinning fiver? Come to think of it, how can you explain the ship's artificial gravity? Would not the inertial field prove that the ship actually rotated, not the cosmos?

If this were the case, general relativity could never have been formulated as a consistent theory. How does Einstein's theory escape from this seeming paradox?

28
BOUNCING SUPERBALLS

Intuitively it seems impossible, but our idealized superball will bounce as high as you please if you give it enough time!

To find out how high the ball goes on its *n*th bounce, we must add the first *n* terms (this is called a "partial sum") of the following infinite series:

$$1/1 + 1/2 + 1/3 + 1/4 + 1/5 + \ldots .$$

This is known as the "harmonic series." It has many important applications in physics and technology, and a raft of curious properties. The most astonishing property is that it does not converge; that is, it has no limit sum. Although the amount by which the partial sum grows with each new term is always diminishing, nevertheless, if you add enough terms, the sum can be made as large as you wish!

Here is a ridiculously simple way to prove by *reductio ad absurdum* that the series diverges. First group the terms as follows:

$$(1/1 + 1/2) + (1/3 + 1/4) + (1/5 + 1/6) + \ldots .$$

If the harmonic series converges, then the series given above must converge to the same sum. Observe, however, that the first parenthetical term of the above series is greater than 1, the first term of the harmonic series. Likewise, the second parenthetical term is greater than 1/2, the second term of the harmonic series. The third parenthetical term is greater than 1/3, the third term of the harmonic series. And so on for all the other terms. Therefore, the limit sum of the above series must be larger than the sum of the harmonic series. But this contradicts our assumption that the sums of the two series are equal. Therefore our original assumption, that the harmonic series converges, is false.

When does the ball reach a height of at least two feet? The answer is: after the fourth bounce. This carries the ball to a height of 25/12, or 2 and 1/12 feet. It can be shown that the partial sum of any number of finite terms in the harmonic series is never an integer. The ball requires 83 bounces to exceed a height of five feet, and 12,367 bounces to exceed ten feet. To go higher than 100 feet requires 15,092,688,622,113,788,323,693,563,264,538,101,449,859,497 bounces. The time required would far exceed the age of the universe.

Some notion of how close the harmonic series comes to converging may be gained from the fact that if you eliminate from the series just those fractions that have in their denominator one or more of any specified digit, the series will converge. The following table gives the sum, to two decimal places, of the series for each omitted digit:

Omitted digit	*Sum*
1	16.18
2	19.26
3	20.57
4	21.33
5	21.83
6	22.21
7	22.49
8	22.73
9	22.92
0	23.10

Suppose we strike out of the harmonic series all terms with denominators that are not prime. We are left with the following series of increasing reciprocals of primes:

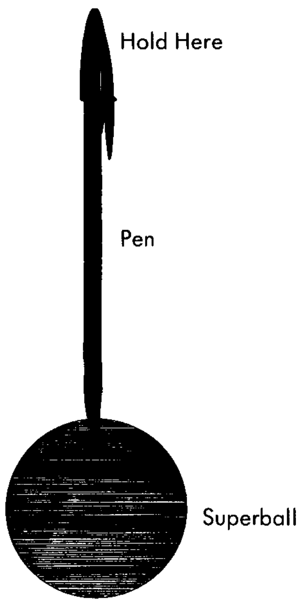
$$1/2 + 1/3 + 1/5 + 1/7 + 1/11 + \dots$$

It is hard to believe, but this series, like the harmonic series, also diverges! Of course it diverges much more slowly.

Although our harmonically bouncing superball is only a thought experiment, here is an amazing experiment you can actually perform with an ordinary hard rubber superball of the sort now on sale in toy stores.

The larger the ball the better. One with a three-inch diameter is ideal. Cut a tiny slot in the rubber so that if you push the point of a long ball-point pen into the slot, the ball will hang suspended when you hold the other end of the pen as shown in Figure 39. Drop the ball and pen on a hard floor or cement sidewalk. What happens to the pen?

FIGURE 39



29

RUN, ROBOT, RUN!

It makes not the slightest difference how the man throws the ball. Since we know the dog trotted continually and at a constant rate during the man's walk around the reservoir, he will cover the same distance no matter what paths he takes. Let x be the distance in miles around the reservoir. The man will finish the circuit in $x/5$ hours. During this time the dog will have trotted $2x$ miles regardless of how hard the man tosses the ball or in what direction.

After finishing his walk, the man encountered an attractive dark-eyed girl who was taking a stroll through the park with her own robot dog, Pasta. Walking dogs in Central Park, then as now, was an excellent way to meet strangers.

"I see our pets are made by different companies," said the man. "I wonder how accurate the manufacturers are these days in programing trotting speeds."

"Why don't we let the dogs race to find out?"

"Splendid idea. Let's set them both at fifteen miles per hour and see if they finish the race at the same time."

The man picked up a stick and drew a starting line in the snow. Then about 200 yards away he drew a finish line. When Farfel crossed the finish he was ahead of Pasta by about 10 yards.

"Sorry about that," said Farfel.

"I don't mind," said Pasta.

"It just shows you," said the man, "how careless our corporations are these days. If our pets had been made in Africa, instead of Japan, I'll bet they would have been nose to nose when they finished."

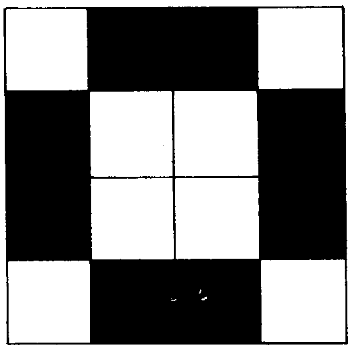
"Let's test them again," said the girl. "This time we'll let Farfel start ten yards behind Pasta. If their running times are consistent, they ought to cross the finish line at the same instant."

To the surprise of both pet owners, one of the dogs finished the race ahead of the other. Each dog ran at the same constant speed as before, and it took the owners some time to figure out why. Which dog finished first? You don't need algebra to decide, just some common sense and insight.

30

THANG, THUNG, AND METAGAME

FIGURE 40



Glance at the eight coins on the shaded cells in Figure 40. If there is an even number of heads among the eight, the game is solvable. If the number of heads is odd, you can turn coins until doomsday without ever making them all heads.

Now see if you can prove why this must always work.

31

THE NUMBER OF THE BEAST

1. Only MONDAY has a sum of 666 (112 + 114 + 113 + 103 + 100 + 124 = 666). This was discovered by one of Steuben's former students. Another former student discovered that PAYDAY also is a 666 word in the same code, which is not surprising considering that payday often comes on Monday.

2. To prove that the Beast is a fox, number the alphabet as shown below, and note the three letters that fall under 6:

1	2	3	4	5	6	7	8	9
A	B	C	D	E	F	G	H	I
J	K	L	M	N	O	P	Q	R
S	T	U	V	W	X	Y	Z	

Another solution, sent to me by Larry T. Green, is to use the code I had given for finding 666 in the name of Hitler. In this code F = 105, O = 114, and X = 123. The digits in each number add to 6.

3. The only other way to get a sum of 666 by inserting three plus or minus signs into 123456789 is:

$$1234 - 567 + 8 - 9 = 666.$$

4. The only way to make 666 by inserting four plus or minus signs into 987654321 is:

$$9 + 87 + 6 + 543 + 21 = 666.$$

5. The only way to make 777 by inserting any number of plus or minus signs into 987654321 is:

$$98 + 7 + 654 - 3 + 21 = 777.$$

After these answers appeared in my column, readers too numerous to list sent me the results of computer programs that gave all solutions for the problem of adding plus and minus signs to the sequences 123456789 and 987654321 so as to make a sum of 666 or 777. R. H. Lyddane was the first to do this, followed by Michael Buchanan and many others.

666 has eight solutions for the ascending sequence:

$$- 1 + 2 - 3 + 4 - 5 + 678 - 9$$
$$+ 1 - 2 - 3 - 4 + 5 + 678 - 9$$
$$+ 1 + 2 + 3 - 4 - 5 + 678 - 9$$
$$+ 1 - 23 - 4 + 5 + 678 + 9$$
$$+ 1 + 2 + 3 + 4 + 567 + 89$$
$$+ 1 + 23 - 45 + 678 + 9$$
$$+ 123 + 456 + 78 + 9$$
$$+ 1234 - 567 + 8 - 9$$

And five solutions for the descending:

$$- 9 + 8 + 7 + 654 + 3 + 2 + 1$$
$$+ 9 - 8 + 7 + 654 + 3 + 2 - 1$$
$$+ 9 + 8 - 7 + 654 + 3 - 2 + 1$$
$$+ 9 - 8 - 7 + 654 - 3 + 21$$
$$+ 9 + 87 + 6 + 543 + 21$$

777 has two solutions for the ascending sequence:

$$- 12 - 3 + 4 + 5 - 6 + 789$$
$$- 12 + 3 - 4 - 5 + 6 + 789$$

And two for the descending:

$$- 9 - 8 + 765 - 4 + 32 + 1$$
$$+ 98 + 7 + 654 - 3 + 21$$

Walker Percy, in his 1983 book *Lost in the Cosmos,* says that the British novelist Graham Greene sometimes is unable to write until he has watched traffic long enough to see a car go by with 777 on its license plate.

32

THE JOCK WHO WANTED TO BE FIFTY

If you try substituting numbers for the three variables in the initial equation, $a = n + d$, you'll find at once that the expression $(a - n - d)$ is equal to zero. When Ophelia canceled $(a - n - d)$ from both sides of an equation she was in effect dividing by zero, an operation verboten in arithmetic because the result has no meaning.

Ophelia's proof is a classic instance of how division by zero can produce false answers. If it were permitted, you could use the proof to show that any variable equals any other variable. Want to prove that a flea weighs the same as an elephant? Just let *n* be the flea's weight, and *a* the weight of the elephant.

Ophelia soon talked Lucky out of his plans by threatening never to see him again if he carried them out. He continued to be lucky in algebra class, not just because Ophelia liked him, but also because the university's president sent Ophelia a memo saying that under no circumstances should Lucky be failed. The football team needed him too much.

With this in mind, Ophelia often gave tests with questions based on how she thought Lucky would try to answer them. For example, one of her test questions was to reduce the following three fractions to lower terms:

$$26/65, 16/64, 49/98.$$

As she anticipated, Lucky lowered the fractions by cancelling out like digits above and below each line:

$$\frac{2\not{6}}{\not{6}5} \quad \frac{1\not{6}}{\not{6}4} \quad \frac{4\not{9}}{\not{9}8}$$

You'll notice that in each case this gives a correct answer! Aside from fractions that equal 1, such as 37/37, or fractions with terminal zeros, like 20/30, there is only one other fraction with 2-digit numerators and denominators for which this illegal cancellation works. See if you can find the fourth example.

33

FIBONACCI BAMBOO

What is called *the* Fibonacci series begins: 1, 1, 2, 3, 5, 8, 13, 21,. . . . A generalized Fibonacci sequence may start with any two positive integers whatever. The second number may be larger, smaller, or the same as the first. One of the most amazing of the many curious properties of such sequences is that the ratio between two consecutive terms approaches closer and closer to an irrational limit as the sequence continues. The limit for all Fibonacci sequences is the same. It is the famous "golden ratio" given by the expression:

$$\frac{\sqrt{5}+1}{2} = 1.61803\quad 39887\quad 49894\quad 84820\ldots$$

The golden ratio provides a simple way to solve our problem. The longest Fibonacci sequence terminating in 100 will have a next-to-last term, x, such that $100/x$ will differ from the golden ratio by less than 1. To determine x we need only solve the following equation:

$$\frac{100}{x} = 1.618 +$$

This gives x a value of 61.80+. The next-to-last term of the sequence, therefore, will be either 61 or 62. If we round x down to 61, then subtract backward through the sequence from 100, we get the terms 100, 61, 39, 22, 17, 5, 12. We were told that the second day's growth exceeded the first day's growth, so we have to discard 12. This leaves a six-term sequence; therefore it is not the sequence we are seeking.

We now try rounding x up to 62. This gives the longer chain: 100, 62, 38, 24, 14, 10, 4, 6. As before, we must drop the 6 because it exceeds the second term, 4. We now have a sequence of seven terms in a chain that ends with 100. The sequence begins with 4 and 10, the two numbers that solve our problem.

The method applies to all top numbers of a Fibonacci chain. If the last number is 1,000, you can easily determine that the longest chain terminating in 1,000 begins with 8, 2, 10, . . . , and has 13 terms. If the last number is one million, the longest chain has 20 terms and starts with 154 and 144.

Hundreds of entertaining puzzles are based on Fibonacci sequences. Suppose, for instance, a rectangle has sides a and b, and that c is the rectangle's area. If a, b, and c are consecutive terms in a generalized Fibonacci sequence, what is the area of the rectangle?

34

TETHERED PURPLE-PEBBLE EATERS

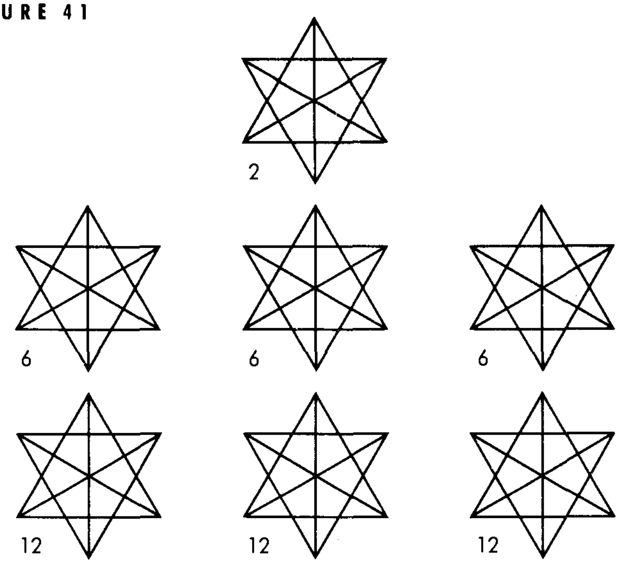
The turple can graze over the quadrant of a circle with an area of 25 square feet—one fourth of the square's area of 100 square feet. The full circle, with the chain's length as its radius, will have an area of 100 square feet. The area of a circle, as everyone should know, is pi times the square of the radius. To obtain the radius we need only divide 100 by pi, then find the square root of the result. This gives the length of the chain as very close to 5.64 feet.

The turple chomped pebbles so rapidly that it was necessary to shift it to other corners of the enclosure, and finally to tether it to the *outside* of a corner, using a longer chain. The feeding provided the crew with dozens of little purple cubes, all identical. One of the mechanics turned a set of them into a handsome beaded necklace for one of the ship's nurses.

The new tether was 20 feet long. You should have little difficulty determining the new grazing area.

35
THE DYBBUK AND THE HEXAGRAM

FIGURE 41



There are 56 triangles. Figure 41 shows the seven kinds, and the number of each. If you enjoyed working on this task, you might try a much more difficult one: counting the number of quadrilaterals in the diagram, including "crossed quadrilaterals" that have a pair of intersecting sides.

For word-play buffs, can you rearrange the letters of FURICLE to discover why I gave my cat that name?

36

1984

The digit that must be omitted is 5.

To understand why, you must know what a "digital root" is. Assume that x is any set of numbers. If you add all the digits in all the numbers, then add the digits in the sum, and keep doing this until only one digit remains, that final digit is called the digital root of the original set x.

One of the basic laws of digital roots is this. No matter how you scramble the digits of x to make a new set of numbers, the digital root of the new set will be the same as before. For example, consider the set of numbers 1, 23, and 931. The sum is 955. Adding the three digits gives 19, and $1 + 9 = 10$, and $1 + 0 = 1$, therefore the digital root of 1, 23, and 931 is 1. Now use the same digits to make a new set of numbers: 12, 13, and 39. The sum is 64. The digits add to 10, and $1 + 0 = 1$, therefore the digital root of 1, 23, and 931 has not altered.

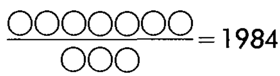
Let's apply our law to the set of ten digits. The sum is 45, and $4 + 5 = 9$, so the digital root is 9. No matter how we use these digits to make a set of numbers, the sum must have a digital root of 9. But 1984 has a digital root of 4. Therefore it is impossible to use just the ten digits to form a set of numbers that add to 1984.

Our problem asked what digit must be left out of the set of ten so that the remaining nine digits can be used to make the sum 1984. Clearly we must omit a digit such that the remaining nine will have a digital root of 4. Only by removing 5 can we do this. The remaining nine digits will solve the problem in many different ways. Here is one:

$$\begin{array}{r} 869 \\ 702 \\ \underline{413} \\ 1984 \end{array}$$

Now for a much more difficult problem. Can you place the ten digits inside the ten circles in Figure 42, a different digit in each circle, to make an improper fraction that equals 1984?

FIGURE 42



There is only one solution.

37
THE CASTRATI OF WOMENSA

To comprehend how the wives reasoned, let us first suppose there had been just one unfaithful husband. By the first posit, every wife in the city would know of the infidelity except the wronged wife. Knowing that no other husband was unfaithful, and believing the decree, the wronged wife would know that her own husband must be the guilty one and would at once report him.

Now consider what would happen had there been just two guilty husbands. Call their wives *A* and *B*. *A* would know that *B*'s husband had been unfaithful. If he were the only guilty husband, *A* would expect *B* (by the reasoning above) to know at once that her husband was guilty and to report him on the first day. However, when the first day went by without an emasculation, *A* would realize there had to be *two* unfaithful husbands. Because *A* knew of only one (*B*'s husband), *A* would reason that the second guilty man had to be her own spouse. Therefore, she would report him on the second day. Naturally, wife *B* would reason exactly the same way. She, too, would report her husband on the second day.

Consider the case of three unfaithful husbands. Each of their wives, knowing of two infidelities, would expect (by the above reasoning) two emasculations on the second day. When the second day passed with no such operations, each of the three wives would know there was a third guilty man, namely her own husband. Hence three reports would be made on the third day. And if there were four guilty men, each of their wives would reduce the situation to the previous case of three and not report her husband until the fourth day.

Mathematicians call this type of sequential reasoning "mathematical induction." Clearly it extends to n cases of infidelities. On the nth day after the decree, n emasculations would take place.

After the ten punishments had been carried out, the husbands of Womensa maintained strict control over their passions for several months. This gave them time to think carefully about the logic involved in the way their wives had reasoned. One day it occurred to them that there was a simple way to escape the consequences of such a decree.

Details of their plan spread quickly among them. Infidelities began to occur again, and Fidelia was forced to issue a second decree exactly like the former one. This time, however, nothing happened. Months and even years went by, but no infidelities were reported. Indeed, the plan was so successful that it soon became apparent that the decree was totally useless in uncovering the identities of unfaithful husbands.

What was the clever plan?

SECOND ANSWERS

1

CHESS BY RAY AND SMULL

Figure 43 shows the only way the five pieces can be placed.

Both versions of the game can be played without a computer. Two players sit back to back, each with a board and five pieces. One places the pieces, the other asks questions, and a record is kept of the number of questions needed to know where all five pieces are. Players then

FIGURE 43

	a	b	c	d	e	f	g	h
8								0
7		3	2	1			R	
6		2	B					
5		1		N				
4					1	2		
3					2	K	2	
2		Q				2	3	
1								

trade places for the next game, and the person with the lowest number of guesses wins.

I am indebted to Jaime Poniachik of Buenos Aires (he edits an Argentine magazine called *Humor & Juegos,* Spanish for *Humor and Games*) for suggesting this game and providing the first problem.

The names Ray and Smull were chosen to honor the mathematician Raymond Smullyan for his two book collections of brilliant chess problems: *The Chess Mysteries of Sherlock Holmes* and *The Chess Mysteries of the Arabian Knights.*

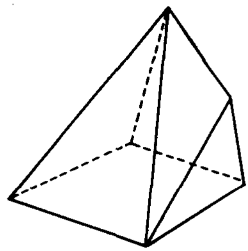
2
THE POLYBUGS OF TITAN

The seventh convex hexahedron is shown in Figure 44.

The general problem of enumerating all distinct convex polyhedra, given the number of corners, faces, or edges, is extremely difficult and remains unsolved except for very low values. There is only one polyhedron with 4 faces (the tetrahedron), two with 5 faces (a pyramid with a quadrilateral base and a tetrahedron with one corner sliced off), and thirty-four with 7 faces.

A proof that there are just seven convex hexahedra is given in *Excursions into Mathematics,* a highly recommended book by Anatole Beck, Michael Bleicher, and Donald Crowe (1969), pages 29–30. Stanley G. Winetree is a play on the name of Stanley G. Weinbaum, an American SF writer best remembered for his weird extraterrestrial life-forms.

FIGURE 44



3

CRACKER'S PARALLEL WORLD

After 1980 the first year with 36 divisors is 2016, which, by a pleasant coincidence, is just 36 years later!

Now for a much harder problem. What will be the first year that has the largest possible number of divisors for any year of no more than four digits?

5

TITAN'S LOCH METH MONSTER

The unjustified assumption is that the two parallel slices of the serpent were perpendicular to the serpent's head-to-tail axis. We have no way of knowing if this was the case. All we are told is that Winetree's cuts were parallel. For all we know they could have been at any angle to the serpent's axis, the angle varying from zero to ninety degrees.

If the angle was zero, the cuts would be parallel to the monster's sides, and each piece would then be as long as the serpent itself. Thus the correct answer to the problem is that the serpent's length could be any value from 20 to 60 meters inclusive. (I want to thank Max Muller, of Cleveland, for sending this amusing switch on an old brainteaser.)

What about the weight of Titan's Loch Meth monster? Let's take weight to mean its weight on earth as expressed in pounds.

An ancient arithmetic riddle says that if a brick weighs a pound plus half its own weight, how much does the brick weigh? Many people carelessly answer 1½ pounds without realizing that this is logically contradictory. First they assume the brick weighs a pound, then they conclude that it weighs a pound and a half. Clearly the only consistent interpretation of the question's wording is that the brick's weight is the sum of one pound and half its own weight. One pound obviously is the "other half." Therefore the brick weighs two pounds.

See how quickly you can answer the following four questions about the serpent. How much does it weigh if its weight is:

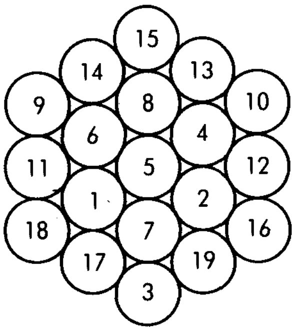
1. 1,000 pounds plus half its own weight?
2. 1,000 pounds minus half its own weight?
3. 1,000 pounds times half its own weight?
4. 1,000 pounds divided by half its own weight?

If you remember your elementary algebra, you should have no difficulty with any of the appropriate simple equations.

6

THE BALLS OF ALEPH-NULL INN

FIGURE 45



The amazing pattern shown in Figure 45 was first discovered in 1895 by one William Radcliffe, a teacher at the Andreas School on the Isle of Man. He patented his "38 Puzzle," as he called it, in 1896 in England and the United States, but was not successful in marketing it. Sixty years later it was rediscovered by Tom Vickers, who published it in *The Mathematical Gazette,* Volume 42, December 1958, page 291. Not until 1963 did Charles W. Trigg, of San Diego, California, prove the solution to be unique, and that no other hexagon, of any size, can be magic.

The pattern has no earthly use, but surely it is a thing of strange and subtle beauty.

7

SCRAMBLED HEADS ON LANGWIDERE

The limit is $1-(1/e)$, where e is the irrational base of natural logarithms. The value of e is 2.718281828. . ., which gives the fraction a value of .63212055866 . . ., or slightly less than 2/3. This can be modeled with playing cards as follows.

One person shuffles a packet of *n* cards while someone else shuffles a duplicate packet consisting of the same *n* cards. They deal their cards one at a time in synchronization. What is the probability that at least one pair of simultaneously dealt cards will be identical? If the packet contains three or more cards, the probability is close to 2/3.

The probability for all packets of five or more cards is .63+. Thus if a self-styled psychic calls out any sequence of the 52 playing cards while a deck is being dealt in another room, at least one call of a card will be correct in about two out of every three repetitions of the test.

Now see if you can answer this simple question. After Dot, Trot, and Zot got their randomized checks from the clerk, what is the probability that exactly *two* of the ladies got her correct head check?

8

ANTIMAGIC AT THE NUMBER WALL

The only pattern is:

```
2 4 5
 3 6
  1
```

Before the day ended, Yin and Yang hit on an even more challenging project. They asked themselves if they could form triangles with consecutive numbers, starting with 1, such that each number below the top row represented the absolute difference between the pair of numbers directly above it. The two patterns for order 2 are trivial:

```
3 2     3 1
 1       2
```

Finding the four solutions for order 3 was not so easy:

```
6 2 5    2 6 5    6 1 4    1 6 4
 4 3      4 1      5 3      5 2
  1        3        2        3
```

And finding the four solutions for order 4 took several hours:

```
6 1 10 8        6 10 1 8
 5 9 2           4 9 7
  4 7             5 2
   3               3

8 3 10 9        8 10 3 9
 5 7 1           2 7 6
  2 6             5 1
   4               4
```

Yin and Yang returned to the Natural Number Wall the following day to tackle the fifteen balls of the order-5 difference triangle—the triangle that is the starting formation for the fifteen balls in a game of pool. Eventually they found the only possible solution. It is difficult to obtain without computer help, and even harder to show it is unique.

9

PARALLEL PASTS

Neither Cracker nor Ada thought to check on which calendars England and Spain were using in 1616. England was then still on the old Julian calendar, but Spain had adopted the reformed Gregorian calendar, which was ten days ahead of the Julian. The two famous writers actually died ten days apart!

Robert Service wrote another poem about the Bard of Avon in which he disclosed a reason, based on the title of one of Shakespeare's plays, for believing that the plays were really written by Francis Bacon. Can you guess what play it is? I will answer by quoting the poem's final stanza.

10

LUKE WARM AT FORTY BELOW

The constriction is so small that mercury cannot get through unless pushed by a strong force, such as the force created by pressure when it expands. When the mercury cools, it breaks at the constriction into two parts. Only inertia, produced by vigorous shaking, will force the upper part down again.

When the thermometer is put in hot water (not *too* hot, or it might break!) the glass expands before the mercury. This enlarges the channel enough to allow the mercury in the upper part to go down a trifle.

Now for those who enjoy word play, what temperature (on either scale) is represented by the following rebus?

$$\frac{\text{BABS}}{0}$$

11

THE GONGS OF GANYMEDE

The Pythagorologist will make 240 clockwise circuits before he goes part of another circuit to stop at the last bronze eagle and hit a gong 100 times. If a woman made the same walk, traveling counterclockwise, I calculate she would hit the gong 100 times, at the gold eagle, after completing 260 full circuits.

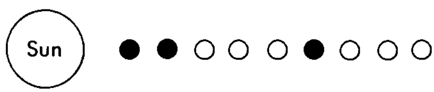
These calculations are simplified by the fact that each nth phase of the walk requires just n circuits except when n is a multiple of 5, in which case there are $n/5$ circuits.

Since my puzzle tale appeared in print, James wrote to tell me that he had verified Singer's conjecture for n through 33, and that the impossibility of a circle not of the form $n = p^k + 1$ is reported (by P. Dembowski, in *Finite Geometrics,* 1968) to have been verified for all n less than 1600. James also learned that the nonexistence of a circle for $n = 7$ had been proved by T. P. Kirkman (of the famous Kirkman schoolgirl problem) in a paper of 1857.

12

TANYA HITS AND MISSES

FIGURE 46



There are nine planets, three barren, three with life, and three about which no information has been given (see Figure 46). As before, the shaded circles are the barren planets. (I am indebted to Michael Steuben for the ideas on which both problems are based.)

Speaking of planets, can you solve the elegant addition cryptarithm shown below? Each letter represents a unique digit, each digit corresponds to only one letter, and (as customary in such puzzles) no number starts with zero.

```
    M A R S
  V E N U S
S A T U R N
U R A N U S
-------------
N E P T U N E
```

13

MYSTERY TILES AT MURRAY HILL

FIGURE 47

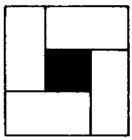


Figure 47 shows the only way to tile a rectangle (in this case a square) so that the average size of a tile is less than 1.83333. . . . The average clearly is $9/5 = 1.8$. Aside from this one case, all rectangular patterns have an average tile-size greater than 1.83333. . . .

Little is known about the problem's generalization to three dimensions, using "tiles" that are "bricks" (rectangular parallelepipeds) with integral edges. As before, we rule out "reducible" patterns containing sub-bricks formed by two or more bricks; otherwise we could tile space with unit cubes.

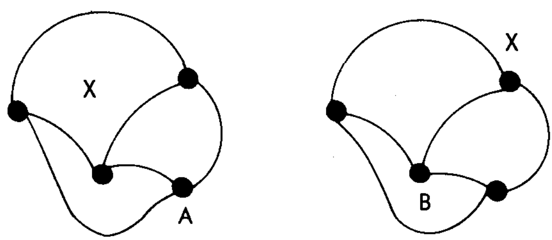
Three $1 \times 1 \times 2$ bricks, and two unit cubes, will form a $2 \times 2 \times 2$ cube with an average brick volume of $8/5 = 1.6$. Aside from this possible exception, a lower bound for the tiling of three-space with integral bricks is not known. If the tiles of Graham's wall are given a unit thickness, the result can be used for tiling large rectangular parallelepipeds with an average volume for the tiles that is arbitrarily close to 11/6. Whether this can be improved, and if so, by how much, remains a challenging unsolved problem.

For more on the tiling that provided the basis for this puzzle tale, see "Tiling Rectangles with Rectangles," by F.R.K. Chung, E. N. Gilbert, R. L. Graham, J. B. Shearer, and J. H. van Lint, in *Mathematics Magazine,* Volume 55, November 1982, pages 286–91. Names in the story are plays on the names of Claude Berge, a French graph theorist, and Doris Schattschneider, an American mathematician.

14

CROSSING NUMBERS ON PHOEBE

FIGURE 48



Any complete graph for four points must divide the plane into three mutually bordering regions as shown in Figure 48. A fifth point, indicated by X, must go either inside a region, as shown on the left, or outside all three as shown on the right. If it is inside, there clearly is no way it can be joined to point A without crossing a line. If it is outside, it cannot be joined to point B without crossing a line.

To prove that no map with five mutually "touching" regions can be drawn, assume that such a map is possible.

We could then put a point within each region, and connect every pair of points without any crossings of lines. We simply draw each line so it passes over the border shared by the two regions that contain the line's two ends. This would complete a graph of five points with a crossing number of zero. Because we proved this impossible, our original map assumption must be false.

Unfortunately, this does *not* prove the famous four-color map theorem, since it is conceivable that a map of many regions could require five colors even though the map has no spot where five regions mutually touch. The two theorems are often confused. I confused them myself many years ago when I wrote a science-fiction story called "The Island of Five Colors." (It is reprinted in Clifton Fadiman's anthology *Fantasia Mathematica.)* Readers frequently send me what they believe to be simple proofs of the four-color theorem, but which actually are no more than the old graph-theory argument outlined above.

The literature on crossing numbers is growing rapidly. For an introduction to the topic, consult my *Scientific American* column for June 1973. The most complete reference is *Crossing Numbers of Graphs,* the 1973 Ph.D. thesis of Roger B. Eggleton, at the University of Calgary, Alberta, Canada, where he worked under Richard K. Guy, a world expert on the topic.

In 1960 Guy conjectured that the formula for the crossing number of a complete graph on the plane for n points is:

$$\frac{1}{4}\left[\frac{n}{2}\right]\left[\frac{n-1}{2}\right]\left[\frac{n-2}{2}\right]\left[\frac{n-3}{2}\right]$$

where the brackets indicate that the number inside is rounded down to the nearest integer. On the torus there are no good conjectures for a formula, although upper and lower bounds have been established.

Arkay Guy is, of course, R. K. Guy, and Frank Hoorayri is the noted graph-theory expert Frank Harary. "Phoebe Snow" is a gambler's slang expression for the five-side of a die.

15

SF'S AND F'S ON FIFTY-FIFTH STREET

There are fourteen **F**s on the sign. If you don't believe it, go back and count more carefully. It's an entertaining test to try on friends. You'll be amazed by how few people find all the **F**s on the first count.

16

HUMPTY FALLS AGAIN

Humpty's use of quotation marks clearly indicates that he is asking for the longest word in the phrase "the English language." The word, of course, is *language*.

The egg was so surprised when I finally guessed his riddle that he fell off the wall. The sound of the crash woke me with a start.

I want to thank Stephen Barr for Humpty's riddle. Incidentally, if we ignore hyphenated words, made-up words, chemical-compound names, medical terms, surnames, place names and other artificially constructed words, the longest common words in English are such 21-letter words as *disproportionableness* and *indistinguishableness*. The longest dictionary word (aside from the exceptions noted above) is still *antidisestablishmentarianism*, which can be lengthened by replacing *-ism* with *-istically*.

The two best known long invented words are Shakespeare's *honorificabilitudinitatibus* (spoken by Costard the

Clown in *Love's Labour's Lost*) and Julie Andrews' *supercalifragilisticexpialidocious.* Note that the vowels in Shakespeare's word alternate with consonants throughout. In his classic work on word play, *Language on Vacation,* Dmitri Borgmann reveals that the letters can be rearranged to spell "Bath is idiotic in bountiful air," and also the Latin sentence, *Hi ludi F. Baconis nati tuiti orbi,* which means "These plays, F. Bacon's offspring, are preserved for the world."

It is often asserted that the longest word in the Oxford English Dictionary is *floccinaucinihilipilification,* meaning the action of estimating as worthless, but the word is usually spelled with four hyphens.

And we shouldn't forget that longest word of all: "And now, a word from our sponsors."

17

PALINDROMES AND PRIMES

To prove that a repunit prime must have a prime number of digits, assume that the number of digits is not prime. Call this composite number *x.* It will have at least two divisors, *a* and *b,* that are neither 1 nor *x.* The numbers represented by *a* and *b* repunits will obviously divide the original repunit number, proving it to be composite. For example: 111111111111111 is divisible by 111 and 11111. In searching for repunit primes, therefore, you need test only those with a prime number of digits.

151 and 1111111511111111 are palindromic primes. Williams proved that the next largest prime of this form consists of 45 ones on each side of 5.

11411, 1114111, and the number formed by 32 units on each side of 4 are palindromic primes. Williams found that the next largest prime of this form consists of 45 ones on each side of 4.

The two mammoth primes, each with 91 digits, are called "twin palindromic primes" because they are identical except for their middle digits which differ by one. This is the largest known pair of twin palindromic primes.

Williams also found an astonishing "almost palindromic prime." It consists of 8 with 252 nines on the left side and 253 nines on the right!

Gigo, as computer hackers may recognize, is a familiar acronym for "Garbage in, garbage out."

18

THIRTY DAYS HATH SEPTEMBER

Myrtle's father was 52 in 2032. He was born in 1980, which would have made him 45 in the year $45^2 = 2025$. The British mathematician Augustus de Morgan, born in 1806, liked to tell people he was x years old (43) in the year x^2(1849). The same claim could be made by anyone born in 1892 (*e.g.,* Oliver Hardy, Basil Rathbone, Margaret Rutherford). During the twenty-first century, only those born in 2070 will be able to boast of being x years old (46) in the year x^2(2116).

Here are eight curious questions involving months:

1. If each date of the month is spelled out, such as "February sixth," and the dates alphabetized, what are the first and last dates on the list?

2. In what sense does DEC 25 (Christmas) equal OCT 31 (Halloween)?

3. OCTOBER has no letter in common with SUNDAY. Find another pair of words, consisting of a month and a week day, with the same property.

4. Jeremiah Jason, a radio disc-jockey, has a business

card that reads: J. JASON, D.J. FM-AM. How does that relate to the calendar?

5. It is easy to find two anagrams for MAY. There are AMY and YAM. Find an anagram for MARCH.

6. How are March and October related by way of NBS, the acronym for the National Bureau of Standards?

7. How are HIP and TUB related by way of November?

8. What unusual property does FORT have that no other month name has?

19

HOME SWEET HOME

The alien is describing a baseball player sliding safely home to win an important game.

Speaking of baseball, here are three simple questions that even baseball fans often answer incorrectly:

1. Suppose the home team, in a game that does not go beyond the standard number of innings, made two runs each time it came to bat, and the visiting team made one run each time at bat. What's the final score?

2. How many outs are in one inning?

3. Mudville won the baseball game with a score of 12 to 1, yet not a single man on the Mudville team touched third base. Explain!

20

FINGERS AND COLORS ON CHROMO

Did you answer 15 without thinking? If 15 are moved from *A* to *B*, that would make 30 more in *B* than *A*. Starting with the same number of people in two rooms, whatever the number, if you want to transfer persons from one room to another to make the difference between rooms equal to *x*, you must transfer half of *x*. There is no way to divide 15 persons in half without cutting one person in half. Therefore the task Coralie had set for herself is impossible.

Now for a classic problem that combines elementary number theory with logical reasoning. I do not know who first thought of it, but it comes to me by way of Michael Steuben.

Assume that more than one Chromo is in a room. Every Chromo has two hands with at least one finger on each hand, and all Chromos have the same number of fingers. The total number of fingers in the room is more than 200 and less than 300. If you knew the exact number of fingers in the room, you could deduce exactly how many Chromos are in the room. How many are there, and how many fingers does each have?

Impossible as it may seem, you now have enough information to determine both the total number of persons in the room, and the number of fingers on each!

"For ten minutes," wrote Steuben, when he gave a version of this problem in a recent Mensa newsletter, "I was convinced the solution was based on a pun or some other type of verbal quibble. . . . Then inspiration struck."

21

VALLEY OF THE APES

There were just two gorillas in the cave. The hunter killed one and left one. Thus he did not kill some gorillas (plural) nor did he leave some gorillas (plural) alive.

We all know the popular song about Lulu ("Lulu's back in town"). Can you think of another popular song—it goes back half a century or more but is still sung occasionally—that puns on the word gorilla in its title and first line?

22

DR. MOREAU'S MOMEATERS

Montgomery used the trick known as casting out nines. If you add all the digits of a number, then add the digits of the sum, and continue this procedure until only one digit remains, it is called the number's digital root. If and only if the digital root is 9, the number is a multiple of 9. If and only if the digital root is 1, the number has a remainder of 1 when divided by nine.

Montgomery realized at once that 5,000 did not have a ditigal root of 1, and therefore could not count the population in the tank at any stage of the breeding process. It was a simple task to determine the two numbers with a digital root of 1 that are the nearest to 5,000. They are 4,996 and 5,005, of which 4,996 is the closer.

When the tank held 4,996 fish, Dr. Moreau asked Montgomery to remove two-thirds of the males. He needed them, he said, for some new experiments.

Male momeaters are easy to recognize. Each has 15 fins, whereas the female has only 5. Montgomery first counted the number of male fish in the tank, and was pleased to learn that the number was a multiple of 3. He removed two thirds of the males, as he had been instructed.

Now we have a very lovely problem. How many fins are there on all the fish that remain in the tank?

It seems impossible to determine this number because we are not told the number of males. In fact, we do not even know if male and female fish are born in about equal numbers. Nevertheless, you have enough information to solve the problem.

23

AND HE BUILT ANOTHER CROOKED HOUSE

1. Three cuts will do the trick. Slice off the base to make a $2 \times 1 \times 1$ block, and cut off two of the arm cubes. You will then have four polycubes that go together in an obvious way to make a $2 \times 2 \times 2$ cube.

It is easy to see that this is the only way to do it. The model's height of 4 units must be cut in half to eliminate a height greater than 2, and each of the other two coordinate lengths must have a cube chopped off (from either end) to reduce the length to 2.

2. Six cuts are necessary and sufficient. To prove that six cuts are necessary, just consider the cube at the interior of the cross. To free this cube, each of its six faces must be cut.

3. Three colors.

4. Four toothpicks.

You should have no trouble finding ways to accomplish the last three tasks.

I have saved our best problem for the last. Imagine that you have eight unit cubes. Each is painted with the same three colors, with pairs of opposite faces the same color. All eight cubes are identical. With these cubes, can you build a model of the house so that no pair of like-colored faces touch along an edge?

There is a quick way to answer this without having to make a set of the colored cubes.

24

PIGGY'S GLASSES AND THE MOON

A moon that rises, just after the sun sets, is on the opposite side of the earth from the sun. It must, therefore, be a full moon, not the thin crescent that Golding describes in the first sentence of the quoted paragraph.

Barry Singer, a psychologist at California State University at Long Beach, likes to give his students a test to bring out how poor a guide intuition is in answering even the simplest scientific questions. One of his test questions is: "From what direction or directions does the moon appear to rise in the sky?" He discovered that very few students answered correctly.

Singer speaks of this in a paper, "To Believe or Not Believe," that can be found in a book I highly recommend, *Science and the Paranormal*. It is an anthology edited by Singer and the astronomer George Abell. Only a small number of people, writes Singer, are aware of the fact that the moon, like the sun and the stars, always rises in the east. "Many people, probably because they occasionally discern a faint image of the moon during the daytime, believe that the moon does not rise and set like the sun, but always floats around the sky until it is dark enough to see it."

There are many good questions that you can spring on people to determine whether their education has included basic science. Asking where the moon rises is almost as good as asking what causes the moon to shine. You wouldn't believe some of the answers that are given by space opera fans as well as by professors of literature!

25

MONORAILS ON MARS

FIGURE 49

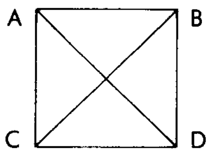
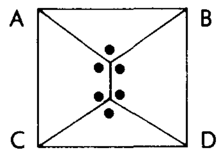


FIGURE 50



Most people suppose that the monorail network of minimum length, joining the four corners of a square, is the one shown in Figure 49. The total length is twice the square root of 2,000, or about 282.84 miles.

But this is not the best possible. The correct pattern is shown in Figure 50, where each angle with a dot is 120 degrees. It is a bit tricky to calculate the total length of the network, but it turns out to be the square root of 3,000 plus 100, or 273.20 + miles. This is slightly less than 282.84 miles.

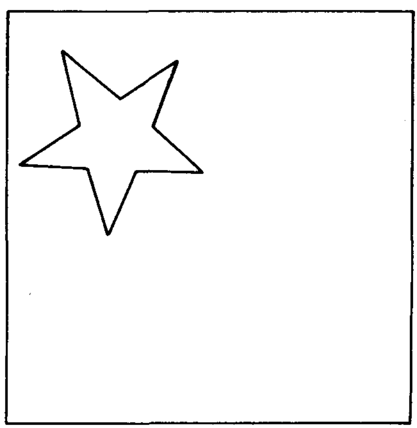
Minimum-length networks joining points on the plane are known as "minimum Steiner trees" after a nineteenth-century German geometer named Jacob Steiner, who studied the problem. The general task of finding such trees for *n* points on the plane is unsolved in the sense that no efficient algorithm is known for obtaining them. (The task belongs to a famous class of unsolved problems known as NP-complete. They are so related that if an efficient algorithm is found for any one of them it can be applied to all the others.) The task is even more difficult for joining points in three dimensions. It is not easy, for example, to determine the shortest-length network joining the eight corners of a cube.

Our final puzzle was sent to me in 1981 by Yasuo Hakinuma, an 18-year-old student in Tokyo. Assume that the four Russian bases, *ABCD* are on the corners of a perfect square in the following sense. If you travel due south 100 miles from A , you arrive at *C*. Going 100 miles due east from *C* puts you at *D*. Another 100 miles due north takes you to *B*, then 100 miles due west brings you back to *A*.

Where on Mars can the four bases be located? The answer is more surprising and more complicated than you may at first suppose.

26
THE DEMON AND THE PENTAGRAM

FIGURE 51



If pentagrams are drawn on non-Euclidean planes, such as the plane modeled by the surface of a sphere, the sum of their angles can vary widely from 180 degrees. A number of readers pointed this out, including Mark S. Fineman, of Santa Clara, California, who wrote as follows:

Based on the information included in your article, I drew a pentagram in the normal way on an empty child's balloon. Then, taking advantage of well-known facts about non-Euclidean geometry, I blew up the balloon. At this point the demon in question appeared. I guess he wasn't paying much attention to who had summoned him, since he had the first edition of *Faustus* in his hand. I took the first edition from him and made some deals of my own. (Perhaps I'll write about them sometime in the future—or past, but that's another story.)

Anyway, as thanks for your help in this matter, I am forwarding the first edition of *Faustus* to you under separate cover since I don't understand German.

27

FLARP FLIPS A FIVER

According to general relativity, the force field inside a rotating spaceship can be regarded as either inertial or gravitational depending on what you choose as your frame of reference. If you make the universe the fixed frame, the field is called inertial. If you make the ship the fixed frame, the cosmos that rotates around it generates a gravity field inside the ship that is indistinguishable from the inertial field of the other interpretation. In this sense, gravity inside the *Bagel* is genuine gravity, not simulated, even though the structure of its field is not the same as that of a gravitational field surrounding a planet or a star.

Strictly speaking, one should not say that the field is caused either by a rotating ship or by a rotating cosmos.

The fundamental reality is the *relative* rotation of ship and cosmos. This relative motion creates the force field that simulates earth's gravity and topples spinning tops. The question of which *really* rotates, ship or universe, is as meaningless as asking whether you are really sitting on top of a chair while you read these lines or whether the chair is supporting you from below.

Einstein used the phrase "principle of equivalence" to stand for the fundamental identity of inertia and gravity. In general relativity they are two names for essentially the same force. The name used depends on the choice of a reference frame. In the case of a rotating spaceship it is much simpler to assume that the universe is fixed and that the ship's spin produces an inertial field. But if the ship is taken as fixed, the rotating universe can be thought of as causing a gravity field that tugs on the spaceship from all directions.

In either interpretation the tensor equations are the same and the field has exactly the same structure. It is not yet known whether the field results from rotation relative to the structure of spacetime itself, or relative to the masses of the stars. The second hypothesis is known as Mach's principle. Einstein favored it for a time, then later abandoned it, and experts today are divided on the matter.

Presumably gravity (or inertia) is transmitted by waves that are quantized. The carrying particle has been called a graviton, but neither gravity waves nor particles have yet been detected by replicable experiments.

After I wrote this column it occured to me that not only would gyroscopic motions play havoc with spinning objects on a rotating spaceship, but coriolis forces would also influence the trajectories of tossed and dropped objects. This probably would have little effect on games such as tennis, when the ball's motion is mostly horizontal, but if a large space station in the form of a rotating torus were large enough for baseball, fielders might have to learn new reflexes in catching high flies.

28

BOUNCING SUPERBALLS

The pen will blast off, going ten feet or more into the air. The height depends on the size of the superball. If you are in a room, the pen will strike the ceiling with considerable force.

By trying sticks of various lengths and weight, you can find a stick of such weight that when you drop the ball and stick, the ball will not bounce at all. It lands with a thud and stays put, sending the stick skyward. The height the stick goes can be greatly increased by gluing two or more superballs together so they hang like a chain.

Caution: Make sure that anyone watching these tests stands at a safe distance. If the ball doesn't hit the ground with the pen vertical, it can propel the pen to one side so rapidly that it might injure someone's eye.

To learn more about this phenomenon—it concerns the conservation of energy-momentum—look up the following paper by its discoverer, physicist William G. Herter: "Velocity Amplification in Collision Experiments Involving Superballs," *American Journal of Physics,* Volume 39, June 1971, pages 656–63. For a similar experiment, put a tennis ball on top of a basketball and drop them.

29

RUN, ROBOT, RUN!

It is easy to see that Farfel would again win the race, and you don't need to know the distance or the dogs' running speeds to prove it. Because Farfel began the race as far behind the finish line as he was ahead when the first race ended, the two dogs will be side by side when Farfel has gone a distance equal to the distance between start and finish lines. In this case, the dogs will be together when they are 10 yards from the finish. Since Farfel is the faster, Farfel will outrun Pasta for the remaining 10 yards.

Now for a confusing third problem. It may startle some readers by revealing how poorly they comprehend the concept of average speed.

Suppose Farfel trots once around the reservoir at a constant speed of 10 mph. Then his speed is raised to 15 mph and he trots around a second time. What is Farfel's average speed for the two circuits? Don't be too hasty in answering.

30

THANG, THUNG, AND METAGAME

Each time you turn a row or column, or a diagonal of three or two coins, you must reverse exactly two coins in the shaded set of eight. And if you turn a main diagonal of four coins, or a corner coin, you will reverse *no* coins among the eight.

At the start, if there are an even number of heads among the eight, there will also be an even number of tails. (Zero counts as an even number.) To make all eight coins heads you must, therefore, reverse an even number of tails in this set. Because your moves allow you to turn over pairs of coins in the set, it is a simple matter to make all eight coins heads. A good strategy is to first make the four central coins heads. Then turn diagonal rows of two, or outside borders of four, to make all the shaded coins heads. Finally, if need be, turn corner coins.

Consider now a starting pattern in which the number of heads (or tails) in the shaded set is each an odd number. Because every move you make must reverse either two coins in this set, or none, there is no way to make all eight coins the same. If two coins that you reverse are alike, you will lose or gain two tails, and the number of heads will remain odd. If the two coins are heads and tails, each changes its face and the situation remains the same as before.

Mathematicians use the word "parity" to describe structures that can be identified with odd or even numbers. The set of eight coins has even parity if it has an even number of heads, and odd parity if it has an odd number of heads. The rules of the game are such as to "conserve the parity" of this set. If the initial parity is odd, there is no way to change it to even.

I found this game in a Russian magazine of popular science. Metagame is explained by Raymond Smullyan in his book *5000 B.C. and Other Philosophical Fantasies*, a book of dazzling speculations that should interest any science-fiction fan. The game was invented by William S. Zwicker, a mathematician at M.I.T.

32
THE JOCK WHO WANTED TO BE FIFTY

The fourth fraction is 19/95. It becomes 1/5 after the two nines are canceled. For those who are interested, a tougher problem is to prove that there are no other fractions (with two digits above and two below the line) with this curious property.

"If you can take 7 from 100 as many times as you can," Ophelia asked her class one day, "what do you get?"

Lucky raised his hand. "I get 93 every time," he said.

33

FIBONACCI BAMBOO

We know that the rectangle's area is both the product of *a* and *b* and the sum of *a* and *b*. A little algebra shows that *a* must equal *b* divided by $b - 1$. It is easy to see that the only solution in positive integers is $a = 2$, $b = 2$; therefore the rectangle is a 2-by-2 square with an area of 4.

Now see how long it takes you to answer an even simpler question. If the three sides of a triangle, *a, b, c,* are three consecutive integers in a generalized Fibonacci sequence, what expression gives the triangle's area?

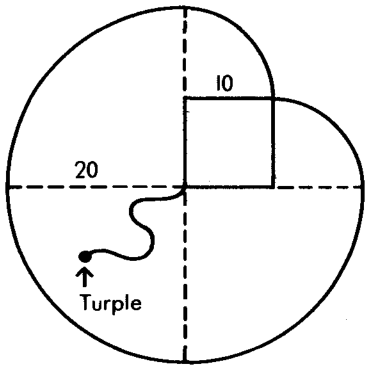
34

TETHERED PURPLE-PEBBLE EATERS

As Figure 52 makes clear, the grazing area consists of three quadrants of a circle with radius 20, and two quadrants of a circle with radius 10. Each large quadrant has an area of 314.159+, so the three together make 942.47+ square feet. The small quadrant, of radius 10, has an area of 78.539+, so the two make about 157.08 square feet. Adding the areas of all five quadrants gives a total grazing area close to 1,099.55 square feet.

Watch for a curve on our next problem. Suppose that each of the turple's excreted cubes has a volume exactly equal to its surface area. How long is the cube's side?

FIGURE 52



35

THE DYBBUK AND THE HEXAGRAM

FURICLE is an anagram of LUCIFER.

The problem of placing numbers 1 through 19 on the hexagram to make the star magic was first posed by Harold B. Reiter in his article "A Magic Pentagram" in *Mathematics Teacher* (March 1983, pages 174–77). Reiter gave it as an unsolved problem, and I had the pleasure of being the first to find two solutions.

The second solution is an inverse of the one shown in Figure 19. It is obtained by subtracting each number from 20. This puts 7 in the center, with the six largest numbers on the outside points. Numbers on opposite corners of the star total 33, opposite numbers on the hexagon total 14, and the magic constant is 54. Note that the two constants, 46 and 54, add to 100. It is not yet known if there are other solutions.

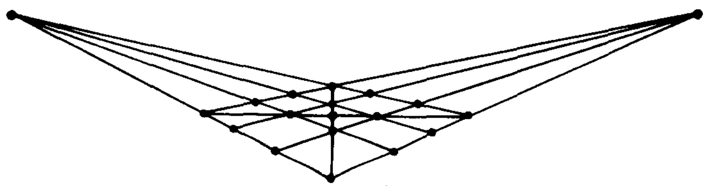
For a discussion of tree-plant problems, see my Mathematical Games column in *Scientific American,* August 1976. An earlier column on magic stars is reprinted in my book

Mathematical Carnival. If you are curious about the use of pentagrams and hexagrams in medieval black art, two good references are A. E. Waite, *The Book of Ceremonial Magic* (1911), and Francis Barrett, *Magus* (1801).

Surprisingly, nine is not the maximum number of rows obtainable by planting nineteen trees so that each row contains five. Figure 53 shows how ten (the maximum) can be achieved.

FIGURE 53

Nineteen trees in ten rows of five eoch



36
1984

$$\frac{1857024}{936} = 1984$$

I am indebted to Stewart Metchette for this problem. It is one of many similar problems in his article "Years Expressed as Distinct-digit Fractions" in the *Journal of Recreational Mathematics,* Volume 13, Number 1, 1980–81, pages 26–28.

One final problem. Can you explain why the magic square that forces 1984 must always work?

37

THE CASTRATI OF WOMENSA

The great plan was this: The husbands increased the number of their extramarital adventures at a steady rate. Thus each day after the decree was issued, the wives would learn of more infidelities than they knew about on the previous day. It is easy to see that this completely destroys the logic that the wives had used before, which had assumed that the number of unfaithful husbands remained constant after the decree.

The increasing infidelities would tend to enlarge the population of Womensa, which already had been growing rapidly. When you consider also the constant shifting of families in and out of the capital city, it is apparent that a steady rate of growth in the number of infidelities would postpone any action on the situation until the unfaithful husbands had died of old age without being punished.

The original problem (with a story line about forty unfaithful wives in an Arabian city ruled by a Sultan) first appeared in 1958 in an amusing little book called *Puzzle-Math*, by the physicist George Gamow in collaboration with Marvin Stern. The second problem is given here for the first time. It was sent to me many years ago by Thomas H. O'Beirne, a Glasgow mathematician, who credited it to his assistant Duncan P. Goudie.

THIRD ANSWERS

3
CRACKER'S PARALLEL WORLD

No number of four or fewer digits can have more than 64 divisors. The first year with that many divisors (including 1 and the number itself) is 7560. It will happen only once again before the year 10,000, in 9240.

Plato, in Book 5 of *The Laws,* recommends that an ideal city have just 5,040 citizens, each owning a plot of land, because 5,040 has as many as 60 divisors, making it easy to divide the citizens and their property into various equal sets. Plato probably did not know that 7,560 and 9,240 would be better by four divisors.

In case you wondered why Professor Cracker and Ada Loveface, when they arrived in the parallel world, did not find a duplicate transport machine, or encounter replicas of themselves, the answer is obvious. Their replicas had just entered their own machine and transported themselves to the *next* parallel world.

One of my good friends from Tulsa, Robert W. Murray (he is no longer living), enjoyed inventing practical jokes that never harmed anybody. One day in the nineteen thirties, when he was a student at Columbia University, he passed Rodin's statue late one night after having had a few drinks at a nearby bar on Broadway. It occurred to him that he might be able to climb the statue and sit on the head. He found the climb easy.

Murray assumed a position identical with that of the thinker. Occasionally a student would stroll by, then suddenly notice the young man perched on the statue's head, himself deep in thought. Murray announced that he was an oracle capable of answering any question. A crowd accumulated, and for some thirty minutes Murray gave wise, witty replies to all questions. Then he circumspectly climbed down and went home.

I once wrote a short story based on this incident, but I was unable to sell it. Editors complained that it was too contrived and improbable.

5

TITAN'S LOCH METH MONSTER

1. 2,000 pounds.
2. 666 and 2/3 pounds.
3. Zero.
4. The square root of 2,000, or 44.721 + pounds.

7

SCRAMBLED HEADS ON LANGWIDERE

The answer is zero. Did you waste time going over all six permutations of three objects looking for those with just two symbols in the right place? If two ladies got their own checks, the third check has to belong the third lady. In other words, it is not possible for just two to be correct.

No Oz buff need be told that Langwidere honors Princess Langwidere of Ev, who can choose to wear any of an assortment of thirty heads that she owns.

8

ANTIMAGIC AT THE NUMBER WALL

Here is the only way fifteen balls, numbered 1 through 15, will form a difference triangle:

```
6 14 15 3 13
 8  1 12 10
   7 11 2
    4  9
     5
```

The problem was invented by Colonel George Sicherman and first published in my *Scientific American* column of April 1977. Since then several mathematicians have proved that the solution is unique and that no higher-order difference triangles are possible. The only published proof of these facts known to me is in "Exact Difference Triangles," by G. J. Chang, M. C. Hu, K. W. Lih, and T. C. Shieh in the *Bulletin of the Institute of Mathematics Academia Sinica,* Volume 5, June 1977, pages 191–97.

Now if I tell you that Colonel Sicherman lives in a well-known American city that has the name of an equally well-known mammal, can you guess the city's name? It's an amusing question to spring on friends. Some guess the animal at once, and others never think of it.

9

PARALLEL PASTS

Said Jock McBrown to Tam McSmith,
"Come on, ye'll pay a braw wee dramlet;
Bacon's my bet—the proof herewith . . .
He called his greatest hero—***Ham****let.*"

10

LUKE WARM AT FORTY BELOW

Two degrees (bachelor of arts and bachelor of science) above zero.

Parts 1 and 3 are based on questions in Michael Steuben's excellent puzzle feature in *Capital M,* a newsletter for the Mensa organization in the Washington, D.C., area. Part 2 is from Jearl Walker's delightful book of recreational science, *The Flying Circus of Physics* (1975).

12

TANYA HITS AND MISSES

$$\begin{array}{r} 4593 \\ 20163 \\ 358691 \\ 695163 \\ \hline 1078610 \end{array}$$

This cryptarithm, devised by Willy Engren of Valby, Denmark, was first published in the *Journal of Recreational Mathematics,* Volume 13, 1980/81, page 293.

18

THIRTY DAYS HATH SEPTEMBER

1. April eighteenth heads the list. September twenty-third closes it.

2. In octal (base 8) notation, 31 is equal to 25 in decimal notation.

3. JUNE, FRIDAY.

4. The letters on the business card are the initials of the months, taken in order and starting with June.

5. CHARM.

6. Shift each letter of NBS back one step in the alphabet to get MAR, and forward one step to get OCT.

7. Shift NOV back six steps in the alphabet to get HIP, forward six steps to get TUB.

8. The letters of FORT are in alphabetical order. The longest English word known to me with letters in alphabetical order is AEGILOPS. Among seven-letter words of this sort, BILLOWY is a good example, and there are many common words with six letters: ALMOST, ABHORS, BEGINS, BIOPSY, CHIMPS, CHINTZ.

The longest word I know with letters in reverse alphabetical order is the hyphenated nine-letter word SPOON-FEED. TROLLIED is a good example with eight letters, and among the many seven-letter words are WRONGED, SPOOFED, SNIFFED, and SPOONED.

Among the names for numbers, only FORTY is in alphabetical order, and only ONE is in reverse order.

Hans Wright Bohlmann made the keyboard discovery. Of the above problems, thanks to Nicholas Temperley for the first one, Solomon Golomb for the second, David Silverman for the third, Edwin McMillan for the fourth, and me for the rest.

The Fortean calendar, adopted by the Fortean Society when Tiffany Thayer headed it, is based on a calendar proposed in 1923 by Moses Bruines Cotsworth, a British statistician, when the League of Nations called for calendar reform schemes. Cotsworth named his thirteenth month Sol, to honor the sun. Thirteen-month calendars are much older than Cotsworth's. One version was adopted by Auguste Comte's Church of Humanity, a cult that he founded in France in the nineteenth century.

After this puzzle appeared in *Isaac Asimov's Science Fic-*

tion Magazine, several readers reminded me of an old method of using the knuckles to determine the number of days in each month. Close your hands into fists, then put the fists together thumb to thumb. Mentally label the knuckles and the spaces between them, from left to right, with the names of the months in order. Thus the knuckle of your left pinkie is January, the adjacent space is February, the next knuckle is March, and so on to July for the knuckle of the left forefinger. The knuckle of the right forefinger becomes August, and December falls on the right ring-finger knuckle. All knuckle months have 31 days. The months that label the spaces have less than 31.

19

HOME SWEET HOME

1. Did you guess 18 to 9? Wrong! It's 16 to 9. The winning home team would not come to bat in the final inning.

2. Six

3. No, it wasn't a team of girls. All the men on the Mudville team who scored were married.

20

FINGERS AND COLORS ON CHROMO

You were told that *if* you knew the total number of fingers, you would know the number of persons, and the number of fingers on each. This could be the case if, and only if, the number of fingers has only one divisor other than 1 and itself. Now the only way a number can have this property is for it to be the square of a prime.

You can see this by considering some random numbers. Take 24 for instance. If there were 24 fingers in the room, there could be six persons with four fingers each, or four persons with six fingers each, or three with eight fingers, or eight with three fingers, or two with twelve, or twelve with two. Knowing the total number of fingers would not, therefore, provide a unique solution. But if there are 25 fingers altogether, there can only be five persons with five fingers.

We were told that the total number of fingers is more than 200 and less than 300. Therefore, to solve the problem we must find a number between 200 and 300 that is the square of a prime. There is only one such number: 289, the square of 17. Consequently, the room contains 17 Chromos, each with 17 fingers.

The first two problems of our puzzle tale are variations on the first two "chestnuts" in Raymond Smullyan's puzzle book, *The Lady or the Tiger?* I cannot recommend this collection too highly to anyone who enjoys brilliant logic problems, and who wants to learn more about the revolutionary discoveries of Kurt Gödel. Tourmaline and Coralie are from the country of the Pinks in L. Frank Baum's *Sky Island*.

21

VALLEY OF THE APES

Gorilla my dreams, I love you.
Honest I do. . . .

The first puzzle is based on a problem I found in a recent book, *The Universe Within,* by Morton Hunt. The second problem derives from a puzzle by Michael Steuben. Lulu, who is especially fond of awful puns, sent me the third.

Fanzine Patterfanny is a play on Dr. Francine Patterson, an ape researcher who claims that Koko, her talking gorilla, likes to make up puns.

22

DR. MOREAU'S MOMEATERS

Incredible as it may seem, the number of fins left in the tank is independent of the number of males!

Let x be the number of male fish, and $(4{,}996 - x)$ the number of females.

After 2/3 of the males are removed, the number of male fins left will be $5x$. To get the total number of fins now in the tank, both male and female, we add $5x$ to $5(4{,}996 - x)$:

$$5x + 5(4{,}996 - x)$$
$$5x + 24{,}980 - 5x$$

The x terms cancel out, leaving a total of 24,980 fins in the tank.

You could have guessed this, if you had the right insight, without doing any algebra at all! How?

23

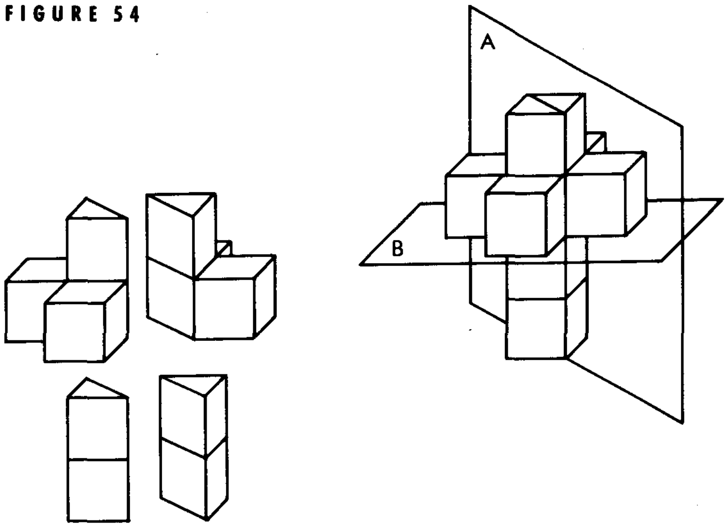
AND HE BUILT ANOTHER CROOKED HOUSE

The answer is no. Consider any edge of the interior cube. Four different faces touch along that edge; therefore four colors are required.

Maybe you can think of some other good puzzles based on the shape of Teal's house. If so, send them to me in care of the publisher. And if you never read Heinlein's 1940 story "—And He Built a Crooked House," you'll find it in Clifton Fadiman's *Fantasia Mathematica* and in many other anthologies.

After this puzzle appeared, I received an interesting letter from Bob Ulbrich of Philadelphia. In the first problem that is answered in the second answer section, I had asked for the minimum number of cuts. If I had asked instead for the minimum number of *pieces,* my solution would have been correct, but as I worded the problem, it can be solved in two cuts if they are made simultaneously. Figure 54 shows how cuts *A* and *B* produce four pieces which go together neatly to make a 2 × 2 × 2 cube.

FIGURE 54



25
MONORAILS ON MARS

One obvious answer is that the square can be anywhere straddling Mars's equator, with base *A* 50 miles north of the equator. But this is not the only answer. There are an infinite number of other spots!

For example, *A* could be so close to the North Pole that when you go due west from *B* to *A*, the monorail circles completely around the pole as shown in Figure 55. Of course *A* could be even closer to the pole, so that it takes two trips to go due west around the pole from *B* to *A,* or three trips, or four, and so on.

The same scheme applies to locations of the square near the South Pole. As shown in Figure 56, *A* could be so close to the pole that if you travel due east from *C* to *D* you circle the pole once, or twice, or three times, and so on.

To summarize: there is an infinite set of solutions on the equator, an infinite set near the North Pole, and an infinite set near the South Pole. If you are skilled in geometry and algebra, you'll find it a challenging task to derive a formula that gives all the distances *A* can be from the North Pole, and a similar formula for all of *A*'s distances from the South Pole.

FIGURE 55

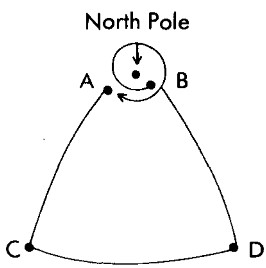
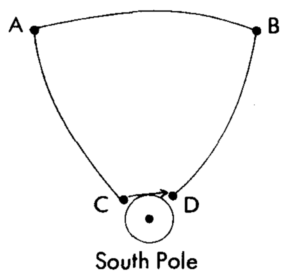


FIGURE 56



29

RUN, ROBOT, RUN!

The average speed for the two trips is *not* 12½ mph as most people guess. Again, let *x* be the distance around the reservoir. At 10 mph it takes Farfel $x/10$ hours to make one circuit. The two circuits take $x/10 + x/15$ hours, which reduces to $x/6$ hours. Farfel, therefore, runs $2x$ miles in $x/6$ hours. Speed equals distance divided by time, and time equals distance divided by speed. Therefore the average speed for Farfel, for the two circuits, is $2x$ divided by $x/6$, which comes to $12x/x$. The x's cancel, giving an answer of 12 mph.

One final question—an old chestnut that still amazes those who haven't heard it before. If Farfel completes one trip at 10 mph, how fast must he travel on a second trip to raise his average speed for the two trips to 20 mph?

33

FIBONACCI BAMBOO

The area of the Fibonacci triangle is zero. Any triangle with one side equal to the sum of the other two sides is a triangle that has degenerated into a straight line segment.

A little more thinking and you will see that no triangle of positive area can have sides that are distinct integers in any Fibonacci sequence even if they are not consecutive.

If you want to know more about Fibonacci sequences and how they are related to biological growth, games, and other curious matters, see the chapter on them in my 1979 book, *Mathematical Circus*. There is a bibliography of selected references for further reading. Beatrice Mince is my old friend Dr. Beatrice Mintz, a distinguished geneticist at the Institute for Cancer Research in Philadelphia.

34

TETHERED PURPLE-PEBBLE EATERS

Did you realize that it is impossible to answer the question without being told what unit of length is used for measuring the cube? Call the unit x. The volume of the cube is x^3. The surface area is clearly $6x^2$. We are told that the two values are equal, so we write the equation $6x^2 = x^3$, and solve for *x*. The answer is 6. We know the cube has a side of six units, but this could be six inches, six feet, or six miles. The size of the cube depends on the choice of a measuring unit. A cube of *any* size has the property called for if we measure it with a unit that is one-sixth of the cube's side! Curiously, 6 is also the answer if we ask for the diameter of the sphere, in *x* units, whose surface area equals its volume.

Our final problem, also involving surface area, is independent of the measuring system. Suppose a cube is sliced by three planes to make eight identical small cubes, each with a side just half that of the original. The total volume of the eight cubes obviously remains the same, but their combined surface area increases. This is why smaller ice cubes melt much faster in a drink than their equivalent volume in larger cubes.

How quickly can you determine how much larger is the total surface area of the eight small cubes than the surface area of the original cube?

36

1984

You see in Figure 57, along the top and left sides of the Orwellian square, ten numbers that are called the "generators" of the magic square. Observe that each number inside a cell is the product of the generator directly above it and the generator on the left.

If you multiply the ten generators, you'll find that the product is 1984. The procedure of circling cells and crossing out numbers guarantees that no two of the five circled numbers will share a row or column. Because each selected number is the product of a different pair of generators, the five selected numbers must have a product equal to the product of the ten generators, namely 1984.

If you would like to know more about how to construct magic squares of this sort—they can be based on addition as well as multiplication—see the second chapter of my *Scientific American Book of Mathematical Puzzles & Diversions.*

FIGURE 57

	1	2	1.6	4	2.5
4	4	8	6.4	16	10
2.5	2.5	5	4	10	6.25
2	2	4	3.2	8	5
1	1	2	1.6	4	2.5
3.1	3.1	6.2	4.96	12.4	7.75

FOURTH ANSWERS

8
ANTIMAGIC AT THE NUMBER WALL

The city is Buffalo.

22
DR. MOREAU'S MOMEATERS

You were told that the problem could be solved without knowing the number of male fish. Therefore the answer must be the same regardless of how many males. So let the number of males be 0. The tank will then contain 4,996 females. Since each female has five fins, the total number of fins will be 5 times 4,996, or 24,980 fins.

After I wrote this puzzle tale I learned, to my surprise, that the offspring of certain insect species do literally devour their mothers. You'll find the shocking details in "Organic Wisdom, or Why Should a Fly Eat Its Mother from Inside," in Stephen Jay Gould's splendid collection of essays *Ever Since Darwin.*

29
RUN, ROBOT, RUN!

We won't go through the algebra this time, but if you try any value for x (the distance around the reservoir), you'll find that Farfel would have to make the second trip in *no time at all!* Regardless of the distance, or the speed on the first circuit, in order to raise the average speed for two circuits to twice the first speed, the dog would have to complete the second trip with infinite speed.

34

TETHERED PURPLE-PEBBLE EATERS

No algebra is needed. Just reflect on the fact that each cut exposes two new surfaces, each a square of the same size as one of the cube's faces. Because there are three such cuts, six new square surfaces are exposed. The total surface area, therefore, jumps from six squares to twelve squares, an increase of 100 percent. In other words, the surface area precisely doubles.

An even simpler insight was sent to me by Sander Eller. Each of the eight small cubes has three sides facing out (forming the surface area of the original cube), and three turned inward. Therefore, the total surface area has doubled.

Problems concerning tethered animals (usually goats or cows) and their grazing areas are common in puzzle literature. They range from simple ones, like the two given here, to difficult questions about animals tethered to the inside or outside of circles, ellipses, and enclosures of other shapes. Consider our second problem. Had the chain been of length 40 instead of 20, the calculation of the grazing area would at once have become more difficult, as you will see if you attempt it.

The difficulty arises from the fact that the grazing areas in the two directions, clockwise and counterclockwise around the square, overlap outside the corner opposite to the one where the animal is tethered. It then becomes necessary to measure the area of the overlap so it won't be counted twice. In the example just given, the total grazing area is (I think) about 30,290 square feet.

For a good discussion of two tethering problems involving a circular enclosure, see "A Tale of Two Goats" by Marshall Fraser, in *Mathematics Magazine,* Volume 55, September 1982, pages 221–27. Additions to this paper's bibliography (including a goat-grazing problem in *The Ladies Diary* for 1748) are given by Fraser in a letter in the March 1983 issue of the same journal, page 123.

ABOUT THE AUTHOR

Martin Gardner has written prolifically on science, philosophy and mathematics, as well os numerous game books, works of literary criticism, and one novel. His most recent book is *The Whys of a Philosophical Scrivener.* He also writes reviews for the *New York Times Book Review* and the *New York Review of Books,* and was for many years a regular contributor to *Scientific American.* He currently has a column in *Isaac Asimov's Science Fiction Magazine.* He lives in Hendersonville, North Carolina.

PUZZLES

Each of these thirty-seven science-fiction puzzle tales, including *The Polybugs of Titan, Piggy's Glasses and the Moon, The Jinn of Hyperspace,* and *Tethered Purple-Pebble Eaters*, contains a challenging puzzle involving logic, wordplay, palindromes, geometry, probability or magic numbers. Written by a master puzzle maker, this is an irresistible book for both science fiction and puzzle fans.