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The number of elementary particles in a fractal M-theory of 11.2360667977 dimensions

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Abstract

It is generally accepted that there are 60 experimentally found particles. The standard model strongly predicts two more hypothetical particles, the Higgs and the graviton. This paper reveals other possible scenario for predicting 69 particles at different energy scales in $11 + \phi^3$ fractal dimensions of a fractal M theory, where $\phi = (\sqrt{5} - 1)/2$. A modified Newton's law is suggested to experimentally verify our predictions at extremely small quantum scales. The modified Newton's law is in harmony with Heisenberg's uncertainty principle.

1. Introduction

As pointed out for instance by El Naschie the standard model of elementary particles could not stand as a consistent theory without the addition of two more hypothetical particles to the 60 experimentally confirmed degrees of freedom [1]. These two particles are thought to be a massive spin(0) boson named Higgs boson and another massless spin(2) particle to mediate gravity at the quantum level, termed graviton. Only when calculating the number of particles using a string-like theory, then gravity is implicitly included. For instance we know that the super Yang–Mills spectrum is found from a multiplicative operation to be $(\mathrm{Dim}\, E_8 \otimes E_8)(8) = (496)(8) = 3968$ particle-like states. This must include a graviton because it is related to Heterotic super string theory and the exceptional Lie group $E_8 \otimes E_8$. Therefore in the sub Yang–Mills theory which is equivalent to the standard model, we use the dual operation namely division leading to $(\mathrm{Dim} E_8 \otimes E_8)/8 = 62$. This may be interpreted as the 60 experimentally found particles plus 1 Higgs plus 1 graviton. The graviton is of course outside the scope of the standard model but is essential for a consistent picture of particle physics. However there are other possible scenarios for predicting 66 or 69 particles at different energy scales. These and various other points are discussed in [1–24]. In the present short paper, based on a fractal 11-dimensional spacetime theory, an alternative derivation to the 69 prediction will be given.

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2. *P*-Brane in 11 dimensions

Following the theory of *P*-Branes [1,2] in 11 dimensions, it is easily shown that the number of particles-like states is given by the following simple formula [2]:

$$N_0 = P_\mu + Z^{(2)} + Z^{(5)} = \binom{11}{1} + \binom{11}{2} + \binom{11}{5} = 11 + 55 + 462 = 528, \tag{1}$$

where the contribution of the strings and membrane part is obviously

$$N(\mathrm{SM}) = 11 + 55 = 66. \tag{2}$$

This is consistent with the result of the three steps symmetry breaking of *E*-infinity theory which leads to

$$N(\mathrm{SM}) = \frac{N_0}{D^{(8)}} = \frac{528}{8} = 66. \tag{3}$$

In turn this value may be interpreted as one graviton, and five Higgs particles, namely three neutrals H_1^0, H_2^0 and H_3^0, and two charged particles H^+ and H^-, which together with the 60 experimentally found particles make exactly 66 particles.

3. $E^{(\infty)}$ theory in 11 + ϕ^3 dimensions

In our personal opinion, *E*-infinity theory may be considered to be possibly an epoch-making theory following, in a natural way, the work of Albert Einstein. The main application of *E*-infinity theory shows miraculous exactness compared to experimental measurements, especially in determining the number of the elementary particles and their mass spectrum. We can also find widely open possibilities for applying *E*-infinity theory in biology [25,26], ecology [27], turbulence [28], as well as cosmology [29,30]. We should also point out a recent and quite unexpected realization that *E*-infinity theory could be of a considerable help in deciphering one of the greatest secrets and impenetrable questions of our own existence, namely what is consciousness and how does it relate to the brain [25,31]. In fact the present author will discuss the possible application of *E*-infinity to nanoscience, angstrom-science (A-science for short) and economics in a forthcoming publication entitled "*Scaling is everything, and E-infinity theory is everywhere".*

Noting that $D^{(11)} = 11$ may be written as

$$D^{(11)} = D^{(7)} \oplus D^{(4)} = 7 + 4, \tag{4}$$

where $D^{(7)}$ may be viewed as the compactified manifold of *M* theory, then we could imagine that the 11-dimensional spacetime could be compactified on a small seven-dimensional sphere, leaving 4 spacetime dimensions visible to observer at large distances equivalent to lower energy scale.

In the standard model, as well as string theory, the dimensions are limited to integers. This restriction imposed on string theory is anything but advantageous. By contrast in $E^{(\infty)}$ theory El Naschie admits formally hierarchical infinite-dimensional Cantorian space-time. However the expectation value of the Hausdorff dimension of this Cantorian manifold is finite and resemble the four-dimensional spacetime of our sensual "real" world

$$4 + \phi^3 = 4 + \frac{1}{4 + \frac{1}{4 + \cdots}} = 4.2360667977, \tag{5}$$

where $\phi = (\sqrt{5} - 1)/2$.

The fractal interpretation of Eq. (5) is that our "real" world mimics the appearance of a four-dimensional space-time manifold.

Replacing $D^{(4)}$ by $E^{(4)}$ in Eq. (4), we have the expectation value of the Hausdorff dimension in 11-Branes

$$D^{(11)} = D^{(7)} \oplus E^{(4)} = 7 + 4 + \phi^3 = 11.2360667977. \tag{6}$$

The 11.2360667977-dimensional El Naschie spacetime can now be compactified on a small seven-dimensional sphere, leaving $4 + \phi^3$ spacetime dimensions mimicking the 4 "real" dimensions visible to observers at large distances.

In view of $E^{(11+\phi^3)}$, using fractal combination, Eq. (2) should be rewritten in the form

$$N(\mathrm{SM}) \approx (11 + \phi^3) + (11 + \phi^3)(10 + \phi^3)/2 = 68.742657 \approx 69. \tag{7}$$

These are the 69 particles predicted by El Naschie [11,13,23,24].

This result reminds us of an old paper of the great American mathematical physicist Dyson's which showed that quantum electrodynamics changes its character for systems with more than 137 particle–antiparticle pairs [32], while we have here

$$N(\mathrm{SM}) \times 2 = 68.74 \times 2 = 137.48 \approx \bar{\alpha}_0 = 137 + \phi^5(1-\phi^5) = 137.082039325. \tag{8}$$

Here $\bar{\alpha}_0 = 137.082039325$ is a somewhat mysterious number, for example [5], the absolute zero temperature T_0 can be calculated as $T_0 = 2\bar{\alpha}_0 - 1 = 273.1640788$ K, which is in astonishing agreement with the widely used number 273.16 K. This could be explained as follows: temperature is a scale for thermodynamics, and the absolute zero temperature is the smallest scale in thermodynamics like quantum scale in high energy particle physics. In addition different scales in temperature lead to different laws in thermodynamics and plasma. That is why E-infinity is an ubiquitous theory for all scales. We shall discuss this topic in a forthcoming paper.

Another intriguing and enduring problem related to both $\bar{\alpha}_0$ and 11.2360667977 fractal dimensions is given by the following remarkable equation [33]:

$$d^{(11+\phi^3)} = (1/\phi)^{11+\phi^3-1} = (1/\phi)^{10+\phi^3} = 137.7880937 \approx \bar{\alpha}_0.$$

Now let us rewrite Eq. (11) in the following form using $D = 11 + \phi^3$:

$$N_0 \approx \binom{11+\phi^3}{1} + \binom{11++\phi^3}{2} + \binom{11++\phi^3}{5}. \tag{9}$$

Using fractal combination, we have

$$\begin{aligned} N_0 &\approx (11+\phi^3) + \frac{(11+\phi^3)(10+\phi^3)}{(1)(2)} + \frac{(11+\phi^3)(10+\phi^3)(9+\phi^3)(8+\phi^3)(7+\phi^3)}{(1)(2)(3)(4)(5)} \\ &\approx 11.2360668 + 57.506578 + 527.56497 \approx 596.30762. \end{aligned}$$

This indicates that in $E^{(11+\phi^3)}$ theory we should expect the existence of 596 massless gauge bosons.

For 10 dimensions, we have $(10)(\phi^2) = 4.18033989 = 4 + k$, $D^{(8)} \Rightarrow (4-k)(2) = 8 - 2k$, thus for $D = 11 + \phi^3$, we have $(11+\phi^3)(\phi^2) = 4.291796068$, $D^{(8)} \Rightarrow 8.58359213$, and consequently by analogy to Eq. (3) we find

$$N(\mathrm{SM}) = \frac{N_0}{D^{(8)}} = \frac{596.30762}{8.58359213} = 69.47063776 \approx 69 \text{ particles}. \tag{10}$$

4. Possible experimental verification of the extra dimensions of *E*-infinity spacetime

A short description of possible experiments to verify E-infinity predictions was given by El Naschie in Ref. [20]. Hereby we provide an alternative way to verify our predictions.

First we replace Newton's law

$$F = Gm_1m_2r^{-2} \tag{11}$$

by a more general form for all forces including the electromagnetic force, the weak force, the strong force, and gravitational force [34]

$$F^{(D)} = G^{(D)}m_1m_2r^{2-D}, \tag{12}$$

where $F^{(D)}$ and $G^{(D)}$ are, respectively, the force and the gravitational constant in D-dimensional spacetime. In case of $D = 4$, Eq. (12) reduces to the classical case of Eq. (11).

For large scales, on the other hand, which could be visible to observers using for instance nanotechnology, we have $D = 4 + \phi^3$. Thus the following modified Newton's law becomes feasible:

$$F^{(4+\phi^3)} = G^{(4+\phi^3)}m_1m_2r^{2-(4+\phi^3)} = G^{(4+\phi^3)}m_1m_2r^{-(2+\phi^3)}. \tag{13}$$

We point out that the direction of the attraction of the Sun for the Earth is not always toward the center of the Sun, and this is why the Earth moves in an elliptic circle as illustrated in Fig. 1. To explain this, we rewrite Eq. (12) in a vector form

$$\mathbf{F}^{(D)} = G^{(D)}m_1m_2\nabla\Phi, \tag{14}$$

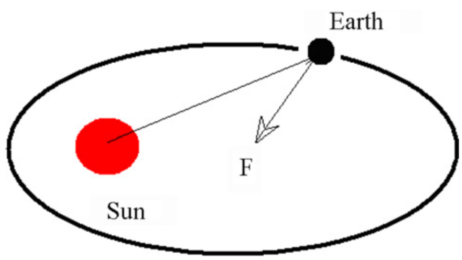

Fig. 1. The direction of the attraction of the Sun exerted on the Earth.

where Φ is the potential. For four-dimensional spacetime $\Phi = -1/r$, $\nabla\Phi = (1/r^2)\mathbf{i}_r$, and $\mathbf{i}_r$ is the unit vector from, for example, the Earth to the Sun. Similarly, for fractal-dimensional spacetime, the direction of $\nabla\Phi$ does not always coincide with $\mathbf{i}_r$, an effect which is relatively easier to detect than extra dimensions. We predict that fractal geometry, the golden mean and KAM theorem will play an important role in astronomy and cosmology as in E-infinity theory high energy particle physics.

Let us consider another important point regarding a string world sheet or equivalently $D = d_{\mathrm{H}} = 2$, where $d_{\mathrm{H}} = 2$ is the Hausdorff dimension of the path of a quantum particle, we have thus [34]

$$F^{(2)} = G^{(2)} m_1 m_2 r^{2-d_{\mathrm{H}}} = G^{(2)} m_1 m_2. \tag{15}$$

In this hypothetical case the force does not depend upon the distance. This is reminiscence of the spooky quantum mechanical action at distance and nonlocality. This can be interpreted as a consequence of Heisenberg's uncertainty principle, which implies that $d_{\mathrm{H}} = 2$. In other words, a quantum path caused by the force given by Eq. (15) is not line-like or circle-like paths as in classical mechanics but an area-like path similar to a string world sheet. It then turned out that the only Hausdorff dimension for these quantum paths which is compatible with Heisenberg's uncertainty principle is $d_{\mathrm{H}} = 2$. In view of E-infinity theory, a more accurate statement than $d_{\mathrm{H}} = 2$ is that the expectation value of Hausdorff dimension of a quantum path is a membrane-like and is given by

$$\langle d_{\mathrm{H}} \rangle = \langle 1/d_{\mathrm{c}}^{(0)} \rangle = 2. \tag{16}$$

This value was calculated using purely mathematical and very general geometrical and dimensional considerations without resorting to any quantum mechanical postulates [20].

Before proceeding any further, let us give an entropy interpretation of $D = 4 + \phi^3$ of E-infinity. It is well known that topological entropy is given by [35]

$$P = \ln(1/\phi). \tag{17}$$

Thus the topological entropic content of a quantum path is given by

$$\frac{\langle d_{\mathrm{H}} \rangle}{P} = \frac{2}{\ln(1/\phi)} = 4.156173841. \tag{18}$$

The linear part of $2/\ln(1/\phi)$ is given by the expression

$$\frac{2}{\ln(1/x)} \approx \frac{1+x}{1-x}. \tag{19}$$

Thus

$$\frac{2}{\ln(1/\phi)} \to \frac{1+\phi}{1-\phi} = 4 + \phi^3. \tag{20}$$

That means the topological entropic content of a quantum path is given by the expectation value of the Hausdorff dimension of E-infinity $\sim\langle n \rangle = 4 + \phi^3$.

Considering now the case $D = 1$ for 1-Brane or one-dimensional string, we have

$$F^{(1)} = G^{(1)} m_1 m_2 r^{2-1} = G^{(1)} m_1 m_2 r. \tag{21}$$

If the energy density in the electromagnetic field was distributed along a line in spacetime, this one-dimensional line would be considered a P-Brane with $P = 1$. In such case Eq. (21) is similar to a linear spring.

Finally for $E^{(11+\phi^3)}$, we have

$$F^{(11+\phi^3)} = G^{(11+\phi^3)} m_1 m_2 r^{2-(11+\phi^3)} = G^{(11+\phi^3)} m_1 m_2 r^{-(9+\phi^3)}. \tag{22}$$

Here r is at extremely small quantum scale, the force F is, therefore, extremely strong. To make such an experiment at such a high observational resolution may be quite difficult at present time [36,37].

5. Conclusions

El Naschie's discovery of *E*-infinity theory is in our opinion a landmark achievement for a new century that will eventually change the way we do high energy physics.

We conclude that scaling is of utter importance in physics as well as sciences in general. Different scalings lead to different laws and thus results. This is the main power of *E*-infinity theory.

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References

[1] El Naschie MS. On 336 kissing spheres in 10 dimensions, 528 *P*-Brane states in 11 dimensions and the 60 elementary particles of the standard model. Chaos, Solitons & Fractals 2005;24(2):447–57.
[2] El Naschie MS. A *P*-Brane vindication of the two Higgs-doublet minimally super-symmetric standard model and related issues. Chaos, Solitons & Fractals 2005;23(5):1511–4.
[3] El Naschie MS. On Einstein's super symmetric tensor and the number of elementary particles of the standard model. Chaos, Solitons & Fractals 2005;23(5):1521–5.
[4] El Naschie MS. Light cone quantization, Heterotic strings and *E*-infinity derivation of the number of Higgs bosons. Chaos, Solitons & Fractals 2005;23(5):1931–3.
[5] El Naschie MS. The unreasonable effectiveness of the electron-volt units system in high energy physics and the role played by $\bar{\alpha}_0 = 137$. Int J Nonlinear Sci Numer Simul 2006;7(2):119–28.
[6] El Naschie MS. Thomas Mann and Heinrich Mann, dual brothers and complimentary genius embraced by complex reality. Int J Nonlinear Sci Numer Simul 2006;7(1):1–6.
[7] El Naschie MS. Linderhof Room of Mirrors, Thurston three-manifolds and the geometry of our universe. Int J Nonlinear Sci Numer Simul 2006;7(1):97–100.
[8] El Naschie MS. The missing particles of the standard model via a unified particle-field framework. Int J Nonlinear Sci Numer Simul 2006;7(1):101–4.
[9] El Naschie MS. The Feynman path integral and *e*-infinity from the two-slit Gedanken experiment. Int J Nonlinear Sci Numer Simul 2005;6(4):335–42.
[10] El Naschie MS. Einstein in a complex time – some very personal thoughts about *E*-infinity theory and modern physics. Int J Nonlinear Sci Numer Simul 2005;6(3):331–3.
[11] El Naschie MS. On a fuzzy Kahler-like manifold which is consistent with the two slit experiment. Int J Nonlinear Sci Numer Simul 2005;6(2):95–8.
[12] El Naschie MS. Superstrings, entropy and the elementary particles content of the standard model. Chaos, Solitons & Fractals 2006;29(1):48–54.
[13] El Naschie MS. An elementary proof for the nine missing particles of the standard model. Chaos, Solitons & Fractals 2006;28(5):1136–8.
[14] El Naschie MS. Elementary number theory in superstrings, loop quantum mechanics, twistors and *E*-infinity high energy physics. Chaos, Solitons & Fractals 2006;27(2):297–330.
[15] El Naschie MS. Hilbert space, the number of Higgs particles and the quantum two-slit experiment. Chaos, Solitons & Fractals 2006;27(1):9–13.
[16] El Naschie MS. An elementary model based method for determining the number of possible Higgs bosons in the standard model. Chaos, Solitons & Fractals 2005;26(3):701–6.
[17] El Naschie MS. From the two-slit experiments to the expected number of Higgs particles in the standard model. Chaos, Solitons & Fractals 2005;26(1):67–70.
[18] El Naschie MS. Tadpoles, anomaly cancellation and the expectation value of the number of the Higgs particles in the standard model. Chaos, Solitons & Fractals 2005;24(3):659–63.

[19] El Naschie MS. Higgs number from anomaly cancellation and super Riemann tensor. Chaos, Solitons & Fractals 2005;24(2):653–7.
[20] El Naschie MS. Supergravity and the number of fundamental particles in the standard model. Chaos, Solitons & Fractals 2005;23(5):1941–3.
[21] El Naschie MS. The fractal dimension of spacetime-Remaks on theoretical derivation and experimental verifications. Chaos, Solitons & Fractals 1998;9(7):1211–17.
[22] Goldfain E. Higgs-free derivation of gauge boson masses using complex dynamics of Levy flows. Int J Nonlinear Sci Numer Simul 2005;6(4):351–6.
[23] He JH. In search of 9 hidden particles. Int J Nonlinear Sci Numer Simul 2005;6(2):93–4.
[24] He JH. Space, time and beyond. Int J Nonlinear Sci Numer Simul 2005;6(4):343–6.
[25] He JH, Huang ZD. A novel model for allometric scaling laws for different organs. Chaos, Solitons & Fractals 2006;27(4):1108–14.
[26] He JH. Application of *E*-infinity theory to biology. Chaos, Solitons & Fractals 2006;28(2):285–9.
[27] He JH, Shrinkage of body size of small insects: a possible link to global warming? Chaos, Solitons & Fractals, in press, doi:10.1016/j.chaos.2006.04.052.
[28] He JH. Application of *E*-infinity theory to turbulence. Chaos, Solitons & Fractals 2006;30(2):506–11.
[29] El Naschie MS. The brain and *E*-Infinity. Int J Nonlinear Sci Numer Simul 2006;7(2):129–32.
[30] Iovane G. El Naschie *E*-infinity cantorian spacetime and lengths scales in cosmology. Int J Nonlinear Sci Numer Simul 2006;7(2):155–62.
[31] Iovane G, Benedetto E. El Naschie *E*-infinity cantorian space-time, Fantappie's group and applications in cosmology. Int J Nonlinear Sci Numer Simul 2005;6(4):357–70.
[32] Dyson FJ. Divergence of perturbation theory in quantum electrodynamics. Phys Rev 1952;85:631–2.
[33] El Naschie MS. The fine structure constant, vacuum polarization and the geometry of $E(\infty)$ space. Chaos, Solitons & Fractals 1999;10(11):1947–54.
[34] He JH. *E*-infinity theory and the Higgs field. Chaos, Solitons & Fractals, in press, doi:10.1016/j.chaos.2006.04.041.
[35] Hao BL, Zheng WM. Applied symbolic dynamics and chaos. Singapore: World Scientific; 1998. see in particular page 83 and Eq. (2.117).
[36] Brumfiel G. In search of hidden dimensions. Nature 2005;433(7021):10.
[37] Bijnens J, Maul M. Large extra dimensions in rare decays. J High Energy Phys 2000;10. Art. No. 003.